The University of Chicago School Mathematics Project

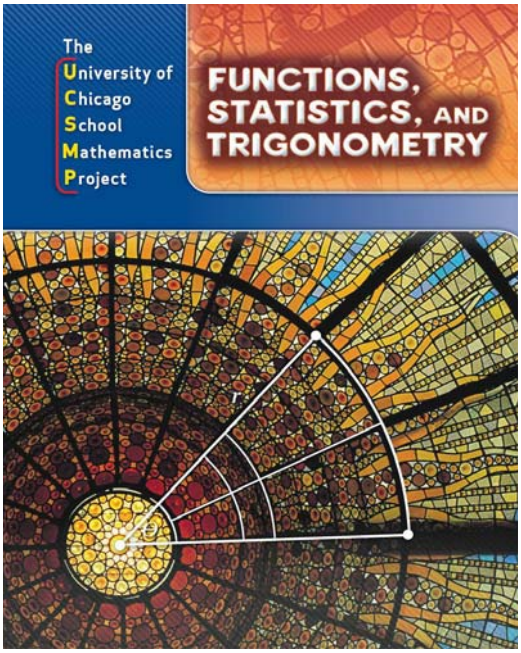
# Functions, Statistics, and Trigonometry

Correlated to Colorado Academic Standards for Mathematics

High School

Colorado Academic Standards
Mathematics
High School

Colorado




# FUNCTIONS, STATISTICS, AND TRIGONOMETRY



| High School STANDARDS | PAGE REFERENCES |
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| **Number Sense, Properties, and Operations**<br>Number sense provides students with a firm foundation in mathematics. Students build a deep understanding of quantity, ways of representing numbers, relationships among numbers, and number systems. Students learn that numbers are governed by properties, and understanding these properties leads to fluency with operations.<br><br>**Prepared Graduates**<br>**The prepared graduate competencies are the preschool through twelfth-grade concepts and skills that all students who complete the Colorado education system must master to ensure their success in a postsecondary and workforce setting.** | |
| 1. The complex number system includes real numbers and imaginary numbers | |
| a. Show that between any two rational numbers there are an infinite number of rational numbers, and that between any two irrational numbers there are also an infinite number of irrational numbers | |
| b. Express the square root of a negative number using imaginary numbers | SE: 466, 470<br>TE: 466 |
| 2. Formulate, represent, and use algorithms with real numbers flexibly, accurately, and efficiently | |

**SE:** Student Edition **TE:** Teacher's Edition **LM:** Lesson Master **AR:** Assessment Resources **RM:** Resource Master

| High School STANDARDS | PAGE REFERENCES |
|---|---|
| a. Use appropriate computation methods that encompass estimation and calculation | Students apply methods of estimation and calculation throughout the *Functions, Statistics, and Trigonometry* program. Representative pages:<br>SE: 11, 14, 18, 27, 50–51, 300, 308, 452, 565–567, 635–636, 639, 702, 705 |
| b. Use technology to perform operations (addition, subtraction, multiplication, and division) on numbers written in scientific notation | |
| c. Describe factors affecting take-home pay and calculate the impact (PFL) | SE: 536–539, 540<br>TE: 536–539 |
| d. Design and use a budget, including income (net take-home pay) and expenses (mortgage, car loans, and living expenses) to demonstrate how living within your means is essential for a secure financial future (PFL) | SE: 536–541<br>TE: 536–541 |
| 3. Systematic counting techniques are used to describe and solve problems | |
| a. Use combinatorics (Fundamental Counting Principle, permutations and combinations) to solve problems in real-world contexts | SE: 381–386, 618–623, 624–630, 631–632<br>TE: 381–386, 618–623, 624–630, 631<br>LM: 6-4, 10-1, 10-2<br>RM: 134, 135, 221, 222, 223, 224<br>AR: 25, 41 |

## Patterns, Functions, and Algebraic Structures

Pattern sense gives students a lens with which to understand trends and commonalities. Being a student of mathematics involves recognizing and representing mathematical relationships and analyzing change. Students learn that the structures of algebra allow complex ideas to be expressed succinctly.

**Prepared Graduates**

The prepared graduate competencies are the preschool through twelfth-grade concepts and skills that all students who complete the Colorado education system must have to ensure success in a postsecondary and workforce setting.

| | |
|---|---|
| 1. Functions model situations where one quantity determines another and can be represented algebraically, graphically, and using tables | |
| a. Determine* when a relation is a function using a table, a graph, or an equation | SE: 80<br>TE: 80 |

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| b. Demonstrate the relationship between all representations of linear functions using point-slope, slope-intercept, and standard form of a line | SE: 87–93, 94–101<br>TE: 87–93, 94–101<br>LM: 2-2, 2-3<br>RM: 29, 30, 31, 34, 36<br>AR: 6 |
| c. Represent* linear, quadratic, absolute value, power, exponential, logarithmic, rational, trigonometric (sine and cosine), and step functions in a table, graph, and equation and convert from one representation to another | SE: 87–93, 102–109, 110–116, 117–124, 152–153, 247–251, 252–256, 257–262, 263–268, 269–275<br>TE: 87–93, 102–109, 110–116, 117–124, 152–153, 247–251, 252–256, 257–262, 263–268, 269–275<br>LM: 2-2, 2-4, 2-5, 2-6, 4-5, 4-6, 4-7, 4-8, 4-9<br>RM: 29, 30, 31, 35, 36, 37, 38, 39, 40, 41, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91<br>AR: 10, 16 |
| d. Determine the inverse (expressed graphically or in tabular form) of a function from a graph or table | SE: 199–205<br>TE: 199–205<br>LM: 3-8<br>RM: 65, 66<br>AR: 13 |
| e. Categorize sequences as arithmetic, geometric, or neither and develop formulas for the general terms related to arithmetic and geometric sequences | SE: 502–508, 509–514<br>TE: 502–508, 509–514<br>LM: 8-1, 8-2<br>RM: 183, 184, 185, 186<br>AR: 33, 35 |
| 2. Graphs and tables are used to describe the qualitative behavior of common types of functions | |
| a. Evaluate* a function at a given point in its domain given an equation (including function notation), a table, and a graph | SE: 299–303, 309–313, 320–324<br>TE: 299–303, 309–313, 320–324<br>LM: 5-2, 5-4, 5-6<br>RM: 98, 99, 104, 105<br>AR: 20, 22 |
| b. Identify* the domain and range of a function given an equation (including function notation), a table, and a graph | SE: 81–86, 128, 249<br>TE: 81–86<br>LM: 2-1, 3-1, 4-5<br>RM: 27, 28<br>AR: 6, 8, 13, 17 |

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| c. Identify* intercepts, zeros (or roots), maxima, minima, and intervals of increase and decrease, and asymptotes of a function given an equation (including function notation), a table, and a graph | SE: 105–108, 117–118, 126–128, 155–158, 249, 440–445, 459–464<br>TE: 105–108, 117–118, 126–128, 155–158, 440–445, 459–464<br>LM: 2-6, 2-7, 3-1, 7-1, 7-4<br>RM: 155, 156<br>AR: 31 |
| d. Make qualitative statements about the rate of change of a function, based on its graph or table | |
| 3. Parameters influence the shape of the graphs of functions | |
| a. Apply* transformations (translation, reflection, dilation) to a parent function, f(x) | SE: 159–164, 172–178, 179–184, 185–192, 257–262, 263–268, 269–275<br>TE: 159–164, 172–178, 179–184, 185–192, 257–262, 263–268, 269–275<br>LM: 3-2, 3-3, 3-4, 3-5, 4-7, 4-8, 4-9<br>RM: 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 83, 87<br>AR: 11–14, 17–19 |
| b. Interpret the results of these transformations verbally, graphically, and symbolically | SE: 159–164, 172–178, 179–184, 185–192, 257–262, 263–268, 269–275<br>TE: 159–164, 172–178, 179–184, 185–192, 257–262, 263–268, 269–275<br>LM: 3-2, 3-3, 3-4, 3-5, 4-7, 4-8, 4-9<br>RM: 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 83, 87<br>AR: 11–14, 17–19 |
| 4. Expressions, equations, and inequalities can be expressed in multiple, equivalent forms | |
| a. Perform and justify steps in generating equivalent expressions by identifying properties used including the commutative, associative, inverse, identity, and distributive properties | |
| b. Apply the properties of positive and negative rational exponents to generate equivalent algebraic expressions including those involving nth roots | SE: 568–574<br>TE: 568–574<br>LM: 9-2<br>RM: 206 |

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| c. Solve equations for one variable in terms of the others | SE: 299–303, 309–313, 320–324, 325–330, 466–472<br>TE: 299–303, 309–313, 320–324, 325–330, 466–472<br>LM: 5-2, 5-4, 5-6, 5-7<br>RM: 114 |
| 5. Solutions to equations, inequalities and systems of equations are found using a variety of tools | |
| a. Find* solutions to quadratic and cubic equations and inequalities by using appropriate algebraic methods such as factoring, completing the square, graphing or using the quadratic formula | SE: 466–472<br>TE: 466–472<br>LM: 7-5 |
| b. Find* solutions to equations involving power, exponential, rational and radical functions | SE: 104, 107–109, 110–116, 565, 568–574, 575–580, 581–586<br>TE: 104, 107–109, 110–116, 565, 568–574, 575–580, 581–586<br>LM: 2-5, 9-2, 9-3, 9-4<br>RM: 206, 207, 208, 209, 210<br>AR: 37, 38 |
| c. Solve* systems of linear equations and inequalities with two variables | SE: 449–451<br>TE: 449–451 |
| 6. Quantitative relationships in the real world can be modeled and solved using functions | |
| a. Represent, solve*, and interpret problems in various contexts using linear, quadratic, and exponential functions | SE: 87–93, 95, 102–109, 110–116, 117–124, 473, 584<br>TE: 87–93, 95, 102–109, 110–116, 117–124, 473, 584<br>LM: 2-2, 2-4, 2-5, 2-6<br>RM: 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41<br>AR: 6–10 |
| b. Represent, solve*, and interpret problems involving direct and inverse variations and a combination of direct and inverse variation | SE: 125–131<br>TE: 125–131<br>LM: 2-7<br>RM: 42, 43<br>AR: 9 |

| High School STANDARDS | PAGE REFERENCES |
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| c. Analyze* the impact of interest rates on a personal financial plan (PFL) | SE: 536–541<br>TE: 536–541<br>LM: 8-6<br>RM: 196, 197<br>AR: 34, 36 |
| d. Evaluate* the costs and benefits of credit (PFL) | |
| e. Analyze various lending sources, services, and financial institutions (PFL) | SE: 536–539 |
| **Data Analysis, Statistics, and Probability**<br>Data and probability sense provides students with tools to understand information and uncertainty. Students ask questions and gather and use data to answer them. Students use a variety of data analysis and statistics strategies to analyze, develop and evaluate inferences based on data. Probability provides the foundation for collecting, describing, and interpreting data.<br><br>**Prepared Graduates**<br>The prepared graduate competencies are the preschool through twelfth-grade concepts and skills that all students who complete the Colorado education system must master to ensure their success in a postsecondary and workforce setting. | |
| 1. Statistical methods take variability into account, supporting informed decision-making through quantitative studies designed to answer specific questions | |
| a. Formulate appropriate research questions that can be answered with statistical analysis | SE: 6, 67, 68<br>TE: 6, 67, 68 |
| b. Determine appropriate data collection methods to answer a research question | SE: 4–5, 6–13<br>TE: 4–5, 6–13<br>LM: 1-1<br>RM: 2, 3, 4 |
| c. Explain how data might be analyzed to provide answers to a research question | SE: 4–5, 6–13, 14–21, 22–30, 31–37, 38–44, 45–53, 54–60, 61–66<br>TE: 4–5, 6–13, 14–21, 22–30, 31–37, 38–44, 45–53, 54–60, 61–66<br>LM: 1-1, 1-2, 1-3, 1-4, 1-5, 1-6, 1-7, 1-8<br>RM: 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24<br>AR: 1–5 |
| 2. The design of an experiment or sample survey is of critical importance to analyzing the data and drawing conclusions | |

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| a. Identify the characteristics of a well-designed and well-conducted survey | SE: 6–7<br>TE: 6–7 |
| b. Identify the characteristics of a well-designed and well-conducted experiment | SE: 360<br>TE: 358 |
| c. Differentiate between the inferences that can be drawn in experiments versus observational studies | |
| 3. Visual displays and summary statistics condense the information in data sets into usable knowledge | |
| a. Identify and choose appropriate ways to summarize numerical or categorical data using tables, graphical displays, and numerical summary statistics (describing shape, center and spread) and accounting for outliers when appropriate | SE: 358–359, 667–668, 714–716<br>TE: 667–668, 714–716 |
| b. Define and explain how sampling distributions (developed through simulation) are used to describe the sample-to-sample variability of sample statistics | SE: 708–711<br>TE: 708–711 |
| c. Describe the relationship between two categorical variables using percents | SE: 7–8<br>TE: 7–8 |
| d. When the relationship between two numerical variables is reasonably linear, apply* the least-squares criterion for line fitting, use Pearson's correlation coefficient as a measure of strength, and interpret the slope and y-intercept in the context of the problem | SE: 94–101<br>TE: 94–101<br>LM: 2-3<br>RM: 32, 33<br>AR: 9 |
| 4. Randomness is the foundation for using statistics to draw conclusions when testing a claim or estimating plausible values for a population characteristic | |
| a. Define and explain the meaning of significance (both practical and statistical) | SE: 420, 668–671<br>TE: 421, 668–671 |
| b. Explain the role of p-values in determining statistical significance | SE: 666–668<br>TE: 666–668 |

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| c. Determine the margin of error associated with an estimate of a population characteristic | SE: 721–724<br>TE: 721–724<br>LM: 11-6 |
| 5. Probability models outcomes for situations in which there is inherent randomness, quantifying the degree of certainty in terms of relative frequency of occurrence | |
| a. Develop simulations that demonstrate probability as a long-run relative frequency | SE: 402–409<br>TE: 402–409<br>LM: 6-7<br>RM: 143, 144 |
| b. Apply and solve problems using the concepts of independence and conditional probability | SE: 374–380, 395–401<br>TE: 374–380, 395–401<br>LM: 6-3, 6-6<br>RM: 131, 132, 133, 140, 141, 142<br>AR: 24–27 |
| c. Apply and solve problems using the concept of mutually exclusive properties when combining probabilities | SE: 368–373<br>TE: 368–373<br>LM: 6-2<br>RM: 128, 129<br>AR: 26 |
| d. Evaluate and interpret probabilities using a normal distribution | SE: 663, 684–685, 692–697<br>TE: 684–685, 692–697<br>LM: 11-2<br>RM: 245, 246<br>AR: 45, 47 |
| e. Find and interpret the expected value and standard deviation of a discrete random variable X | SE: 640–644<br>TE: 640–644<br>LM: 10-4<br>RM: 227, 228<br>AR: 42, 43 |
| f. Analyze the cost of insurance as a method to offset the risk of a situation (PFL) | |
| **Shape, Dimension, and Geometric Relationships** | |

| High School STANDARDS | PAGE REFERENCES |
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| Geometric sense allows students to comprehend space and shape. Students analyze the characteristics and relationships of shapes and structures, engage in logical reasoning, and use tools and techniques to determine measurement. Students learn that geometry and measurement are useful in representing and solving problems in the real world as well as in mathematics.<br><br>**Prepared Graduates**<br>The prepared graduate competencies are the preschool through twelfth-grade concepts and skills that all students who complete the Colorado education system must master to ensure their success in a postsecondary and workforce setting. | |
| 1. Attributes of two- and three-dimensional objects are measurable and can be quantified | |
| a. Calculate (or estimate when appropriate) the perimeter and area of a two-dimensional irregular shape | |
| b. Justify, interpret, and apply the use of formulas for the surface area, and volume of cones, pyramids, and spheres including real-world situations | SE: 562<br>TE: 562 |
| c. Solve for unknown quantities in relationships involving perimeter, area, surface area, and volume | SE: 439, 562<br>TE: 562 |
| d. Apply the effect of dimensional change, utilizing appropriate units and scales in problem-solving situations involving perimeter, area, and volume | SE: 179–184, 332–335, 753–754<br>TE: 179–184, 332–335<br>LM: 3-5, 5-8<br>RM: 56, 57, 58, 59, 116, 117, 118 |
| 2. Objects in the plane and their parts, attributes, and measurements can be analyzed deductively | |
| a. Classify polygons according to their similarities and differences | SE: 472 |
| b. Solve for unknown attributes of geometric shapes based on their congruence, similarity, or symmetry | |
| c. Know and apply properties of angles including corresponding, exterior, interior, vertical, complementary, and supplementary angles to solve problems. Justify the results using two-column proofs, paragraph proofs, flow charts, or illustrations | SE: 222–228<br>TE: 222–228 |
| d. Develop conjectures and solve problems about geometric figures including definitions and properties (congruence, similarity, and symmetry). Justify these conjectures using two-column proofs, paragraph proofs, flow charts, or illustrations | |

| High School STANDARDS | PAGE REFERENCES |
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| 3. Objects in the plane can be transformed, and those transformations can be described and analyzed mathematically | |
| a. Make conjectures involving two-dimensional objects represented with Cartesian coordinates. Justify these conjectures using two- column proofs, paragraph proofs, flow charts, and/or illustrations | SE: 751–756, 757–761, 762–765<br>TE: 751–756, 757–761, 762–765<br>LM: 12-2, 12-3, 12-4<br>RM: 260, 261, 262, 263, 264, 265, 267, 268<br>AR: 49, 50 |
| b. Represent transformations (reflection, translation, rotation, and dilation) using Cartesian coordinates | SE: 751–756, 757–761, 762–765<br>TE: 751–756, 757–761, 762–765<br>LM: 12-2, 12-3, 12-4<br>RM: 260, 261, 262, 263, 264, 265, 267, 268<br>AR: 49, 50 |
| c. Develop arguments to establish what remains invariant and what changes after a transformation (reflection, translation, rotation, and dilations). Justify these conjectures using two-column proofs, paragraph proofs, flow charts, and/or illustrations | SE: 751–756, 757–761, 762–765<br>TE: 751–756, 757–761, 762–765<br>LM: 12-2, 12-3, 12-4<br>RM: 260, 261, 262, 263, 264, 265, 267, 268<br>AR: 49, 50 |
| d. Using construction tools, including technology, make conjectures about relationships among properties of shapes in the plane including those formed through transformation. Justify these conjectures using two-column proofs, paragraph proofs, flow charts, and/or illustrations | SE: 751–756, 757–761, 762–765<br>TE: 751–756, 757–761, 762–765<br>LM: 12-2, 12-3, 12-4<br>RM: 260, 261, 262, 263, 264, 265, 267, 268<br>AR: 49, 50 |
| 4. Right triangles are central to geometry and its applications | |
| a. Apply right triangle trigonometry (sine, cosine, and tangent) to find indirect measures of lengths and angles | SE: 294–298<br>TE: 294–298<br>LM: 5-1<br>RM: 96, 97<br>AR: 20, 22, 23 |
| b. Apply the Pythagorean theorem and its converse to solve real-world problems | SE: 235, 306, 508<br>TE: 235, 306, 508 |
| c. Determine the midpoint of a line segment and the distance between two points in the Cartesian coordinate plane | |