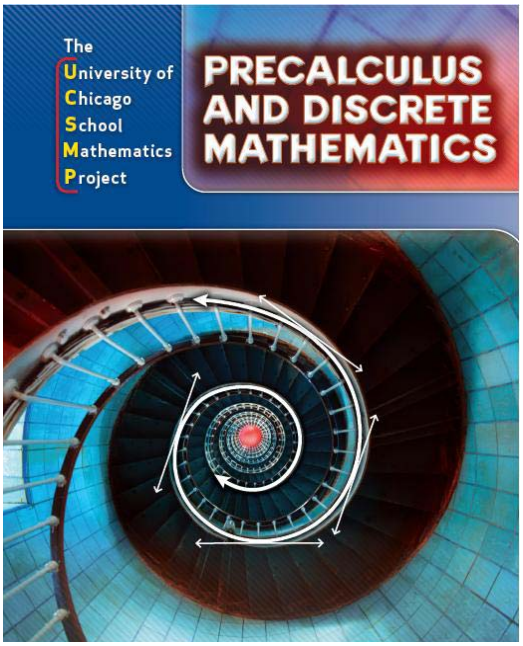
The University of Chicago School Mathematics Project

Precalculus and Discrete Mathematics

Correlated to Colorado Academic Standards for Mathematics

High School



PRECALCULUS AND DISCRETE MATHEMATICS

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High School STANDARDS	PAGE REFERENCES
Number Sense, Properties, and Operations	
Number sense provides students with a firm foundation in mathematics. Students build a deep understanding of quantity, ways of representing numbers, relationships among numbers, and number systems. Students learn that numbers are governed by properties, and understanding these properties leads to fluency with operations. **Prepared Graduates** **The prepared graduate competencies are the preschool through twelfth-grade concepts and skills that all students who complete the Colorado education system must master to ensure their success in a postsecondary and workforce setting.**	
1. The complex number system includes real numbers and imaginary numbers	
a. Show that between any two rational numbers there are an infinite number of rational numbers, and that between any two irrational numbers there are also an infinite number of irrational numbers	Students learn about rational and irrational numbers. See examples below: SE: 278–284 TE: 278–284 LM: 5-1

SE: Student Edition LM: Lesson Master RM: Resource Master
TE: Teacher's Edition AR: Assessment Resources

High School STANDARDS	PAGE REFERENCES
b. Express the square root of a negative number using imaginary numbers	Students learn about square roots. See examples below: SE: 280–284, 335, 562–570 TE: 280–284, 335, 562–570 LM: 5-1, 9-7 AR: 20, 22, 39 RM: 95
2. Formulate, represent, and use algorithms with real numbers flexibly, accurately, and efficiently	
a. Use appropriate computation methods that encompass estimation and calculation	Students use appropriate computation methods to calculate throughout the program. Representative pages: SE: 27–29, 102–104, 175–177, 232–234, 302–304, 396–398, 448–450, 503–505, 568–570, 584–586, 661–662, 721–723, 774–776, 843–844 TE: 27–29, 102–104, 175–177, 232–234, 302–304, 396–398, 448–450, 503–505, 568–570, 584–586, 661–662, 721–723, 774–776, 843–844 LM: 1-3, 2-4, 3-5, 4-2, 5-4, 6-8, 7-5, 8-6, 9-7, 10-1, 11-4, 12-6, 13-4, 14-5 AR: 3–5, 8–10, 13–15, 18–19, 22–23, 26–28, 30–32, 35–36, 39–40, 43–44, 46–47, 50–51, 54–56, 59–61 RM: 6, 37, 61, 78, 99, 129, 147, 229, 255, 257, 289, 311, 333, 365
b. Use technology to perform operations (addition, subtraction, multiplication, and division) on numbers written in scientific notation	
c. Describe factors affecting take-home pay and calculate the impact (PFL)	
d. Design and use a budget, including income (net take-home pay) and expenses (mortgage, car loans, and living expenses) to demonstrate how living within your means is essential for a secure financial future (PFL)	
3. Systematic counting techniques are used to describe and solve problems	

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High School STANDARDS	PAGE REFERENCES
a. Use combinatorics (Fundamental Counting Principle, permutations and combinations) to solve problems in real-world contexts	SE: 684–685, 686–690, 691–697, 698–703, 704–710, 717–723, 724–729, 730–735 TE: 684–685, 686–690, 691–697, 698–703, 704–710, 717–723, 724–729, 730–735 LM: 12-1, 12-2, 13-3, 12-4, 12-5, 12-7, 12-8 AR: 48, 49, 50–51 RM: 300, 301, 302, 303, 304, 305, 306, 307, 311, 312, 313, 314, 315, 316, 317, 318

Patterns, Functions, and Algebraic Structures

Pattern sense gives students a lens with which to understand trends and commonalities. Being a student of mathematics involves recognizing and representing mathematical relationships and analyzing change. Students learn that the structures of algebra allow complex ideas to be expressed succinctly.

Prepared Graduates

The prepared graduate competencies are the preschool through twelfth-grade concepts and skills that all students who complete the Colorado education system must have to ensure success in a postsecondary and workforce setting.

1. Functions model situations where one quantity determines another and can be represented algebraically, graphically, and using tables	
a. Determine* when a relation is a function using a table, a graph, or an equation	SE: 78–79 TE: 78–79 LM: 2-1 RM: 29
b. Demonstrate the relationship between all representations of linear functions using point-slope, slope-intercept, and standard form of a line	SE: 616–617 LM: 10-6 RM: 274, 277

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High School STANDARDS	PAGE REFERENCES
c. Represent* linear, quadratic, absolute value, power, exponential, logarithmic, rational, trigonometric (sine and cosine), and step functions in a table, graph, and equation and convert from one representation to another	SE: 99, 102, 106, 111, 130–133, 150–151, 204, 328, 481–487, 500–505, 580–583, 588, 594–597, 613–615, 823–827, 828–832, 833–838, 841–843, 857 TE: 98–99, 102–103, 106, 110–111, 130–131, 150–151, 204–205, 328, 481–487, 500–505, 580–583, 588, 594–596, 613–615, 823–825, 828–832, 833–838, 841–843, 857 LM: 2-4, 3-1, 8-3, 8-6, 10-3, 10-6, 14-2, 14-3, 14-4, 14-5 AR: 6, 8, 11, 14, 22–23, 33, 34, 35–36, 41, 43–44, 57, 58, 59–61 RM: 37, 40, 42, 43, 50, 51, 52, 53, 74, 110, 222, 223, 231, 232, 233, 234, 257, 258, 259, 260, 262, 264, 265, 266, 275, 276, 355, 356, 358, 359, 360, 362, 364, 366, 367, 368
d. Determine the inverse (expressed graphically or in tabular form) of a function from a graph or table	SE: 158–163, 192, 468–473 TE: 158–163, 192, 468–473 LM: 3-3, 8-1 AR: 11, 13 RM: 56, 57, 67, 68, 217
e. Categorize sequences as arithmetic, geometric, or neither and develop formulas for the general terms related to arithmetic and geometric sequences	SE: 109, 117, 118–122, 346–353, 406 TE: 118, 122, 346–353 LM: 2-7 AR: 7, 10, 24, 26–27 RM: 45, 114, 115
2. Graphs and tables are used to describe the qualitative behavior of common types of functions	
a. Evaluate* a function at a given point in its domain given an equation (including function notation), a table, and a graph	SE: 86–92, 130–135, 148–153, 172–177 TE: 86–92, 130–135, 148–153, 172–177 LM: 2-2, 2-9, 3-1, 3-5 AR: 6, 8–10, 13–15 RM: 51, 52, 61
b. Identify* the domain and range of a function given an equation (including function notation), a table, and a graph	SE: 78–85 TE: 78–85 LM: 2-1 AR: 9 RM: 28

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High School STANDARDS	PAGE REFERENCES
c. Identify* intercepts, zeros (or roots), maxima, minima, and intervals of increase and decrease, and asymptotes of a function given an equation (including function notation), a table, and a graph	SE: 78–85, 86–92, 93–97, 99, 241–247, 305–312, 445–450 TE: 78–85, 86–92, 93–97, 98–99, 241–247, 305–312, 445–450 LM: 2-2, 2-3, 2-4, 4-4, 5-5, 7-5 AR: 6, 8–9, 17, 21, 22, 30–31 RM: 30, 31, 32, 33, 35, 82, 83, 84, 103, 147, 148
d. Make qualitative statements about the rate of change of a function, based on its graph or table	SE: 301–302 TE: 302 RM: 101
3. Parameters influence the shape of the graphs of functions	
a. Apply* transformations (translation, reflection, dilation) to a parent function, f(x)	SE: 178–183, 541–546 TE: 178–183, 541–546 LM: 3-6, 9-4 AR: 38, 39 RM: 62, 63, 64, 246, 247, 248, 249
b. Interpret the results of these transformations verbally, graphically, and symbolically	SE: 178–183, 541–546 TE: 178–183, 541–546 LM: 3-6, 9-4 AR: 38, 39 RM: 62, 63, 64, 246, 247, 248, 249
4. Expressions, equations, and inequalities can be expressed in multiple, equivalent forms	
a. Perform and justify steps in generating equivalent expressions by identifying properties used including the commutative, associative, inverse, identity, and distributive properties	SE: 29, 60, 61–66, 110–117, 123–129, 164–171, 190–196, 222–228, 256–262, 711–716, 833–838 TE: 60, 61–66, 110–117, 123–129, 164–171, 190–196, 222–228, 256–262, 711–716, 833–838 LM: 1-8, 2-6, 3-4, 3-8, 4-1, 4-6, 14-4 RM: 42, 47, 59, 68, 69, 77, 89, 310, 363
b. Apply the properties of positive and negative rational exponents to generate equivalent algebraic expressions including those involving nth roots	SE: 105–109 TE: 105–109 LM: 2-5 AR: 7

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High School STANDARDS	PAGE REFERENCES
c. Solve equations for one variable in terms of the others	Representative pages: SE: 61–62, 102–104, 172–177, 241–247, 285–290, 313–317, 432–438, 663–669 TE: 62, 102–103, 172–177, 241–247, 285–290, 313–317, 432–438, 663–669 LM: 1-8, 2-4, 3-5, 4-4, 5-2, 5-6, 7-3, 11-5 AR: 6, 8, 9, 12, 13, 15, 17, 18, 19, 20, 21, 22, 47 RM: 37, 61, 82, 96, 106
5. Solutions to equations, inequalities and systems of equations are found using a variety of tools	
a. Find* solutions to quadratic and cubic equations and inequalities by using appropriate algebraic methods such as factoring, completing the square, graphing or using the quadratic formula	SE: 13, 29, 38, 62–65, 98–104, 506–510 TE: 62, 98–104, 159, 506–510 LM: 1-8, 2-4, 8-7 AR: 8, 35–36 RM: 37, 235, 236
b. Find* solutions to equations involving power, exponential, rational and radical functions	SE: 81, 101–102, 105–109, 285, 305–312, 451–455 TE: 105–109, 305–312, 451–455 LM: 2-5, 5-5, 7-6 AR: 7, 8–10, 21, 22–23, 30–32 RM: 39, 40, 103, 104, 105, 150, 151
c. Solve* systems of linear equations and inequalities with two variables	
6. Quantitative relationships in the real world can be modeled and solved using functions	
a. Represent, solve*, and interpret problems in various contexts using linear, quadratic, and exponential functions	SE: 105–109, 114–115, 434–435, 451–455, 841–842 TE: 105–109, 114–115, 434, 451–455, 842 LM: 2-5, 7-6 AR: 7, 8–10, 30–32 RM: 39, 151
b. Represent, solve*, and interpret problems involving direct and inverse variations and a combination of direct and inverse variation	
c. Analyze* the impact of interest rates on a personal financial plan (PFL)	SE: 153, 377, 406, 452, 454 TE: 406
d. Evaluate* the costs and benefits of credit (PFL)	

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High School STANDARDS	PAGE REFERENCES
e. Analyze various lending sources, services, and financial institutions (PFL)	SE: 153, 406 TE: 406

Data Analysis, Statistics, and Probability

Data and probability sense provides students with tools to understand information and uncertainty. Students ask questions and gather and use data to answer them. Students use a variety of data analysis and statistics strategies to analyze, develop and evaluate inferences based on data. Probability provides the foundation for collecting, describing, and interpreting data.

Prepared Graduates

The prepared graduate competencies are the preschool through twelfth-grade concepts and skills that all students who complete the Colorado education system must master to ensure their success in a postsecondary and workforce setting.

1. Statistical methods take variability into account, supporting informed decision-making through quantitative studies designed to answer specific questions	
a. Formulate appropriate research questions that can be answered with statistical analysis	
b. Determine appropriate data collection methods to answer a research question	
c. Explain how data might be analyzed to provide answers to a research question	
2. The design of an experiment or sample survey is of critical importance to analyzing the data and drawing conclusions	
a. Identify the characteristics of a well-designed and well-conducted survey	
b. Identify the characteristics of a well-designed and well-conducted experiment	
c. Differentiate between the inferences that can be drawn in experiments versus observational studies	
3. Visual displays and summary statistics condense the information in data sets into usable knowledge	
a. Identify and choose appropriate ways to summarize numerical or categorical data using tables, graphical displays, and numerical summary statistics (describing shape, center and spread) and accounting for outliers when appropriate	

High School STANDARDS	PAGE REFERENCES
b. Define and explain how sampling distributions (developed through simulation) are used to describe the sample-to-sample variability of sample statistics	
c. Describe the relationship between two categorical variables using percents	
d. When the relationship between two numerical variables is reasonably linear, apply* the least-squares criterion for line fitting, use Pearson's correlation coefficient as a measure of strength, and interpret the slope and y-intercept in the context of the problem	
4. Randomness is the foundation for using statistics to draw conclusions when testing a claim or estimating plausible values for a population characteristic	
a. Define and explain the meaning of significance (both practical and statistical)	
b. Explain the role of p-values in determining statistical significance	
c. Determine the margin of error associated with an estimate of a population characteristic	
5. Probability models outcomes for situations in which there is inherent randomness, quantifying the degree of certainty in terms of relative frequency of occurrence	
a. Develop simulations that demonstrate probability as a long-run relative frequency	
b. Apply and solve problems using the concepts of independence and conditional probability	
c. Apply and solve problems using the concept of mutually exclusive properties when combining probabilities	
d. Evaluate and interpret probabilities using a normal distribution	
e. Find and interpret the expected value and standard deviation of a discrete random variable X	
f. Analyze the cost of insurance as a method to offset the risk of a situation (PFL)	
Shape, Dimension, and Geometric Relationships	

High School STANDARDS	PAGE REFERENCES
Geometric sense allows students to comprehend space and shape. Students analyze the characteristics and relationships of shapes and structures, engage in logical reasoning, and use tools and techniques to determine measurement. Students learn that geometry and measurement are useful in representing and solving problems in the real world as well as in mathematics. **Prepared Graduates** The prepared graduate competencies are the preschool through twelfth-grade concepts and skills that all students who complete the Colorado education system must master to ensure their success in a postsecondary and workforce setting.	
1. Attributes of two- and three-dimensional objects are measurable and can be quantified	
a. Calculate (or estimate when appropriate) the perimeter and area of a two-dimensional irregular shape	SE: 447, 814–820, 821–827, 828–832, 833–838, 839–844 TE: 447, 814–820, 821–827, 828–832, 833–838, 839–844 LM: 14-1, 14-2, 14-3, 14-4, 14-5 AR: 57, 58, 59–61 RM: 351, 352, 353, 355, 356, 357, 358, 359, 360, 362, 364, 365, 366, 367, 368
b. Justify, interpret, and apply the use of formulas for the surface area, and volume of cones, pyramids, and spheres including real-world situations	SE: 80–81, 86–87, 240, 435–436, 845–850 TE: 80, 87, 435, 845–850 LM: 2-2, 7-3, 14-6 AR: 6, 58, 60 RM: 28, 141, 369, 370, 371
c. Solve for unknown quantities in relationships involving perimeter, area, surface area, and volume	SE: 80–81, 86–87, 240, 435–436, 447, 814–820, 821–827, 828–832, 833–838, 839–844, 845–850 TE: 80, 87, 435, 447, 814–820, 821–827, 828–832, 833–838, 839–844, 845–850 LM: 2-2, 7-3, 14-1, 14-2, 14-3, 14-4, 14-5, 14-6 AR: 6, 57, 58, 59–61 RM: 28, 141, 351, 352, 353, 355, 356, 357, 358, 359, 360, 362, 364, 365, 366, 367, 368, 369, 370, 371
d. Apply the effect of dimensional change, utilizing appropriate units and scales in problem-solving situations involving perimeter, area, and volume	
2. Objects in the plane and their parts, attributes, and measurements can be analyzed deductively	
a. Classify polygons according to their similarities and differences	

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High School STANDARDS	PAGE REFERENCES
b. Solve for unknown attributes of geometric shapes based on their congruence, similarity, or symmetry	SE: 541–546 TE: 541–546 LM: 9-4 AR: 38, 39 RM: 246, 248, 249
c. Know and apply properties of angles including corresponding, exterior, interior, vertical, complementary, and supplementary angles to solve problems. Justify the results using two-column proofs, paragraph proofs, flow charts, or illustrations	SE: 622–624 TE: 622–624 LM: 10-7 AR: 43–44 RM: 278, 279, 280
d. Develop conjectures and solve problems about geometric figures including definitions and properties (congruence, similarity, and symmetry). Justify these conjectures using two-column proofs, paragraph proofs, flow charts, or illustrations	
3. Objects in the plane can be transformed, and those transformations can be described and analyzed mathematically	
a. Make conjectures involving two-dimensional objects represented with Cartesian coordinates. Justify these conjectures using two- column proofs, paragraph proofs, flow charts, and/or illustrations	
b. Represent transformations (reflection, translation, rotation, and dilation) using Cartesian coordinates	
c. Develop arguments to establish what remains invariant and what changes after a transformation (reflection, translation, rotation, and dilations). Justify these conjectures using two-column proofs, paragraph proofs, flow charts, and/or illustrations	
d. Using construction tools, including technology, make conjectures about relationships among properties of shapes in the plane including those formed through transformation. Justify these conjectures using two-column proofs, paragraph proofs, flow charts, and/or illustrations	
4. Right triangles are central to geometry and its applications	

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High School STANDARDS	PAGE REFERENCES
a. Apply right triangle trigonometry (sine, cosine, and tangent) to find indirect measures of lengths and angles	SE: 127 TE: 127 RM: 45, 47
b. Apply the Pythagorean theorem and its converse to solve real-world problems	
c. Determine the midpoint of a line segment and the distance between two points in the Cartesian coordinate plane	