



NEW JERSEY
Core Curriculum Content Standards for Mathematics
Grade 12
Advanced Mathematical Concepts
Precalculus with Applications © 2004

CONTENT STANDARDS	PAGE REFERENCES
Standard 4.1 (Number and Numerical Operations) All students will develop number sense and will perform standard numerical operations and estimations on all types of numbers in a variety of ways.	
Building upon knowledge and skills gained in preceding grades, by the end of Grade 12 , students will:	
A. Number Sense	
1. Extend understanding of the number system to all real numbers.	SE: 8, 206 <i>Graphing Calculator Exploration 86</i> <i>Exercises 210-211#32-#47</i> TWE: TT 206, 208 PRS 206 AIN 208 PA 209 MTL 580 NOTE: Study of imaginary numbers, by contrast, helps to further students' understanding of real numbers.
2. Compare and order rational and irrational numbers.	SE: 206, 229-233, 712-713 <i>Real World Application 229</i> <i>Example 232-233</i> <i>Check for Understanding 714</i> TWE: PA 233 EC 235 MTL 580 TT 713
3. Develop conjectures and informal proofs of properties of number systems and sets of numbers.	SE: <i>Graphing Calculator Exploration 86</i>
B. Numerical Operations	
1. Extend understanding and use of operations to real numbers and algebraic procedures.	SE: <i>Graphing Calculator Exploration 86</i>
2. Develop, apply, and explain methods for solving problems involving rational and negative exponents.	SE: 695-703, 716 #18, 732 #67, 750 #19-#20, 814 #46 TWE: GCE 696
3. Perform operations on matrices. <ul style="list-style-type: none"> • Addition and subtraction • Scalar multiplication 	SE: 78-86, 88-96, 98-105, 120 #20-#27, 179 #46, 250 #54 TWE: A 86, 105
4. Understand and apply the laws of exponents to simplify expressions involving numbers raised to powers.	SE: 695-703, 716 #18, 732 #67, 750 #19-#20, 814 #46 TWE: GCE 696

CONTENT STANDARDS	PAGE REFERENCES
C. Estimation	
1. Recognize the limitations of estimation, assess the amount of error resulting from estimation, and determine whether the error is within acceptable tolerance limits.	SE: 272 ex 2, 744 #6c, 745 #14c, 747 #21d, 974 #45
Standard 4.2 (Geometry and Measurement) All students will develop spatial sense and the ability to use geometric properties, relationships, and measurement to model, describe and analyze phenomena.	
A. Geometric Properties	
1. Use geometric models to represent real-world situations and objects and to solve problems using those models (e.g., use Pythagorean theorem to decide whether an object can fit through a doorway).	SE: 168 #46, 178 #35, 187 #44, 192-193 ex 5, 226 #13, 229, 318 #42, 358 #51, 510 #38, 701 #68
2. Draw perspective views of 3D objects on isometric dot paper, given 2D representations (e.g., nets or projective views).	This objective can be found in Glencoe's <i>Geometry</i> © 2004 pages 636-642 and 643-648.
3. Apply the properties of geometric shapes. <ul style="list-style-type: none"> • Parallel lines - transversal, alternate interior angles, corresponding angles • Triangles <ul style="list-style-type: none"> a. Conditions for congruence b. Segment joining midpoints of two sides is parallel to and half the length of the third side c. Triangle inequality • Minimal conditions for a shape to be a special quadrilateral • Circles - arcs, central and inscribed angles, chords, tangents • Self-similarity 	SE: 36 #11, 273 #3, 483 #3, 613 #3, 617 ex 3, 620 #23-#27, 652 #3, 887 #2 TWE: EC 621 TT 617
4. Use reasoning and some form of proof to verify or refute conjectures and theorems. <ul style="list-style-type: none"> • Verification or refutation of proposed proofs • Simple proofs involving congruent triangles • Counterexamples to incorrect conjectures 	SE: 429 #58, 436 #45, 764 #54
B. Transforming Shapes	
1. Determine, describe, and draw the effect of a transformation, or a sequence of transformations, on a geometric or algebraic object, and, conversely, determine whether and how one object can be transformed to another by a transformation or a sequence of transformations.	SE: 88-96, 145 #46, 535-542

CONTENT STANDARDS	PAGE REFERENCES
2. Recognize three-dimensional figures obtained through transformations of two-dimensional figures (e.g., cone as rotating an isosceles triangle about an altitude), using software as an aid to visualization.	SE: 88-92, 535-539 <i>Real World Application 535</i> <i>Example 536</i> TWE: MTL 88 TT 89, 90 ICE 90, 91 PA 92 NOTE: Computer animation obtained through matrix manipulation and translation is an application of two-dimensional transformation into three-dimensional objects.
3. Determine whether two or more given shapes can be used to generate a tessellation.	This objective can be found in Glencoe's <i>Geometry</i> © 2004 page 483-488.
4. Generate and analyze iterative geometric patterns. <ul style="list-style-type: none"> • Fractals (e.g., Sierpinski's Triangle) • Patterns in areas and perimeters of self-similar figures • Outcome of extending iterative process indefinitely 	SE: 815-821 TWE: AIN 818 EC 821
C. Coordinate Geometry	
1. Use coordinate geometry to represent and verify properties of lines. <ul style="list-style-type: none"> • Distance between two points • Midpoint and slope of a line segment • Finding the intersection of two lines • Lines with the same slope are parallel • Lines that are perpendicular have slopes whose product is -1 	SE: 135 #40, 272-273, 617 ex 3, 618 ex 5, 619 #4, 620 #8-#9, 621 #33, 640 #55, 652 #51, 688 #11-#13
2. Show position and represent motion in the coordinate plane using vectors. <ul style="list-style-type: none"> • Addition and subtraction of vectors 	SE: 487 ex 3, 490 #13, 491 #40, 492 #41, 495 ex 3, 497 #13, 498 #45, 503 #11, 504 #41-#42, 508 ex 4
D. Units of Measurement	
1. Understand and use the concept of significant digits.	This objective can be found in Glencoe's <i>Geometry</i> © 2004 page 14.
2. Choose appropriate tools and techniques to achieve the specified degree of precision and error needed in a situation. <ul style="list-style-type: none"> • Degree of accuracy of a given measurement tool • Finding the interval in which a computed measure (e.g., area or volume) lies, given the degree of precision of linear measurements 	Standard error of the mean can be found on pages 927-928.

CONTENT STANDARDS	PAGE REFERENCES
E. Measuring Geometric Objects	
1. Use techniques of indirect measurement to represent and solve problems. <ul style="list-style-type: none"> • Similar triangles • Pythagorean theorem • Right triangle trigonometry (sine, cosine, tangent) 	SE: 299 ex 2, 301 ex 4, 302 #9, 303 #23, 304 #29, 317 #31, 326 #41, 332 #34, 936 #1-#2, 961-968
2. Use a variety of strategies to determine perimeter and area of plane figures and surface area and volume of 3D figures. <ul style="list-style-type: none"> • Approximation of area using grids of different sizes • Finding which shape has minimal (or maximal) area, perimeter, volume, or surface area under given conditions using graphing calculators, dynamic geometric software, and/or spreadsheets • Estimation of area, perimeter, volume, and surface area 	SE: 257 #51, 533 #29, 549 #6, 684 #50
Standard 4.3 (Patterns and Algebra) All students will represent and analyze relationships among variable quantities and solve problems involving patterns, functions, and algebraic concepts and processes.	
A. Patterns	
1. Use models and algebraic formulas to represent and analyze sequences and series. <ul style="list-style-type: none"> • Explicit formulas for n^{th} terms • Sums of finite arithmetic series • Sums of finite and infinite geometric series 	SE: 759-765, 766-773, 774-783, 793 #37-#38, 800 #53, 830 #11-#13 TWE: AIN 761, 779 EC 764, 773
2. Develop an informal notion of limit.	SE: 774-783, 786-793, 805 #39, 821 #38, 831 #22-#27 TWE: AIN 789 MTL 775 TT 778
3. Use inductive reasoning to form generalizations.	SE: 822-828 TWE: AIN 825 MTL 822
B. Functions and Relationships	
1. Understand relations and functions and select, convert flexibly among, and use various representations for them, including equations or inequalities, tables, and graphs.	SE: 20-25, 29 #4, 30 #27, 44 #19, 46 ex 1-ex 2, 49 #23, 50 #29, 59 #31-#38 TWE: EC 25

CONTENT STANDARDS	PAGE REFERENCES
<p>2. Analyze and explain the general properties and behavior of functions of one variable, using appropriate graphing technologies.</p> <ul style="list-style-type: none"> • Slope of a line or curve • Domain and range • Intercepts • Continuity • Maximum/minimum • Estimating roots of equations • Intersecting points as solutions of systems of equations • Rates of change 	<p>SE: 5-11, 171-179, 229-235, 236-242 TWE: AIN 232 EC 235 FTC 237 MTL 236 TT 230, 231</p>
<p>3. Understand and perform transformations on commonly-used functions.</p> <ul style="list-style-type: none"> • Translations, reflections, dilations • Effects on linear and quadratic graphs of parameter changes in equations • Using graphing calculators or computers for more complex functions 	<p>SE: 88-96, 137-145 TWE: A 96 EC 96 MTL 88</p>
<p>4. Understand and compare the properties of classes of functions, including exponential, polynomial, rational, and trigonometric functions.</p> <ul style="list-style-type: none"> • Linear vs. non-linear • Symmetry • Increasing/decreasing on an interval 	<p>SE: 88-96, 137-145 TWE: A 96 EC 96 MTL 88</p>
C. Modeling	
<p>1. Use functions to model real-world phenomena and solve problems that involve varying quantities.</p> <ul style="list-style-type: none"> • Linear, quadratic, exponential, periodic (sine and cosine), and step functions (e.g., price of mailing a first-class letter over the past 200 years) • Direct and inverse variation • Absolute value • Expressions, equations and inequalities • Same function can model variety of phenomena • Growth/decay and change in the natural world • Applications in mathematics, biology, and economics (including compound interest) 	<p>SE: 154 ex 4, 156 #14, 157 #42, 183 ex 3, 186 #13, 209 ex 5, 211 #51, 239 ex 4, 240 #11, 241 #33</p>
<p>2. Analyze and describe how a change in an independent variable leads to change in a dependent one.</p>	<p>SE: 137-145, 151 #46, 168 #41, 198 #19-#22, 709 #22-#23 TWE: AIN 141, 145 FTC 139</p>

CONTENT STANDARDS	PAGE REFERENCES
3. Convert recursive formulas to linear or exponential functions (e.g., Tower of Hanoi and doubling).	SE: 806-814, 821 #36, 832 #41-#44 TWE: EC 814 TT 808
D. Procedures	
1. Evaluate and simplify expressions. <ul style="list-style-type: none"> Add and subtract polynomials Multiply a polynomial by a monomial or binomial Divide a polynomial by a monomial 	SE: 14 ex 1, 17 #11-#14, 19 #39, 31 #33, 51 #34, 58 #18-#23, 86 #62, 202 ex 2, 203 #6, 581-582
2. Select and use appropriate methods to solve equations and inequalities. <ul style="list-style-type: none"> Linear equations - algebraically Quadratic equations - factoring (when the coefficient of x^2 is 1) and using the quadratic formula All types of equations using graphing, computer, and graphing calculator techniques 	SE: 67-72, 73-77, 86 #55, 96 #36, 104 #55, 107-111, 118 #28, 125 #6 TWE: EC 72, 76
3. Judge the meaning, utility, and reasonableness of the results of symbol manipulations, including those carried out by technology.	SE: <i>Graphing Calculator Exploration</i> 69, 86, 87, 133, 284, 333, 378, 433, 695, 705
Standard 4.4 (Data Analysis, Probability, and Discrete Mathematics) All students will develop an understanding of the concepts and techniques of data analysis, probability, and discrete mathematics, and will use them to model situations, solve problems, and analyze and draw appropriate inferences from data.	
A. Data Analysis	
1. Use surveys and sampling techniques to generate data and draw conclusions about large groups. <ul style="list-style-type: none"> Advantages/disadvantages of sample selection methods (e.g., convenience sampling, responses to survey, random sampling) 	SE: 84 #49 <i>Internet Project</i> 937
2. Evaluate the use of data in real-world contexts. <ul style="list-style-type: none"> Accuracy and reasonableness of conclusions drawn Bias in conclusions drawn (e.g., influence of how data is displayed) Statistical claims based on sampling 	SE: 927 <i>Graphing Calculator Exploration</i> 877
3. Design a statistical experiment, conduct the experiment, and interpret and communicate the outcome.	TWE: A 880 AIN 854, 877 FTC 876
4. Estimate or determine lines of best fit (or curves of best fit if appropriate) with technology, and use them to interpolate within the range of the data.	SE: 38-44, 51 #31, 60 #53, 61 #69, 145 #49, 151 #51, 258-264 TWE: AIN 40, 260 EC 44

CONTENT STANDARDS	PAGE REFERENCES
5. Analyze data using technology, and use statistical terminology to describe conclusions. <ul style="list-style-type: none"> • Measures of dispersion: variance, standard deviation, outliers • Correlation coefficient • Normal distribution (e.g., approximately 95% of the sample lies between two standard deviations on either side of the mean) 	SE: 911 ex 3, 912 ex 4, 913 ex 5, 918-925 <i>Graphing Calculator Exploration</i> 926 TWE: AIN 921
B. Probability	
1. Calculate the expected value of a probability-based game, given the probabilities and payoffs of the various outcomes, and determine whether the game is fair.	SE: 852-855, 859-863 <i>Exercises</i> 856 #13-#16, 865 #34-#37 <i>Example</i> 859 <i>Check for Understanding</i> 863 #4 TWE: MTL 852 ICE 853, 860 AIN 854
2. Use concepts and formulas of area to calculate geometric probabilities.	SE: 861, 862, 870 <i>Exercises</i> 857 #42, 866 #52, 873 #38 <i>Check for Understanding</i> 863 #2 TWE: EM 858 AA 863 #2, 866 #52, 873 #38 NOTE: Use of Venn diagrams is an application of use of area to calculate probability.
3. Model situations involving probability with simulations (using spinners, dice, calculators and computers) and theoretical models, and solve problems using these models.	SE: <i>Graphing Calculator Exploration</i> 877 TWE: A 880
4. Determine probabilities in complex situations. <ul style="list-style-type: none"> • Conditional events • Complementary events • Dependent and independent events 	SE: 837-845, 846-851, 852-858, 859-867, 868-874, 875-880, 883 #27-#34, 884 #37-#40, 886 ex 1 TWE: MTL 846
5. Estimate probabilities and make predictions based on experimental and theoretical probabilities.	SE: 837-845, 883 #35-#36 TWE: TT 838
6. Understand and use the “law of large numbers” (that experimental results tend to approach theoretical probabilities after a large number of trials).	SE: <i>Graphing Calculator Exploration</i> 877 TWE: AIN 854
C. Discrete Mathematics—Systematic Listing and Counting	
1. Calculate combinations with replacement (e.g., the number of possible ways of tossing a coin 5 times and getting 3 heads) and without replacement (e.g., number of possible delegations of 3 out of 23 students).	SE: 837-845 TWE: AIN 841 FTC 838 MTL 837

CONTENT STANDARDS	PAGE REFERENCES
2. Apply the multiplication rule of counting in complex situations, recognize the difference between situations with replacement and without replacement, and recognize the difference between ordered and unordered counting situations.	SE: 837, 839 ex 2 TWE: FTC 838
3. Justify solutions to counting problems.	SE: 837, 839 ex 2 TWE: FTC 838
4. Recognize and explain relationships involving combinations and Pascal's Triangle, and apply those methods to situations involving probability.	SE: 837-845
D. Discrete Mathematics–Vertex-Edge Graphs and Algorithms	
1. Use vertex-edge graphs and algorithmic thinking to represent and solve practical problems. <ul style="list-style-type: none"> • Circuits that include every edge in a graph • Circuits that include every vertex in a graph • Scheduling problems (e.g., when project meetings should be scheduled to avoid conflicts) using graph coloring • Applications to science (e.g., who-eats-whom graphs, genetic trees, molecular structures) 	SE: 837 <i>Real World Application</i> 500 <i>Example</i> 501 <i>Exercises</i> 504 #37, 504 #39, 606 #38 TWE: TT 501 FTC 501 ICE 502 PA 502 EC 504 NOTE: These references teach students about vectors in three-dimensional space; classroom application can then demonstrate the use of these skills to solve optimization problems with techniques such as vertex-edge graphing.
2. Explore strategies for making fair decisions. <ul style="list-style-type: none"> • Combining individual preferences into a group decision (e.g., determining winner of an election or selection process) • Determining how many Student Council representatives each class (9th, 10th, 11th, and 12th grade) gets when the classes have unequal sizes (apportionment) 	SE: 504 #39, 606 #38
Standard 4.5 (Mathematical Processes) All students will use mathematical processes of problem solving, communication, connections, reasoning, representations, and technology to solve problems and communicate mathematical ideas.	
Cumulative Progress Indicators	
At each grade level, with respect to content appropriate for that grade level, students will:	
A. Problem Solving	
1. Learn mathematics through problem solving, inquiry, and discovery.	SE: 48 #4, 82 #4, 109 #2, 156 #5, 226 #4, 240 #4, 308 #4, 316 #4, 383 #5, 410 #5

CONTENT STANDARDS	PAGE REFERENCES
2. Solve problems that arise in mathematics and in other contexts (cf. workplace readiness standard 8.3). <ul style="list-style-type: none"> • Open-ended problems • Non-routine problems • Problems with multiple solutions • Problems that can be solved in several ways 	SE: 48 #4, 82 #4, 109 #2, 156 #5, 226 #4, 240 #4, 308 #4, 316 #4, 383 #5, 410 #5
3. Select and apply a variety of appropriate problem-solving strategies (e.g., try a simpler problem or make a diagram) to solve problems.	SE: 389 ex 1b, 459 #4, 467 #4
4. Pose problems of various types and levels of difficulty.	TWE: AIN 432, 457, 502, 530, 539, 841
5. Monitor their progress and reflect on the process of their problem-solving activity.	SE: 48 #4, 82 #4, 109 #2, 156 #5, 226 #4, 240 #4, 308 #4, 316 #4, 383 #5, 410 #5
B. Communication	
1. Use communication to organize and clarify their mathematical thinking. <ul style="list-style-type: none"> • Reading and writing • Discussion, listening, and questioning 	SE: 319 #2, 367 #2, 534 #2, 812 #38, 820 #34, 917 #27 <i>Internet Project</i> 123, 271, 339, 691
2. Communicate their mathematical thinking coherently and clearly to peers, teachers, and others, both orally and in writing.	SE: 17 #3, 18 #29, 23 #4, 37 #32, 110 #24, 118 #22, 133 #4, 149 #3, 820 #34 TWE: MTL 45
3. Analyze and evaluate the mathematical thinking and strategies of others.	TWE: AIN 432, 457, 502, 530, 539, 841
4. Use the language of mathematics to express mathematical ideas precisely.	SE: 17 #3, 18 #29, 23 #4, 37 #32, 110 #24, 118 #22, 133 #4, 149 #3, 820 #34 TWE: MTL 45
C. Connections	
1. Recognize recurring themes across mathematical domains (e.g., patterns in number, algebra, and geometry).	SE: 35 ex 5, 36 #11, 71 #33, 104 #50, 110 #23, 123 #53, 135 #40, 144 #41, 145 #46, 226 #13
2. Use connections among mathematical ideas to explain concepts (e.g., two linear equations have a unique solution because the lines they represent intersect at a single point).	TWE: MTL 52, 88, 107, 152, 181, 243, 251, 320, 527, 535
3. Recognize that mathematics is used in a variety of contexts outside of mathematics.	SE: 103 #48, 150 #40, 187 #41, 468 #33, 504 #48, 611 #65, 639 #49, 790 ex 6, 893 #6
4. Apply mathematics in practical situations and in other disciplines.	SE: 103 #48, 150 #40, 187 #41, 468 #33, 504 #48, 611 #65, 639 #49, 790 ex 6, 893 #6
5. Trace the development of mathematical concepts over time and across cultures (cf. world languages and social studies standards).	SE: 716 #14, 807, 827 #23 <i>History of Mathematics</i> 97, 319, 367, 462, 534, 969
6. Understand how mathematical ideas interconnect and build on one another to produce a coherent whole.	SE: <i>History of Mathematics</i> 97, 319, 367, 462, 534, 969

CONTENT STANDARDS	PAGE REFERENCES
D. Reasoning	
1. Recognize that mathematical facts, procedures, and claims must be justified.	SE: 94 #25, 498 #43, 510 #40, 621 #28-#32, 724 #61, 822-828, 832 #51-#53 TWE: EC 828
2. Use reasoning to support their mathematical conclusions and problem solutions.	SE: 94 #25, 498 #43, 510 #40, 621 #28-#32, 724 #61, 822-828, 832 #51-#53 TWE: EC 828
3. Select and use various types of reasoning and methods of proof.	SE: 94 #25, 498 #43, 510 #40, 621 #28-#32, 724 #61, 822-828, 832 #51-#53 TWE: EC 828
4. Rely on reasoning, rather than answer keys, teachers, or peers, to check the correctness of their problem solutions.	SE: 15 ex 3, 48 #4, 76 #3, 115 #3, 149 #3, 155 #1 TWE: A 77, 151, 168, 228
5. Make and investigate mathematical conjectures. <ul style="list-style-type: none"> • Counterexamples as a means of disproving conjectures • Verifying conjectures using informal reasoning or proofs. 	SE: 17 #3, 308 #3, 363 #1, 410 #11-#12, 411 #32-#37, 421-422, 427 #1, 589 #3, 627 #4, 848 #3
6. Evaluate examples of mathematical reasoning and determine whether they are valid.	SE: 15 ex 3, 48 #4, 76 #3, 115 #3, 149 #3, 155 #1 TWE: A 77, 151, 168, 228
E. Representations	
1. Create and use representations to organize, record, and communicate mathematical ideas. <ul style="list-style-type: none"> • Concrete representations (e.g., base-ten blocks or algebra tiles) • Pictorial representations (e.g., diagrams, charts, or tables) • Symbolic representations (e.g., a formula) • Graphical representations (e.g., a line graph) 	TWE: A 26, 37, 56, 87, 96, 105, 111, 179, 250, 351
2. Select, apply, and translate among mathematical representations to solve problems.	TWE: A 26, 37, 56, 87, 96, 105, 111, 179, 250, 351
3. Use representations to model and interpret physical, social, and mathematical phenomena.	SE: 38-44, 258-264, 740-748 <i>Graphing Calculator Exploration 265</i> TWE: A 44, 264 AIN 40, 260 EC 44 FTC 39
F. Technology	
1. Use technology to gather, analyze, and communicate mathematical information.	SE: <i>Graphing Calculator Exploration 69, 86, 87, 133, 284, 333, 378, 433, 695, 705</i>
2. Use computer spreadsheets, software, and graphing utilities to organize and display quantitative information.	SE: <i>Graphing Calculator Exploration 265-266, 404, 526, 685</i>

CONTENT STANDARDS	PAGE REFERENCES
3. Use graphing calculators and computer software to investigate properties of functions and their graphs.	SE: 143 #35-#37, 166 #26-#31, 173, 176 #2, 237 ex 1-ex 2 <i>Graphing Calculator Exploration</i> 169, 232 <i>Internet Connections</i> 133
4. Use calculators as problem-solving tools (e.g., to explore patterns, to validate solutions).	SE: <i>Graphing Calculator Exploration</i> 69, 86, 87, 133, 284, 333, 378, 433, 695, 705
5. Use computer software to make and verify conjectures about geometric objects.	This objective can be found in Glencoe's <i>Geometry</i> © 2004 pages 132, 343, 477, 448, and 552.
6. Use computer-based laboratory technology for mathematical applications in the sciences.	SE: <i>interNET Projects</i> 275, 339, 417, 481, 611, 691, 753 TWE: IP 481, 691, 753

Codes Used for TWE Pages

A	Assess
AA	Additional Answers
AIN	Addressing Individual Needs
EC	Extra Credit
EM	Enrichment Masters
FTC	From the Classroom of...
GCE	Graphing Calculator Exploration
ICE	In-Class Example(s)
IP	interNET Project
MTL	Motivating the Lesson
PA	Practice/Apply
PRS	Prerequisite Skills
TT	Teaching Tip