

**Common Core State Standards Correlated to Glencoe’s Core-Plus Mathematics:  
Course 1, Course 2, Course 3, and Course 4: Preparation for Calculus\*\***

Standards for Mathematical Practice	Student Edition Lessons			
	Course 1	Course 2	Course 3	Course 4: Preparation for Calculus
<p>The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important “processes and proficiencies” with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council’s report <i>Adding It Up</i>: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy).</p>				
<b>1. Make sense of problems and persevere in solving them.</b>				
<p>Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.</p>	<p>Throughout Units 1-8</p> <p>Examples: 8-10, 56-58, 61 #11, 93 #8, 111, 131, 159, 239-242, 297 STM, 413 #3, 420-421</p>	<p>Throughout Units 1-8</p> <p>Examples: 61-64, 121 #7, 124 #11, 141 Check Your Understandi ng (CYU) 280-285, 314 #13, 364-367, 372 #17, 474 #2, #3</p>	<p>Throughout Units 1-8</p> <p>Examples: 32-33, 149 #14, 233 #7, 360 #15, 432 CYU, 435 #7, 440 #8, 450 #33, 459-461, 470 #4, 554 #22</p>	<p>Throughout Units 1-8</p> <p>Examples: 71 #22, 178- 180, 231 CYU, 288 #7, 300-301, 318-321, 394-398, 470-473, 496-501, 591-595, 611-614</p>

<b>2. Reason abstractly and quantitatively.</b>				
Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to <i>decontextualize</i> —to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to <i>contextualize</i> , to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.	Throughout Units 1-3, 5, 8  Examples: 27-31, 116- 123, 194- 197, 250- 254, 307- 311, 369 CYU, 469- 472	Throughout Units 1-8  Examples: 70-71, 104- 117, 145 #2, 223 #15, 331, 349- 350 #17, 472 #4, 489-497, 552-553	Throughout Units 1-8  Examples: 118 #1, #2, 145-150, 220 #18, 338-339, 353 #1, 562 CYU, 468- 471	Throughout Units 1-8  Examples: 68 #10, 146 #3, 157-161, 239-242, 305-306, 364-367, 399-404, 439-442, 490-495
<b>3. Construct viable arguments and critique the reasoning of others.</b>				
Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.	Throughout Units 1-7  Examples: 180 #30, 194 #4, 335-337, 348 #29, 374-377, 387, 493 #2, 504 #13, 517 CYU	Throughout Units 1-7  Examples: 22 #14, 54- 55, 111#7, 123 #10, 168 #9, 170-180, 179 #8, 207 #3, #4, 490- 491	Throughout Units 1-8  Examples: 2-15, 166 #5, 171 #5, 204-208, 233 #7, 352 #9, 405- 406, 487 #11, 566- 567	Throughout Units 1-8  Examples: 124 #19, 287-294, 315 #5, 316 #10, 326 #22, 434 #6, 472 #8, 594-595, 598 #8, 604-614

<b>4. Model with mathematics.</b>				
Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.	Throughout Units 1-8  Examples: 161-167, 280 #10, 323-332, 363-369, 383, 463- 468, 551- 561	Throughout Units 1-8  Examples: 158-160, 232-242, 260-268, 439-442, 518-519, 522-531, 569 CYU, 573-576	Throughout Units 1-8  Examples: 81-88, 132- 143, 218 #12, 231 #5, 242-247, 360 #14, 435-437, 495-506	Throughout Units 1-8  Examples: 126 #23, 157-161, 182-187, 425-429, 436 #9, 464-470, 482 #26, 501, 512-516
<b>5. Use appropriate tools strategically.</b>				
Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.	Throughout Units 1-8  Examples: 52-56, 137 #16, 161- 167, 281 #13, 297, 480-482, 568-570	Throughout Units 1-8  Examples: 49-57, 97 #18, 280- 298, 385, 405 #12, 498-501, 507 #11, 581	Throughout Units 1-8  Examples: 93-95, 220 #19, 254 #13, 260- 265, 462- 467, 475 #11, 519- 522	Throughout Units 1-8  Examples: 138-144, 451 #31, 468- 470, 483 #28, 505-507, 518 #20, 536- 537, 591-595

<b>6. Attend to precision.</b>				
Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.	Throughout Units 1-8  Examples: 20-21 #15, 175 #17, 374-377, 387 #10, 390 #17, 391 #19, 466 #5, 482 #9	Throughout Units 1-8  Examples: 32-33, 92 #7, 172 #6, 261 #3, 382-383, 467-477, 531	Throughout Units 1-8  Examples: 123 #16, 165-168, 245-247, 283-296, 321-323, 581-583	Throughout Units 1-8  Examples: 66 #7, 110 #4, 239 #10, 366, 493- 495, 517 #15, 535 #3c, 576 #11
<b>7. Look for and make use of structure.</b>				
Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see $7 \times 8$ equals the well remembered $7 \times 5 + 7 \times 3$ , in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$ , older students can see the 14 as $2 \times 7$ and the 9 as $2 + 7$ . They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers $x$ and $y$ .	Throughout Units 1-8  Examples: 178 #24, 195, 239- 247, 291- 297, 303 STM, 328 STM, 468, 478	Throughout Units 1-8  Examples: 26-29, 42 #21, 60 STMd, 96 CYU, 178 #6, 332- 340, 351 #22, 481 #20	Throughout Units 1-8  Examples: 60 #5, 62- 71, 112- 117, 334, 348-352, 387 #28, 489-495, 506 #19, 597 #29	Throughout Units 1-8  Examples: 59-64, 141- 144, 188- 193, 202 #14, 250-256, 260 #14, 262 #24, 369-372, 391-393, 464-470

<b>8. Look for and express regularity in repeated reasoning.</b>				
Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$ . Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$ , $(x - 1)(x^2 + x + 1)$ , and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.	Throughout Units 1-7	Throughout Units 1-7	Throughout Units 1-8	Throughout Units 1-8
	Examples: 27-35, 152 #1e, 332-334, 404-406, 437-438 #6, 473-478, 497 #5	Examples: 23 #20, 40 #15, 210-216, 371 #13, 570-572, 578 #14	Examples: 62 #4, #5, 103 #3, 329-331, 341 #12, 482-489, 507, 539-542	Examples: 107 #7, 165-168, 205 #28, 245 #26, 318-321, 361 #7, 499 10, 501-504, 535 #3, 591-595

## Number and Quantity

Standards	Student Edition Lessons			
	Course 1	Course 2	Course 3	Course 4: Preparation for Calculus
<b>The Real Number System, N-RN</b>				
<b>Extend the properties of exponents to rational exponents.</b>				
1. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. <i>For example, we define <math>5^{1/3}</math> to be the cube root of 5 because we want <math>(5^{1/3})^3 = 5^{(1/3)3}</math> to hold, so <math>(5^{1/3})^3</math> must equal 5.</i>	332-337, 344, 351 #35, 358, 359			
2. Rewrite expressions involving radicals and rational exponents using the properties of exponents.	335-337, 358			

Standards	Student Edition Lessons			
	Course 1	Course 2	Course 3	Course 4: Preparation for Calculus
<b>Use properties of rational and irrational numbers.</b>				
3. Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.	NA*	NA	NA	NA
<b>Quantities ★, N-Q</b>				
<b>Reason quantitatively and use units to solve problems.</b>				
1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.	83 #9 & CYU, 110-112, 153 #3, 154 #4, 155 #5, 157, 170 #5, 171 #10, 190 STM, 191, 233, 292-303, 324-331, 469-472, 569 #5	5-9, 24 #27, 25-29, 44 #26	435-436, 441-442	
2. Define appropriate quantities for the purpose of descriptive modeling.	4-5, 324			
3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.	49-51	460, 464 #8, 467, 471-477, 481 #18		
<b>The Complex Number System, N-CN</b>				
<b>Perform arithmetic operations with complex numbers.</b>				
1. Know there is a complex number $i$ such that $i^2 = -1$ , and every complex number has the form $a + bi$ with $a$ and $b$ real.			354-356	210-218, 223 #20
2. Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.			360 #14, 362 #25	
3. (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.				212, 324

\* NA means Not Addressed

Standards	Student Edition Lessons			
	Course 1	Course 2	Course 3	Course 4: Preparation for Calculus
<b>Represent complex numbers and their operations on the complex plane.</b>				
4. (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.			360 #14	213-215, 218, 220, 222, 224 #28, 313-317
5. (+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. <i>For example, <math>(1 - \sqrt{3}i)^3 = 8</math> because <math>(1 - \sqrt{3}i)</math> has modulus 2 and argument <math>120^\circ</math>.</i>				213-215, 225 #30, 313-327
6. (+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.				315, 322
<b>Use complex numbers in polynomial identities and equations.</b>				
7. Solve quadratic equations with real coefficients that have complex solutions.			353-356, 358, 361 #19	209-214, 219, 221 #12, 224 #28
8. (+) Extend polynomial identities to the complex numbers. <i>For example, rewrite <math>x^2 + 4</math> as <math>(x + 2i)(x - 2i)</math>.</i>				224 #28, 224 #29
9. (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.			353-356, 358 #8, 362 #24	
<b>Vector and Matrix Quantities, N-VM</b>				
<b>Represent and model with vector quantities.</b>				
1. (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., $\mathbf{v}$ , $ \mathbf{v} $ , $\ \mathbf{v}\ $ , $\mathbf{v}$ ).				102-108, 117-120, 123 #15
2. (+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.				109-112
3. (+) Solve problems involving velocity and other quantities that can be represented by vectors.				113-116, 117-120, 125, 126, 145-147, 178-180

<b>Perform operations on vectors.</b>				
4. (+) Add and subtract vectors.				
a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.				113-116, 121 #9, 122 #12, 123 #17,
b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.				113-116, 120 8, 121 #9, 146 #5, 180
c. Understand vector subtraction $\mathbf{v} - \mathbf{w}$ as $\mathbf{v} + (-\mathbf{w})$ , where $-\mathbf{w}$ is the additive inverse of $\mathbf{w}$ , with the same magnitude as $\mathbf{w}$ and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.				121 #10, 122 #12, 132-137, 145 #2, 180
5. (+) Multiply a vector by a scalar.				
a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$ .				106-110, 122 #13, 129-137, 180
b. Compute the magnitude of a scalar multiple $c\mathbf{v}$ using $\ c\mathbf{v}\  =  c \mathbf{v}$ . Compute the direction of $c\mathbf{v}$ knowing that when $ c \mathbf{v} \neq 0$ , the direction of $c\mathbf{v}$ is either along $\mathbf{v}$ (for $c > 0$ ) or against $\mathbf{v}$ (for $c < 0$ ).				110 #4, 111 #6, 129-137, 145-146, 180
<b>Perform operations on matrices and use matrices in applications.</b>				
6. (+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.		74-100, 103-129, 157-160, 400-425	541 #6	
7. (+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.		82-86, 91, 92		149 #10
8. (+) Add, subtract, and multiply matrices of appropriate dimensions.		82-100, 103-129		86 #10, 89 #20,
9. (+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.		132-138, 148, #8, 150 #12	474 #10, 497, 556 #28	
10. (+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.		132-138, 147 #7		

11. (+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.			444 #15, 556 #28	71 #18, 87 #10, 89 #20, 149 #10, 451 #33
12. (+) Work with $2 \times 2$ matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.		231-250, 252-256	444 #15, 556 #28	71 #18, 451 #33

## Algebra

Standards	Student Edition Lessons			
	Course 1	Course 2	Course 3	Course 4: Preparation for Calculus
<b>Seeing Structure in Expressions A-SSE</b>				
<b>Interpret the structure of expressions</b>				
1. Interpret expressions that represent a quantity in terms of its context. ★	150-174, 175 #14, 177 #23, 186-191, 192 #4, 204, 289-350, 462-472, 491-494	2-23, 25-29, 34-46, 326-331, 359-366, 368-369, 377-386	6-9, 60, 127-155, 159, 338,-339, 390-394, 458-494, 499-502, 559-564, 568, 594, 599 #34	2-17, 84-85, 91 #22, 94-100, 138-147, 156-171, 366, 514-520
a. Interpret parts of an expression, such as terms, factors, and coefficients.	157-158, 161-174, 176 # 19, 178 #24, 474-489	336-344, 348, 353 #30-#33, 354 #37, 355 #38, 393-398, 488-497	129-155, 347-362, 364-388, 390-394, 458-494, 499-510	188-193
b. Interpret complicated expressions by viewing one or more of their parts as a single entity. <i>For example, interpret <math>P(1+r)^n</math> as the product of <math>P</math> and a factor not depending on <math>P</math>.</i>	26-45, 298-301, 491-508, 529 STM c	336, 342-343, 348, 355 #39, #41, 359-367, 396 #6	319-345, 353, 364-388, 390-394, 458-494, 597 #29, #30	138-147, 156-171, 225 #29, 270-294, 358-363, 374, 501-506, 517 #15, 520 #26

2. Use the structure of an expression to identify ways to rewrite it. <i>For example, see <math>x^4 - y^4</math> as <math>(x^2)^2 - (y^2)^2</math>, thus recognizing it as a difference of squares that can be factored as <math>(x^2 - y^2)(x^2 + y^2)</math>.</i>	494-498, 501-508	336-344, 348, 355 #41, 382- 390, 395-398	192 #32, 332- 338, 362 #23, 392 #5, 559- 574, 577-583, 585 #3, 587 #8, 594 #20	188-207, 210-224, 225 #29, 313-327, 367-372
<b>Write expressions in equivalent forms to solve problems</b>				
3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. ★				
a. Factor a quadratic expression to reveal the zeros of the function it defines.	475-479, 491-508, 510-514, 518-523	278 #20, 321 #33, 336-344, 348, 353 #31, #32, 364-370	51 #29, 125 #26, 258 #27, 314 #28, 345 #22, 353-354, 356, 358, 361 #21, #22	50, 193, 200- 201, 208-211, 219, 303-304, 306, 324 #8
b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.			348-352, 357-359, 362, 511 #26	430-438, 445
c. Use the properties of exponents to transform expressions for exponential functions. <i>For example the expression <math>1.15^t</math> can be rewritten as <math>(1.15^{1/12})^{12t} \approx 1.012^{12t}</math> to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.</i>	304-306, 311-312, 332-337, 343-344, 348, 351, 358-359	377-390, 397-398	557 #33, 559- 562	369-372, 375 #10
4. Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. ★			492-495, 501 #6, 534 #2, #3, 536	
<b>Arithmetic with Polynomials and Rational Expressions A-APR</b>				
<b>Perform arithmetic operations on polynomials</b>				
1. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.			327-335, 337 #3, 338 #5	75-78, 84-85, 88, 99-100, 188-207
<b>Understand the relationship between zeros and factors of polynomials</b>				
2. Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number $a$ , the remainder on division by $x - a$ is $p(a)$ , so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$ .			345 #20	195 #7-197

3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.		332-335, 352 #28, 355 #38, 359-370	101 #24, 319- 345, 532 #22	188-193, 200-207, 252-254, 258
<b>Use polynomial identities to solve problems</b>				
4. Prove polynomial identities and use them to describe numerical relationships. <i>For example, the polynomial identity <math>(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2</math> can be used to generate Pythagorean triples.</i>				188-207
5. (+) Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of $x$ and $y$ for a positive integer $n$ , where $x$ and $y$ are any numbers, with coefficients determined for example by Pascal's Triangle.				591-595, 597, 599, 602, 623 STM a, d
<b>Rewrite rational expressions</b>				
6. Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$ , where $a(x)$ , $b(x)$ , $q(x)$ , and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$ , using inspection, long division, or, for the more complicated examples, a computer algebra system.			376-379, 382-383, 385 #22, #23, 387-388	227-246, 254, 256, 259 #9, 267-268
7. (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.			369-379	227-241, 242 #20
<b>Creating Equations ★, A-CED</b>				
<b>Create equations that describe numbers or relationships</b>				
1. Create equations and inequalities in one variable and use them to solve problems. <i>Include equations arising from linear and quadratic functions, and simple rational and exponential functions.</i>	190, 200, 203 #9, 204 #10, #12, 207 #19, 209 #25, 210- 211, 292-303, 307-311	340-344, 359-370, 372 #16, 382-384, 387 #14, 390, 393-398	338 #6, 368- 369, 381 #4, 384 #19, 385 #20, 391 #2	4, 14-15, 94- 95, 227-237, 358-361, 363-367, 373-375
2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.	190, 200, 203 #9, 204 #10, #12, 209 #25, 210-211, 297, 299, 300 STM b, 301 CYU b, 307- 311, 314 #21	30-46, 49-67, 69-72, 359- 370, 382-386, 393-398	132-155, 159 #6	2-10, 14-17, 96, 139-154, 156-176, 198-200, 201 #12, 204 #26, 240 #12, 477 #10

3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. <i>For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.</i>		30-45, 53-54, 58-59, 61-67, 69-71, 139-141, 145-146, 148-152, 159	132-155, 159 #6	182-187, 198-199, 238 #3
4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. <i>For example, rearrange Ohm's law <math>V = IR</math> to highlight resistance <math>R</math>.</i>		25-46, 143-144, 229 #38, 382-383, 387 #14, 390, 396 #11, 397 #11, 494 #4	50 #27, 58, 61 STM b, 63 #8, CYU, 192 #33, 452 #37	221 #15, 250 #3, 262 #23
<b>Reasoning with Equations and Inequalities A-REI</b>				
<b>Understand solving equations as a process of reasoning and explain the reasoning</b>				
1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.	191-197, 203 #8, 205 #15, 206 #16, 208 #22-#24	40 #16, 55-57, 139-141, 340-344, 352 #29-#33, 355 #41, 382-389	58-59, 61 STM a, 65 #10 c, 12 a, 66 #14, 353, 564-567, 569 #13, 583, 585, 587 #8, 588 STM c, f, 589, 590 #4, 591 #9, 592 #13	302-303
2. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.		193 #33, 352 #24, 359-375, 485 #32	115-117, 119 #6, 120 #9, 124 #21	247 #33, 251-253, 258, 261, 452 #37, 547 #31
<b>Solve equations and inequalities in one variable</b>				
3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.	186-208, 210 #28, 232-235, 236 STM e, f, 521 #14	24 #23, 30-35, 39 #13, #14, 45 #27, 47 #32, 156 #28, 357 #47, 388 #19	101 #22, 125 #27, 258 #26, 388 #31	73 #26, 221 #15

4. Solve quadratic equations in one variable.				
a. Use the method of completing the square to transform any quadratic equation in $x$ into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.		355 #41	347-353, 358 #4, 359 #10, 362 #23	431-432, 438
b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$ ), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers $a$ and $b$ .		48 #35, 68 #31, 340-344, 348, 355 #41, 364-370, 388 #19, 391 #30, 433 #31, 451 #22, 486 #36, 585 #28	108-124, 281 #22, 345 #22, 346 #28, 347-362, 419 #37	50, 193, 200-201, 208-211, 213 STM a, 214 CYU, 219, 221 #12, 303-304, 306, 324 #8
<b>Solve systems of equations</b>				
5. Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.		67 #24		
6. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.	197-200, 204-211, 236	49-67, 70-72, 130 #27, 132-154, 159-160, 514 #29	125 #29, 132-155, 159 #6	253 #3, 256 STM c, 258
7. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. <i>For example, find the points of intersection between the line <math>y = -3x</math> and the circle <math>x^2 + y^2 = 3</math>.</i>	209 #26	365-375	115-117, 120 #9, 347	
8. (+) Represent a system of linear equations as a single matrix equation in a vector variable.		139-154, 159-160	497, 498, 501 #7, 502 #8	186-187, 200, 203
9. (+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension $3 \times 3$ or greater).		136-138, 147 #7, 151 #13	497, 502 #8	186-187, 200, 265 #2c
<b>Represent and solve equations and inequalities graphically</b>				
10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).	11-13, 26-31, 52-58, 61, 72, 187	30-33, 37 #8, 42 #22	108-124	

11. Explain why the $x$ -coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$ ; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. ★	186-190, 210 #28	49-53, 57-60, 64 #12, 359-375	114 STM, 115-116, 122 #14	
12. Graph the solutions to a linear inequality in two variables as a halfplane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.			127-132, 137-151, 152 #20, 227 #43, 314 #27, 389 #37	

## Functions

Standards	Student Edition Lessons			
	Course 1	Course 2	Course 3	Course 4: Preparation for Calculus
<b>Interpreting Functions F-IF</b>				
<b>Understand the concept of a function and use function notation</b>				
1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If $f$ is a function and $x$ is an element of its domain, then $f(x)$ denotes the output of $f$ corresponding to the input $x$ . The graph of $f$ is the graph of the equation $y = f(x)$ .		326-335, 345-346, 349-350, 352, 354 #35	386 #25, 556 #27, 558 #37, 601 #40	10-13, 17-18, 20 #22, 227-234, 245 #25, 252, 254
2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.		326-335, 345-346, 349-350, 352, 354 #35, 382, 387, 389, 390, 394, 398	323-345, 357 #2, 360 #17, 364-388	50 #31, 54 #1a, 182-193, 227-231, 237, 240, 243
3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. <i>For example, the Fibonacci sequence is defined recursively by <math>f(0) = f(1) = 1</math>, <math>f(n+1) = f(n) + f(n-1)</math> for <math>n \geq 1</math>.</i>	26-44, 150-161, 168, 175, 290-319, 322-329, 338-343		55 #6, 481-510, 533-536, 575 #37	12 #2, 19 #16, 377 #17, 517 #14, 518 #19

<b>Interpret functions that arise in applications in terms of the context</b>				
4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. <i>Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.</i> <sup>H</sup>	2-17, 152-154, 157-159, 170, 176 #18, 197-204, 290-303, 322-329, 346 #24, 462-481	2-23, 331-333, 338, 346 #7, 353 #34	338, 339, 357 #2, 358 #9, 360 #17, 364-366, 368-370, 380-385, 425-427, 432-437, 441 #11, 442 #12, 447 # 26, 577-589	10-50, 182-193, 227-231, 237, 240, 243
5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. <i>For example, if the function <math>h(n)</math> gives the number of person-hours it takes to assemble <math>n</math> engines in a factory, then the positive integers would be an appropriate domain for the function.</i> <sup>H</sup>	2-17, 26-39, 43 #22, 150-159, 162-183, 186-191, 206 #18, 322-329, 462-472	326-335, 345-346, 349-350, 352, 354 #35, 382, 387, 389, 390, 394, 398	138 #2, 338 #6, 364-386, 543-551, 553 #15, 565 #2, 577-582, 601 #40	10-13, 182-193, 227-231, 237, 240, 243
6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. ★	155-156, 158, 161-168, 169 #3, 170 #6, 175 #16, 177 #22, 181-182	2-9, 387 #13		490-496, 507-511, 514 #3
<b>Analyze functions using different representations</b>				
7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. ★				
a. Graph linear and quadratic functions and show intercepts, maxima, and minima.	150-182, 462-477	10-15, 22 #15, 30-33, 38 #10, 40 #16, 42 #22, 332-335, 348, 352 #28	108-155, 332, 336-337, 347-352, 357-361, 391 #2	11-12, 14, 44, 46, 48, 550

b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.	346 #24		124 #21	17-18, 23, 28, 33, 37, 46-47, 50, 58, 65, 69, 92, 523 #37
c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.		355 #38	320-344	188-193, 200, 202 #14, #15, 203 #16
d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.		2-23	364-369, 381-387, 601 #40	227-237, 241 #14, 242 #17-#19, 243-245, 518 #18
e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.	292-303, 307-311	382-383, 387 #13	124, 432-437, 441-442, 446 #24, 449 #31, 472, 559-562, 573, 576 #43	4-6, 11-13, 15, 56, 61-62, 69, 71 #21, 72, 97, 358-359, 362, 364-365, 367-369
8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.				
a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.	475-479,	332-340, 346-356,	347-352, 357, 362 #23	431-432 #4
b. Use the properties of exponents to interpret expressions for exponential functions. <i>For example, identify percent rate of change in functions such as <math>y = (1.02)^t</math>, <math>y = (0.97)^t</math>, <math>y = (1.01)^{12t}</math>, <math>y = (1.2)^{t/10}</math>, and classify them as representing exponential growth or decay.</i>	27-32, 36-40, 42, 44, 46, 71, 290-297	382-386	559-562	86 #9, 364-372, 375 #10, 391-393
9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). <i>For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.</i>	314 #21, 345 #2	19 #7, 373 #19		10-12

<b>Building Functions F-BF</b>				
<b>Build a function that models a relationship between two quantities</b>				
1. Write a function that describes a relationship between two quantities. ★				
a. Determine an explicit expression, a recursive process, or steps for calculation from a context.	26-44, 150-154, 158-159, 168, 289-319, 322-329, 338-343	8 #9, 16 #2, 17 #3, 32-33, 36-37, 61 #1, 70-71, 100 #23, 139-141, 145-146, 159, 360-363, 368-369, 391, #32	481-510, 533-536	12 #2, 19 #16, 608-610, 612-616, 622 #5
b. Combine standard function types using arithmetic operations. <i>For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.</i>	470-471		327-335, 373-388, 433, 441 #11	75-78, 84-86, 88 #15, 99, 155 #31, 229, 237, 245 #25, 520 #26
c. (+) Compose functions. <i>For example, if <math>T(y)</math> is the temperature in the atmosphere as a function of height, and <math>h(t)</math> is the height of a weather balloon as a function of time, then <math>T(h(t))</math> is the temperature at the location of the weather balloon as a function of time.</i>				79-83, 85-90, 99, 100
2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. ★				
	26-44, 150-161, 168, 290-319, 322-329, 338-343		458-479, 481-510, 535-536	12 #2, 19 #16, 377 #16, 608-610, 612-616, 622 #5
<b>Build new functions from existing functions</b>				
3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$ , $k f(x)$ , $f(kx)$ , and $f(x + k)$ for specific values of $k$ (both positive and negative); find the value of $k$ given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. <i>Include recognizing even and odd functions from their graphs and algebraic expressions for them.</i>	153, 155, 177 #22, 473-479	12, 64, 278 #20, 346 #7, 352 #28		26-50, 52-73, 97-98, 100
4. Find inverse functions.				
a. Solve an equation of the form $f(x) = c$ for a simple function $f$ that has an inverse and write an expression for the inverse. <i>For example, <math>f(x) = 2x^3</math> for <math>x &gt; 0</math> or <math>f(x) = (x+1)/(x-1)</math> for <math>x \neq 1</math>.</i>			538-557 545-548, 553-554	

b. (+) Verify by composition that one function is the inverse of another.				82 #6, 83 STM b, CYU b
c. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse.			538-548, 552 #14	
d. (+) Produce an invertible function from a non-invertible function by restricting the domain.			579-581, 584	299
5. (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents. ★			559-562, 572 #24, 573 #25	365-369, 391-406
<b>Linear, Quadratic, and Exponential Models F-LE</b>				
<b>Construct and compare linear, quadratic, and exponential models and solve problems</b>				
1. Distinguish between situations that can be modeled with linear functions and with exponential functions.				
a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.	175 #16, 303 STMa			
b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.	26-45, 150- 183	5, 11, 100 #23, 391, #32, 508 #14		
c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.	27-32, 36-44, 290-303, 307-319, 322-332, 338-343, 348-350, 355-359	100 #23, 382- 383, 390 #27, 391 #32		
2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).	26-45, 157- 183, 290-303, 307-319, 322-332, 338-343, 348-350, 355-359	5, 11, 100 #23, 382-383, 391 #32, 508 #14, 577 #9, #12		
3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.	310 #9, 311, #11, 314 #21, 483 #12, 487 #22		124 #21	

4. For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where $a$ , $c$ , and $d$ are numbers and the base $b$ is 2, 10, or $e$ ; evaluate the logarithm using technology.			559-564, 568-574, 602-604	364-383, 413-414
<b>Interpret expressions for functions in terms of the situation they model</b>				
5. Interpret the parameters in a linear or exponential function in terms of a context.	26-45, 153-183, 307-319, 322-332, 338-343, 348-350, 355-359	280- 285, 286 CYU b, 305-308, 316 #18, 357 #46, 486 #38		2-10, 14-17, 30, 42-43
<b>Trigonometric Functions F-TF</b>				
<b>Extend the domain of trigonometric functions using the unit circle</b>				
1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.			427-432, 444 #16, #17	173 #9
2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.			427-432	
3. (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$ , $\pi/4$ and $\pi/6$ , and use the unit circle to express the values of sine, cosine, and tangent for $x$ , $\pi + x$ , and $\pi - x$ in terms of their values for $x$ , where $x$ is any real number.			427-432, 441 #9 b, 444 #17	270-275, 279-283, 286
4. (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.				290
<b>Model periodic phenomena with trigonometric functions</b>				
5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. ★			432-437, 441-442, 446 #24, 447 #26, 449 #31	56, 61-62, 66, 68-69, 71 #21, 72, 97
6. (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.			577-589	299
7. (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context. ★			585, 589-599	296-311

<b>Prove and apply trigonometric identities</b>				
8. Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to calculate trigonometric ratios.			68 #20	272
9. (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.				276-280, 287-294

**Modeling** Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (★).

### Geometry

Standards	Student Edition Lessons			
	<i>Course 1</i>	<i>Course 2</i>	<i>Course 3</i>	<i>Course 4: Preparation for Calculus</i>
<b>Congruence, G-CO</b>				
<b>Experiment with transformations in the plane</b>				
1. Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.	363 TATS, 386, 391	170-180	29-40, 401- 403, 415	
2. Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).	370	195-229, 231-250, 254-256	208-214, 218, 224	26-50, 52-73
3. Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.	398-403			
4. Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.		195-205	208-213	
5. Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.		162-163, 195-229, 232-238, 254-256	210 #2, 211 #5, 213, 214, 217, 218, 224, 232, 233	

<b>Understand congruence in terms of rigid motions</b>				
6. Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.	370-371	195-229, 254	208-214, 218, 224	
7. Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.		216, 222		
8. Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.	370-371			
<b>Prove geometric theorems</b>				
9. Prove theorems about lines and angles. <i>Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.</i>	374-377, 385 #7, 387, 391 #18, 392 #21e, 405 #3		29-49	
10. Prove theorems about triangles. <i>Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.</i>	369-382, 391 #19	192 #32	191 #28, 200-225	
11. Prove theorems about parallelograms. <i>Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.</i>	387 #10, 390 #17, 391 #18	184 #10	204-208, 216 #8, 220, #18, #19, 221 #24, 233 #7	
12. Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). <i>Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.</i>	386 #8, #9, 401 #2, #4	192 #32	28 #32, 33-34, 39, 40, 190 #24, 201 #3, 202 #7, 220 #19, 222 #28, #29 399 #4, 402 #3a, 416 #27	
13. Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.	NA	NA	NA	NA

<b>Similarity, Right Triangles, and Trigonometry, G-SRT</b>				
<b>Understand similarity in terms of similarity transformations</b>				
1. Verify experimentally the properties of dilations given by a center and a scale factor:				
a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.		206-209, 221 #10	176-177, 188 #18, 229 #1	
b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.		205-210, 220 #9, 223 #15	173-179, 182 #8, 188 #18, 191 #29, 229 #1	
2. Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.		216, 222	177-178, 188 #18	
3. Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.	NA	NA	NA	NA
<b>Prove theorems involving similarity</b>				
4. Prove theorems about triangles. <i>Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.</i>		184 #9	178 CYU, 185 #13, 187, 191 #28,	
5. Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.	372-377, 384-391	184 #9, #10, 197-222	176-191, 195-225, 257 #25, 389 #40, 532 #23, 557 #31	
<b>Define trigonometric ratios and solve problems involving right triangles</b>				
6. Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.		461-462		
7. Explain and use the relationship between the sine and cosine of complementary angles.		481 #20		276
8. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. ★		458-460, 464, 467-477, 481 #18, 482-484		111-112, 118 #4

<b>Apply trigonometry to general triangles</b>				
9. (+) Derive the formula $A = 1/2 ab \sin (C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.		506 #9		
10. (+) Prove the Laws of Sines and Cosines and use them to solve problems.		488-513, 518-520	60-61, 69 #24, 123 #16, 125 #30, 594 #18	91 #23, 113, 291 #18
11. (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).		488-513, 518-520	125 #30, 594 #18,	91 #23, 113, 291 #18
<b>Circles, G-C</b>				
<b>Understand and apply theorems about circles</b>				
1. Prove that all circles are similar.			189 #22, 397	
2. Identify and describe relationships among inscribed angles, radii, and chords. <i>Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.</i>			397-417, 454-456	
3. Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.			201-203, 410 #10	
4. (+) Construct a tangent line from a point outside a given circle to the circle.			399 #4	
<b>Find arc lengths and areas of sectors of circles</b>				
5. Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.			415, 427-429	
<b>Expressing Geometric Properties with Equations, G-GPE</b>				
<b>Translate between the geometric description and the equation for a conic section</b>				
1. Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.		175-180,		431, 486
2. Derive the equation of a parabola given a focus and directrix.				248-250, 431-433
3. (+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.				433-438, 444-445

<b>Use coordinates to prove simple geometric theorems algebraically</b>				
4. Use coordinates to prove simple geometric theorems algebraically. <i>For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point <math>(1, \sqrt{3})</math> lies on the circle centered at the origin and containing the point <math>(0, 2)</math>.</i>		166, 169, 173-174, 178-179, 184 #9 & #10, 191 #30	187 #16, 220 #18, 221 #24, 223 #30	134-137
5. Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).	177 #22, 180 #30	170-172, 186, 190, 251 #23		
6. Find the point on a directed line segment between two given points that partitions the segment in a given ratio.	NA	NA	NA	NA
7. Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula. ★		181 #2, 208, 210, 218, 221		
<b>Geometric Measurement and Dimension, G-GMD</b>				
<b>Explain volume formulas and use them to solve problems</b>				
1. Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. <i>Use dissection arguments, Cavalieri's principle, and informal limit arguments.</i>	175 #14, 447 #12, 448 #13			478
2. (+) Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.	448 #14, 453 #26			478
3. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems. ★		24 #27, 376 #28, 391 #29, 481 #18	380 #1	8 #5, 21, 16, 17, 19, 231
<b>Visualize relationships between two-dimensional and three-dimensional objects</b>				
4. Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.	453, 484			454-483, 485-488
<b>Modeling with Geometry, G-MG</b>				
<b>Apply geometric concepts in modeling situations</b>				
1. Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder). ★	366-369, 383 #2, #3, 388-395, 407-422, 424-453	163, 169, 181 #2, 458-459, 464 #8, 474 #2, 481 #18, 498-502, 505 #6, 512 #24, 519 #4		145 #1, 146 #4, 147 #6, 201 #12, 231 CYU, 238 #3, 477 #10, 478 #13

2. Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot). ★	112 CYU, 137 #16, 452 #24	376 #28	93 #3	5 #2, #3, 397 #5
3. Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios). ★	237-242, 250, 252-254, 260, 262 #29, 266- 278, 282 #16, 283 #17, 286- 288, 366-369, 383 #2, #3, 390 #17, 395 #30, 410 #6, 415-416, 444 #6, 457 #4	176 #1, 182 #6, 188 #19, 220 #9, 231- 250, 252-256, 498-502, 505 #6, 513 #27, 519 #4	42 #6, 43 #8, 173-174, 182 #8, 191 #29, 198 #6, 210 #3, 216, 217, 218 #12, 229- 233, 513 #33	104, 108-120, 145 #1, 146 #4, 147 #6, 201 #12, 231 CYU, 238 #3, 477 #10, 478 #13

### Statistics and Probability ★

Standards	Student Edition Lessons			
	Course 1	Course 2	Course 3	Course 4: Preparation for Calculus
<b>Interpreting Categorical and Quantitative Data, S-ID</b>				
<b>Summarize, represent, and interpret data on a single count or measurement variable</b>				
1. Represent data with plots on the real number line (dot plots, histograms, and box plots).	67, 73-101, 106, 108-142, 144-147, 231 #29, 454 #31, 554-556, 558, 560-562, 564, 571-575, 587	48 #34, 155 #26, 277 #18, 433 #30	81-99, 104- 105, 228 #45, 236-318, 502 #10	
2. Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.	73-101, 103- 142, 144-147, 397 #39, 490 #31	131 #31, 155 #26, 277 #18, 279 #22, 433 #30	81-88, 157 #41, 227 #44, 228 #45, 253 #9, 255 #17, 256 #21, 262 #5, 274 #7, 306, #8	

Standards	Student Edition Lessons			
	Course 1	Course 2	Course 3	Course 4: Preparation for Calculus
3. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).	73-101, 103-142, 144-147, 454 #31, 561	155 #26, 277 #18, 279 #22	81-83, 259-280, 316-318	
4. Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.			240-242, 245-257, 259-280, 287-289, 294-302, 316-318, 479 #21, 511 #27, 601 #41	
<b>Summarize, represent, and interpret data on two categorical and quantitative variables</b>				
5. Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.	539, 543, 544	392 #33, 521-542, 543 #26, 584 #25		
6. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.				
a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. <i>Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.</i>	5-8, 11-18, 20-24, 27-28, 44, 70-72, 150-154, 156, 159, 161-174, 176, 178, 180, 181-183, 232-234, 236, 290-303, 307-319, 322-332, 338-343, 348-350, 355-359, 462-467, 499-502, 579-580	2-6, 11, 14, 19 #8, 20 #10, 21#12, 22 #17, 23 #19, 100 #23, 250 #22, 279 #26, 280-298, 305-310, 314-315, 317 #21, 390 #27, 391, #32, 508 #14, 577 #9 & #12	482-489, 495-498, 505, 509	385-410, 415

Standards	Student Edition Lessons			
	Course 1	Course 2	Course 3	Course 4: Preparation for Calculus
b. Informally assess the fit of a function by plotting and analyzing residuals.		279-286, 305-306, 313-314, 318 b, 317 #22, 322-323		385-410, 415
c. Fit a linear function for a scatter plot that suggests a linear association.	150-154, 156, 159, 161-174, 181-183, 206, 232-234, 236	280-291, 305-308, 310 #6 312 #8 b., 315 # 15, 486 #38		385-410, 415
<b>Interpret linear models</b>				
7. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.	1-7, 60, 150- 155, 156- 159, 161-174, 176, 181-183, 354 #46	279 #26, 280- 285, 286 CYU b, 305-308, 316 #18, 280- 286, 357 #46, 486 #38	101 #23, 126 #32. 257 #42	
8. Compute (using technology) and interpret the correlation coefficient of a linear fit.		258-277, 286-297, 309-310, 313-316, 319 #26, 486 #38, 584 #25	126 #32	
9. Distinguish between correlation and causation.	44	299-304, 310-31, 452 #26	92 #2	
<b>Making Inferences and Justifying Conclusions, S-IC</b>				
<b>Understand and evaluate random processes underlying statistical experiments</b>				
1. Understand statistics as a process for making inferences about population parameters based on a random sample from that population.			266-280, 511 #27, 601 #41	

Standards	Student Edition Lessons			
	Course 1	Course 2	Course 3	Course 4: Preparation for Calculus
2. Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. <i>For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?</i>		554 #8, 560-565, 573 #1, 574 #3		
<b>Make inferences and justify conclusions from sample surveys, experiments, and observational studies</b>				
3. Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.			74-80, 89-91 STM b, 92, 95, 99 #17	
4. Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.			279 #17, 280 #20	
5. Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.			81-88, 92-99, 104-106	
6. Evaluate reports based on data.		299-304, 310-313	79-80, 95, 99	
<b>Conditional Probability and the Rules of Probability, S-CP</b>				
<b>Understand independence and conditional probability and use them to interpret data</b>				
1. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”).	531-541, 546 #11, 548 #20, 553, 557-558, 571	532-542		
2. Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.		526-528, 534-542, 586		
3. Understand the conditional probability of A given B as $P(A \text{ and } B)/P(B)$ , and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.		528-542, 586		598 #7

4. Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. <i>For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.</i>		521-535, 538 #6, 542, 586		
5. Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. <i>For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.</i>		524-542, 586		588-589, 596
<b>Use the rules of probability to compute probabilities of compound events in a uniform probability model</b>				
6. Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.		528-531		
7. Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ , and interpret the answer in terms of the model.	24,43, 212 #38, 396 #32, 531-544, 586	358 #52, 392 #33, 515 #34, 534 #5, 559 #20	51 #30, 73 #37, 295-296, 304-305, 308	585-590
8. (+) Apply the general Multiplication Rule in a uniform probability model, $P(A \text{ and } B) = P(A)P(B A) = P(B)P(A B)$ , and interpret the answer in terms of the model.	19, 212 #38	358 #52, 392 #33, 524- 542, 586	51 #30, 73 #37, 258 #31, 266 #1, 278 #15, 280 #23, 295-296, 304-305, 308, 310	585-590, 596-597
9. (+) Use permutations and combinations to compute probabilities of compound events and solve problems.	143, 355-356 #2, 547, 581 #22		96 #8, 311 #21, 312 #23	585-590, 596-598, 601, 621
<b>Using Probability to Make Decisions, S-MD</b>				
<b>Calculate expected values and use them to solve problems</b>				
1. (+) Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.	533-536, 542, 544-547, 554, 560-562	549-551, 555 #9, 561-569, 571, 573, 579, 580		

2. (+) Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.	8-11, 544,554	545-558, 562-564, 570-581, 587-589	259-280	
3. (+) Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. <i>For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.</i>	533-536, 542, 544-547, 564, 580, 586	545-558, 563-570, 581, 587-589	259-280	
4. (+) Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. <i>For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?</i>	553-555, 557- 558, 559-567, 571-576, 581- 582,587-588	562	73 #37, 259- 280	
<b>Use probability to evaluate outcomes of decisions</b>				
5. (+) Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.	8-11, 15	545-558, 587		
a. Find the expected payoff for a game of chance. <i>For example, find the expected winnings from a state lottery ticket or a game at a fast-food restaurant.</i>		545-558	280 #21	
b. Evaluate and compare strategies on the basis of expected values. <i>For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.</i>		545-558		621 #2
6. (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).		545-558		
7. (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).		557 # 16 & #17	280 #21	