Research Base of Effective Mathematics Instruction

McGraw-Hill's *Florida Math Connects* K–8 Series

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Introduction

Educational research provides information about what students are likely to learn in specific content areas at specific levels and under specific pedagogical conditions. It provides guidance for effective curriculum and instruction. Extensive efforts during development ensure that results of the latest research in mathematics education form the basis for McGraw-Hill's *Florida Math Connects*.The authors of the program have incorporated the results of educational research in mathematics into this curriculum, enabling teachers to provide their students with excellent learning experiences. The McGraw-Hill *Florida Math Connects* series was designed to assist today's teachers with the challenge of helping students become mathematically proficient according to Florida's Next Generation Sunshine State Standards (NGSSS) and the NCTM Curriculum Focal Points. In creating *Florida Math Connects*, McGraw-Hill's goal was to reflect both the findings from key research on mathematics instruction, instructional best practices and curricular focal points. This white paper documents the research findings to which the *Florida Math Connects* series relates and provides examples of how the program practically applies that research.

The remainder of this paper is divided into three major sections:

1. Attaining Mathematical Proficiency
2. Ensuring Mathematical Proficiency for All Learners
3. Educational Principles for Mathematical Proficiency

Each section presents a synthesis of relevant, scientifically based research on effective K-Algebra mathematics teaching and learning. Also in each section, explanation is given as to how the *Florida Math Connects* series is consistent with the research findings. The paper concludes with a comprehensive listing of research references.

Attaining Mathematical Proficiency

Research shows that a curriculum that enables students to reach high mathematics standards has the following characteristics:

- **Balance**—focuses on both conceptual understanding and procedural fluency;
- **Comprehensiveness**—includes all the important content strands of mathematics, as well as computation and other procedural skills;
- **Alignment with standards**—includes content and strategies which align with state and national standards, external assessments and instruction;
- **Coordination and coherence within and across grades**—presents well-developed concepts that build on and connect with other concepts, both within and across grades (Apthorp, Bodrova, Dean, & Florian, 2001).

Student learning is affected by the written curriculum but also by teacher beliefs, teacher knowledge, teacher interpretation of curriculum, and classroom structures and norms (Stein et al., 2007).

The Five Strands of Mathematical Proficiency

The Mathematics Learning Study Committee of the National Research Council's (NRC, 2001) Center for Education conducted a review and synthesis of relevant research on attainign mathematical proficiency from pre-kindergarten through grade 8. From their review, the committee identified five critical "interwoven and interdependent . . . strands of mathematical proficiency":

1. conceptual understanding: comprehension of mathematical concepts, operations, and relations;
2. procedural fluency: skill in carrying out procedures flexibly, accurately, efficiently, and appropriately;
3. strategic competence: ability to formulate, represent, and solve mathematical problems;
4. adaptive reasoning: capacity for logical thought, reflection, explanation, and justification;
5. productive disposition: habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efforts (National Research Council, 2001, p. 5).

Each of the five strands "supports and reinforces the others," noted the committee, so care should be taken to develop each of them concurrently (National Research Council, 2001, p. 409). Furthermore, "Textbooks and other instructional material should develop the core content of school mathematics in a focused way, in depth, and with continuity in and across all grades, supporting all strands of mathematical proficiency" (National Research Council, 2001, p. 421).

Evidence of the interconnectedness of these five strands of mathematical proficiency can be found throughout the research on mathematical learning in the elementary and middle school grades. For example, studies of successful instructional techniques in high-poverty classrooms found that striking a balance between teaching for conceptual understanding and teaching for procedural fluency contributed to the development of both fluency and problem-solving skills in high- and low-achieving students (Knapp, 1995, Zucker, 1995, as cited by Fuson, 2003). Summarizing the results of these studies, Fuson reported that teachers who were successful in eliciting higher performance in their students were those who focused on alternative methods to finding solutions, used "multiple representations and real-life situations to facilitate meaning-making," modeled skills that could uncover "the meaning of mathematical problems or methods," and were "supplied a 'healthy dose' of skills practice (Fuson, 2003, p. 89). Similarly, a comparative study of seventh graders in two mathematics classes concluded that such a balance matched well to students' effective use of metacognitive strategies (Lester, Garofalo, and Kroll, 1989, as cited by Schoenfeld, 1992).

Furthermore, numerous research studies on elementary school children's learning of arithmetic have found that "instructional programs that emphasize conceptual development, with the goal of developing students' understanding, can facilitate significant mathematics learning without sacrificing skill proficiency" (Carpenter, Fennema, Peterson, Chiang, and Loef, 1989, Cobb et al., 1991, Cognition and Technology Group at Vanderbilt, 1997, Hiebert and Wearne, 1993, 1996, Hiebert et al., 1997, Kamii and Joseph, 1989, Knapp, Shields, and Turnball, 1992, Mack, 1990, Wearne and Hiebert, 1988, Wood and Sellers, 1996, as cited by Hiebert, 1999, p. 14). In addition, research suggests that "students can learn new concepts and skills while they are solving problems" (Hiebert, 1999, p. 14). Studies on alternative ways of teaching elementary mathematics have shown that conceptual understanding can be enhanced when teachers treat the development of a procedural skill, itself, as a problem for students to solve (Hiebert et al. 1996, as cited by Hiebert, 1999, pp. 14-15, Grevemeijer and van Galen, 2003).

Conceptual Understanding and Procedural Proficiency

Research from the cognitive sciences has found that the order in which one teaches concepts and procedures is critical to students' success with mathematics.

- Instruction designed to provide conceptual understanding should come first.

- Learning about and practicing procedures should come second for students.

A review of cognitive-science studies has found that focusing on procedural fluency before teaching for conceptual understanding puts students at a disadvantage for developing their conceptual understanding and problem-solving skills in the areas of multidigit addition and subtraction, for instance (Fuson and Briars, 1990, Hiebert and Wearne, 1996, as cited by Siegler, 2003). In exploring why, related studies have suggested that by memorizing and practicing procedures intensively, students have difficulty going back later and understanding them conceptually (Brownell and Chazal, 1935, Mack, 1990, Resnick and Omanson, 1987, Wearne and Hiebert, 1988, as cited by Hiebert, 1999).

Additional supports for these findings come from a comparative study of six classes of fifth-graders that was designed to explore the effects from the sequence of instruction on learning of area and perimeter of squares, rectangles, triangles and parallelograms (Pesek & Kirshner, 2000). In a qualitative analysis of interviews with students, the researchers found that cognitive, metacognitive, and attitudinal interferences all interrupted their ability to understand concepts for students who were taught procedures before receiving instruction focusing on conceptual understanding. Students exposed only to conceptual instruction demonstrated superior performance on a posttest requiring application of both procedural skill and conceptual understanding, compared to students who received procedural-before-conceptual instruction. This learning advantage approached statistical significance. (Pesek and Kirshner, 2000). Another comparative study, involving fifth graders, found that students who had received conceptual instruction prior to procedural instruction made significantly greater gains in both areas than students who received procedural instruction first as well as students who didn't receive either type of instruction (Rittle-Johnson and Alibali, 1999, as cited by Siegler, 2003). In a report on a study exploring how students learn whole-number mathematics, Fuson (2003) explained why the balanced approach is so effective in improving students' mathematical proficiency: Students can learn to understand and explain computational methods if these methods are approached as sense-making endeavors. Educators can call on problem solving from the beginning to give meaning to computations; then they can continually intertwine the two as methods and problem solving become consolidated. Providing opportunities both for discovery and for practice improves student achievement. Students can learn both concepts and skills by solving problems. Focusing instruction on the meaningful development of mathematical ideas increases the level of student learning. It is important to emphasize meaning and how the idea, concept, or skill is connected to other mathematical ideas (Grouws & Cebulla, 2000).

Research has also explored how to utilize students' prior, informal, mathematical understanding to good advantage. For example, a study of fifth graders learning to multiply and divide fractions found that encouraging students to frequently return to their initial conceptions was a viable way for students to develop understanding of complex content domains (Mack, 2001). Other studies of elementary mathematics instruction have determined that the use of visual supports, such as drawings and/or manipulatives, is an effective way to develop students' understanding of mathematical concepts (Mack, 2001; Beishuizen, Gravemeijer, and van Lieshout, 1997, Carpenter, Franke, Jacobs, and Fennema, 1998, Fuson and Briars, 1990, Fuson et al. 1997, as cited by Fuson, 2003).

A review and synthesis of 30 years of research on computational methods showed that learners "move through progressions of methods from initial, transparent, problem-modeling, concretely represented methods to less-transparent, more problem-independent, mathematically sophisticated, symbolic methods" (Fuson, 2003, p. 71). To assist students in making this gradual progression to mastery of "well-structured skills" such as basic mathematical algorithms, Rosenshine and Stevens reviewed correlational and experimental studies on teacher effects conducted from 1955 to 1980 (Rosenshine & Stevens, 1986, as cited by Rosenshine, 1995, p.264). They concluded that teachers should follow this instructional sequence:

- Start with a "short review of previous learning";
- Summarize the learning objectives for students;
- Present step-by-step instruction, with interspersed practice;
- Offer "clear and detailed…explanations";
- Provide all students with lots of "active practice": first teacher-guided practice, then later on, independent practice;
- Ask many questions and obtain answers from all students; and
- Provide systematic feedback and corrections to students in response to their work (Rosenshine and Stevens, 1986, as cited by Rosenshine, 1995, p.264).

Strategic Competency through Problem-Solving Skills

Research points to several specific strategies as being effective in helping students become mathematical problem solvers. A meta-analysis of 487 research reports on four areas of problem solving found that training students in the subskills of selecting the correct operations, translating verbal statements to mathematics, and using diagrams improved their problem-solving performance. Furthermore, reflecting how important representing the problem is as the initial activity toward a problem's successful solution, the meta-analysis showed that training students in diagram drawing delivered the largest performance improvement, and second was providing physical objects for them to use in modeling (Hembree, 1992).

To become good problem solvers, it is important for students to acquire "a mathematical point of view," noted Schoenfeld in his synthesis of research on cognition and mathematical proficiency (1992, p. 344). This can be achieved by creating a community of mathematical practice in the classroom, characterized by collaboration and discussion (Schoenfeld, 1992). For example, discussing solutions to routine problems can provide students with "learnable and usable" problem-solving strategies (Schoenfeld, 1992, p. 354). Based on their review of research on instruction in mathematical problem-solving, Heller and Hungate (1985, as cited by Schoenfeld, 1992) concluded that discussions about problem-solving should

- Make tacit processes explicit;
- Involve students in talking about processes;
- Provide a balanced focus on both qualitative understanding and knowledge of specific procedures, including their component procedures; and
- Be followed by guided practice (Heller & Huntgate, 1985, as cited by Schoenfeld, 1992).

Schoenfeld (1992) also concluded that continually examining students' progress is another key to success in mathematical problem solving. He found that coaching, guided practice, and active interventions by teachers helped students learn to self-monitor their own problem-solving processes and to more quickly abandon paths that were unlikely to lead to a correct solution.

Mathematical problem-solving tasks are inherently less structured than algorithmic procedures. Research on less structured learning tasks (Collins, Brown, & Newman, 1990, Pressley et al., 1990, and Rosenshine & Meister, 1994, as cited by Rosenshine, 1995) suggests that cognitive strategies—guiding strategies that support students as they develop the ability to solve less structured problems—can be effective if teachers:

- "Provide procedural prompts";
- "Teach the cognitive strategies using small steps";

- "Model the process of using the cognitive strategy";
- "Guide student practice" in using the cognitive strategy (Rosenshine, 1995, p. 266-267).

Finally, a recent review of mathematics education research from a cognitive-science perspective found that students who used a greater variety of strategies for solving problems also tended to learn better subsequently (Alibali & Goldin-Meadow, 1993, Chi, de Leeuw, Chiu, & LaVancher, 1994, Siegler, 1995, as cited by Siegler, 2003). This occurs, in part, noted Siegler (2003), because the greater variety enables students to cope with whatever kinds of problems they encounter.

Adaptive Reasoning

Adaptive reasoning refers to one's "ability to think logically about the relationships among concepts and situations," explain the authors of Adding It Up (National Research Council, 2001, p. 129). In mathematics, they noted, "adaptive reasoning is the glue that holds everything together," used by students "to navigate through the many facts, procedures, concepts, and solution methods and to see that they all fit together in some way" (p. 129). Adaptive reasoning, for example, is how students determine if a particular solution strategy will be effective. Hembree's (1992) meta-analysis of 487 studies found that, in this order, students' skill with analogies, reasoning ability, inferences, and critical thinking most strongly correlated to high problem-solving performance in mathematics. Other research noted that analogical reasoning, metaphors, and similar mental methods served students by first helping them understand mathematical problems and then as sources of problem-solving techniques (English, 1997, as cited by National Research Council, 2001).

Productive Disposition

Integral to the NCTM's vision of math instruction is that all K-8 students can be mathematically proficient (National Research Council, 2001). Teachers and students both have to truly believe it; a classroom's activities and atmosphere have to exude it. Fortunately, research has identified what motivates students to learn: they have to have the expectation that they can learn, and they have to appreciate that what they are learning is valuable (Feather, 1982, as cited by NRC, 2001). It is possible to turn this positive premise into practice. Research has thus determined that teachers will motivate students by:

- Designing tasks and goals to be achievable with a reasonable investment of effort;
- Providing appropriate scaffolding (e.g., physical manipulatives, visuals, dialogue) along the way; and,
- Highlighting why and how students will find the learning valuable (National Research Council, 1999, as cited by National Research Council, 2001).

A "productive disposition towards mathematics is a major factor" in determining students' success, concluded the authors of Adding It Up (NRC, 2001, p. 131).

As noted by the Mathematics Learning Study Committee, "The role of practice in mathematics is to be able to execute procedures automatically, without conscious thought—to achieve automaticity" (National Research Council, 2001, p. 351). Practice doesn't have to be disjointed, noted the committee. Students can easily be reinforcing previous skills within the context of solving new problems during practice (National Research Council, 2001). Practice improves a student's speed and accuracy of performance on cognitive tasks, research on expert performance has shown, when certain conditions are met:

- The student must be motivated to attend to the task;
- The task must be deliberately designed to build on the learner's pre-existing knowledge; and,

- The learner must receive immediate, informative feedback on their performance (Welford, 1968, as cited by Ericsson, Krampe, & Tesch-Römer, 1993).

Cognitive science research also suggests that distributing practice sessions over time leads to improved memory retention of facts (Donovan & Radosevich, 1999, as cited by Willingham, 2002). Furthermore, the longer the time between practice sessions, the fewer practice sessions are needed to achieve retention (Bahrick & Phelps, 1987, as cited by Willingham, 2002). Ericsson et al. (1993) concluded that spacing apart sessions (typically one session per day) of deliberate practice of behaviors specifically "designed to improve current level of performance" allows for assimilation and avoids "motivational burnout" (pp. 368, 371). Children need "scaffolded practice" to become more fluent in orchestrating the steps in any algorithm, noted a review of the research on whole-numbers learning (Fuson, 2003, p. 72). Similarly, Rosenshine's (1995) summary of research on teacher effects deemed guided practice an essential transitional step between demonstration and independent practice. His other conclusions concerning guided practice were as follows:

- All students need to be involved in extensive guided practice, and it should continue until students reach a high level of success.
- Guided practice should start with teacher modeling and include a variety of techniques, including problems worked out in front of the class, checking for understanding, whole-class discussion, and pair or small-group work.
- Teachers should provide frequent, immediate feedback during guided practice (typically after each problem is completed) to limit the development of misconceptions. Differentiated feedback for different categories of student responses should also be provided.
- Independent practice should occur after sufficient guided practice so that students do not end up practicing errors and misconceptions.
- Guided practice is advantageous in helping students to develop cognitive strategies that effectively apply to mathematical problem-solving tasks (Rosenshine, 1995).

How* Florida Math Connects *Reflects the Research on Attaining Mathematical Proficiency

Florida Math Connects provides ample opportunity for students to develop the five strands critical for mathematical proficiency as identified by the research:

- Conceptual understanding
- Procedural fluency
- Strategic competence
- Adaptive reasoning
- Productive disposition

The series develops students' conceptual understanding of core mathematical ideas primarily through hands-on experience involving visual supports and models, and related discourse. The program is rooted in the principle that conceptual understanding is of paramount importance. In the primary grades, the curriculum provides teachers with a related activity in the Teacher Edition for every lesson. For example, specific manipulatives provide a concrete experience and discourse whenever a new concept is introduced. These engaging tasks are implemented before addressing procedural fluency, as recommended by the research.

Also, as recommended by the research, McGraw-Hill's *Florida Math Connects* series takes a balanced approach to addressing the five strands. It presents logical sequence of instruction, typically starting with the conceptual, making the link to procedural algorithms and focusing on procedural fluency, then applying the concepts and procedures in problem-solving situations. Along the way, students have opportunities to reflect, explain, and justify their thinking in discussion and in writing, and to apply what they are learning to real-world scenarios, which help students come to view mathematics as making sense and having value.

The strands of mathematical proficiency are developed concurrently by the *Florida Math Connects* series. While there is an appropriate balance among the strands, conceptual development precedes the focus on procedural proficiency. The series demonstrates these research-based characteristics with respect to practice and feedback:

- Guided practice starts with teacher modeling and includes a variety of techniques.
- Guided practice is provided as an essential transitional step between demonstration and independent practice.
- Practice is distributed over time.
- Student understanding is checked frequently to limit the development of misconceptions.
- Ample background on math concepts and procedures is included in the Teacher's Edition.

Each chapter is well-organized and arrayed in a logical sequence to promote teaching and learning. Multi-part lessons allow for in-depth teaching for enduring understanding.

Another of the *Florida Math Connects* series' guiding principles is that sufficient practice opportunities should be provided to enable students to master essential procedural algorithms and problem-solving strategies, but students should not be overwhelmed with too much mindless and counterproductive drill.

The series logically interweaves the five critical strands of mathematical proficiency and strikes an appropriate balance among the strands. Also, as recommended by the research, key math concepts consistently are introduced before a focus on procedural fluency.

McGraw-Hill's *Florida Math Connects* adheres to specific research-based strategies for developing conceptual understanding, procedural fluency, and strategic competence. Problem-solving lessons provide the experience students need to develop adaptive reasoning skills and a productive disposition towards mathematics. The program provides ample opportunities for meaningful practice and teacher feedback, and it provides teachers with sufficient background to give students meaningful feedback.

Mathematical Concepts and Skills

Also important in attaining mathematical proficiency is attention to the critical mathematical concepts and skills. This section is divided into 5 parts—Number & Operations, Algebra, Geometry, Measurement, and Data Analysis & Probability. Specific points taken from the research are enumerated for each and the *Florida Math Connects* features which are based on this research are listed.

Number & Operations

Number and Operations includes both skill efficiency (also called procedural or computational fluency)—the rapid, accurate performance of mathematical procedures—and conceptual understanding—comprehension of concepts, operations, and relationships (Kilpatrick, Swafford, & Findell, 2001). It also includes the effective organization of this knowledge and the ability to apply it (Donovan & Bransford, 2005).

Number is central to most of mathematics. Not only the skills and concepts, but also the methods students use to develop number sense, such as concrete objects, drawings, explanations, and justifications, affect their understanding of and attitude toward mathematics. Computational fluency with fractions and decimals is a valuable everyday skill for dealing with measurements. Rational number concepts form a basis for ratio and proportion. In algebra, generalized forms of fraction operations are used to manipulate variables.

Teachers should know and build on children's existing knowledge and skills, such as counting (Clements & Sarama, 2007; Griffin, 2005) and emphasize students' understanding and explaining, that is, "math talk" (Fuson, Kalchman, & Bransford, 2005; Fuson, 2003). Classroom activities can and should support both skills and conceptual understanding (Kilpatrick et al., 2001). Activities should encourage metacognitive processes (Griffin, 2005).

Numbers and Counting

- Human infants have a rudimentary ability to perceive and represent numbers (Dehaene, 1997).
- Humans perceive up to three or four objects without having to count them (this is called *subitizing*) (Dehaene, 1997).
- Basic number sense, numerals, and number words are processed in three different parts of the brain: the space and location area, the visual area, and the language area (Dehaene, 1997).
- Children often learn the four interrelated aspects of early numerical knowledge separately but gradually connect them (Clements & Sarama, 2007):
  - recognizing and naming how many objects are in a small group,
  - learning the ordered list of number words,
  - enumerating objects (assigning number words to objects one-to-one), and
  - understanding that the last number word used tells the number of objects.
- Children can use their counting skills to solve simple word problems by acting out the situation, that is, by modeling it (Kilpatrick et al., 2001).

Mental Arithmetic and Estimation

- Mental arithmetic should encourage children to reason about the problem situation and the numbers, to use properties of arithmetic, and to select procedures to simplify computation (Kilpatrick et al., 2001).
- Estimation is a complex activity. Children use several strategies and methods to estimate, including number lines. Estimation strategy use improves with age (Vershaffel et al., 2007).

Whole Numbers—Single-Digit Operations

- Pre-school children without instruction can do simple addition by counting. Multiplication is much more difficult because counting is not as helpful (Dehaene, 1997).
- Children (and adults) use a variety of strategies and reasoning processes to do operations with single-digit numbers (Kilpatrick et al., 2001).
- Children do not simply memorize basic facts but, beginning with concrete methods, move through progressively more advanced, symbolic methods to find answers (Fuson, 2003).
- "Meaningful arithmetic" increases retention, increases use of mathematical understanding and skills, reduces the amount of practice needed, helps prevent errors in calculations, and helps students become independent learners (Brownell, 2004).
- Skills and concepts develop simultaneously (Vershaffel et al., 2007; Baroody, 2003).

- Effective methods begin with *counting all*, and progress to *counting on*, and then using strategies such as *make a ten* and *doubles* (Fuson, 2003; Kilpatrick et al., 2001).
- Understanding relationships, such as the commutative property, promotes learning of basic number facts and also helps organize this knowledge as a network of interconnected relations (Kilpatrick et al., 2001).
- Understanding the inverse relationship between addition and subtraction, *related facts*, is a step toward algebra (Baroody, 1999).
- Children progress through a sequence of multiplication procedures, including *make equal groups*, *skip-counting*, and *finding patterns* (Kilpatrick et al., 2001).
- The practice that is essential for developing fluency with single-digit calculations can occur while solving word problems. Practice should follow initial activities that support understanding and use thinking strategies (Kilpatrick et al., 2001).

Whole Numbers—Multi-Digit Algorithms

- Understanding the base-ten number system is the basis for developing computational fluency. Understanding multi-digit algorithms increases accuracy (Siegler, 2003).
- Instruction should use a variety of approaches, including using base-ten blocks to represent hundreds, tens, and ones, emphasizing counting by tens, and involving mental computation methods (Hiebert et al., 1997).
- Instruction should be a blend of conceptual and problem-solving activities to master basic skills, conceptual learning, and mathematical thinking (Baroody, 2003; Vershaffel et al., 2007).

Integers

- Physical models, like colored counters and arrows on number lines, help students understand negative numbers and at earlier ages than previously thought (English, 1997; Kilpatrick et al., 2001; Moreno & Mayer, 1999; Thompson, 1988).
- Students perform better when problems are presented in context, such as scores or debts, and when using number lines (Moreno & Mayer, 1999).
- Approaches that use appropriate models for integers and operations on integers and that gradually develop these models and link them to symbols are best (Kilpatrick, Swafford, & Findell, 2001).

Fractions and Decimals

- The idea of sharing equal parts or equal amounts is a good starting point for understanding fractions (Streefland, 1993).
- Effective activities begin by helping students develop meaning for representations of fractions, using physical models, pictures, and realistic contexts, and then connecting these with mathematical symbols (Kilpatrick et al., 2001).
- Many students find rational numbers, especially fractions, challenging because a rational number has "multiple personalities." It is a
  - Part-whole relation (3 parts out of 4),
  - Quotient (3 divided by 4),
  - Measure (3/4 of the way from the beginning to the end),
  - Ratio (3 red bicycles for every 4 blue bicycles), and
  - Operation that changes size (3/4 of 20 objects) (Lamon, 2007; Kilpatrick et al., 2001; Kieran, 1988).
- Activities should help students recognize these distinctions and construct relations among them (Kilpatrick et al., 2001).

- The part-whole, measure, and ratio approaches have resulted in higher levels of mastery in proportional reasoning and computation (Lamon, 2007).
- Through many years of research, the Rational Number Project, which focuses on students in 4th and 5th grades, has found that the optimum curriculum for learning fractions includes:
  - Active involvement with multiple concrete models,
  - Concrete models used over long periods of time,
  - Opportunities to talk about the activities, and
  - A focus on concepts prior to work with symbols and algorithms (Behr, Harelk, Post, & Lesh, 1992; Cramer & Henry, 2002; Post, Cramer, Behr, Lesh, & Harel, 1992).
- Researchers recommend emphasizing concepts from the beginning, before introducing rote procedures, and using objects or contexts that help students make sense of fraction operations (Kieran, 1988; Kilpatrick et al., 2001; Mack, 1990).

In the *Florida Math Connects* Grades K–2 Curriculum

- Each chapter in grades K–2 begins with *Are You Ready for Chapter X?*, a diagnostic assessment of pre-requisite skills.
- *Focus on Math Background* in each lesson of the Teacher Edition includes tips on using students' prior knowledge.
- *Common Error!* features for each lesson (K–2) alert teachers to typical student misconceptions and errors, along with tips for correcting them.
- *Talk About It* encourages students to explain their thinking.
- *Problem Solving* is included in each chapter.
- Single-Digit Operations
  - Addition is introduced using colored counters, workmats, and both *part-part-whole* and *joining* approaches.
  - Strategies like *counting on*, *make a ten*, and *doubles* are introduced in concrete terms, and later used mentally.
  - Relationships, such as the commutative property, are introduced early to facilitate calculations and to organize students' thinking about operations.
  - Addition and subtraction are related as *fact families*.
  - Multiplication is presented in a variety of ways, including *equal groups, repeated addition,* and *arrays*.
  - Practice takes place through computations using symbols, games, and problem-solving situations.
- Multi-Digit Algorithms
  - Base-ten blocks are used extensively to help students understand the number system and algorithms.
  - Teachers are reminded that students vary as to which manipulatives or strategies they find most helpful.
  - Hands-on lessons using base-ten blocks, as well as problem-solving activities contribute to basic skills and concept learning.
- Estimation
  - Estimation lessons begin in Kindergarten and use drawings, number lines, and rounding.
  - Estimation is explained in Focus on Math Background in the Teacher Edition.
- Fractions are introduced using pattern blocks and drawings.

In the *Florida Math Connects* Grades 3–5 Curriculum

- Models, applications, and concepts, along with skills practice are involved in whole-number operations.
- Number lines, as well as models, help students compare fractions.
- Fractions as parts-of-a-whole are introduced using concrete models, like paper plates, grid paper, and fraction tiles, as well as with drawings and symbols.
- Scaffolding questions guide teachers in helping students see patterns or general rules for operating with fractions before the procedure is presented.
- *Talk About It* questions help students organize and clarify their thinking on key concepts.
- Concrete models for fractions are used continually to help students make connections between concepts and symbolic representations.
- Integers are introduced with real-world problem situations and number lines.

In the *Florida Math Connects* Courses 1–3 Curriculum

- Fractions and decimals are related using place value and the base-ten system, along with mental math and division procedures.
- Fractions and percents are related using 10x10 grids.
- Scaffolding questions guide teachers in helping students see patterns or general rules for operating with fractions before the procedure is presented.
- *Writing In Math* questions help students organize and clarify their thinking on key fraction concepts.
- Ratios and their representation by fractions are introduced in real-world problem situations.
- Ratios are applied to scale drawings.
- Integer concepts are introduced with real world problem situations and number lines.
- Operations with integers begin with students using both colored counters and number lines.

Algebra

Algebra has been described and defined in many different ways over the past twenty years, including generalized arithmetic, symbolic procedures to solve problems, the study of functions, and a modeling language. Algebra is an activity—something you do. It is a language to express meaning (variables and equations), it is rule-based (factoring, simplifying expressions, equivalence), and it is a tool for problem-solving, modeling, justifying, making conjectures, and studying change using functions (Kieran, 2007). Early algebra provides informal introductions to various approaches to algebra: understanding patterns (algebra as generalized arithmetic); understanding relations and functions; and using mathematical modeling, especially in situations involving change (Kaput, 2007; Kaput, Blanton, & Moreno, 2007). Algebra activities include representing, transforming (using rules), generalizing, and justifying (Kilpatrick, Swafford, & Findell, 2001).

Algebra is a foundation of mathematics because it provides the language and structure for analyzing quantitative relationships, for modeling, for problem solving, and for stating and proving generalizations. It is essential in most areas of science and engineering. It is important for all students and crucial for college-intending students (RAND, 2003). Numerous research studies from the 1980s and 1990s have documented the difficulties students encountered in their first algebra course in high school (Smith, 2003). More recently, several national and international groups have recommended including algebraic concepts and algebraic reasoning throughout the K–12 math curriculum (Carraher & Schliemann, 2007).

Researchers find that arithmetic and algebra are not completely separate; algebraic reasoning is needed to fully understand arithmetic. Algebra can be considered as generalized arithmetic. Algebra for young learners needs different approaches from traditional algebra: use of tables, number sentences, graphs, and verbal descriptions; contexts that include physical quantities; and use of functions to connect many topics (Carraher & Schliemann, 2007). Children in elementary grades can generalize arithmetic properties without using algebraic symbols; for example, missing-addend problems implicitly use the additive inverse property (Schifter et al., 2007). Comparing quantities, even before working with numbers, seems to encourage algebraic reasoning in young children (Dougherty, 2007).

- Algebraic methods are implicit in children's generalizations of number sentences, such as 3 + 0 = 3, 8 + 0 = 8; "when you add zero, you get the number you started with" (Carpenter et al., 2003).
- Recent research with second graders and fourth graders showed that children can find rules for patterns and relationships using various representations and can offer justification for their conjectures (Moss et al., 2006).
- Recognizing a pattern means viewing it as a static object. Extending a pattern involves change and identifying a unit of change (Smith, 2003).
- Students have difficulty interpreting the equals sign, often (mistakenly) thinking that it means "makes" as in 5 + 3 = 8, "five plus three makes eight." Number sentences, such as 9 = 5 + 4 or 3 + 6 = 5 + 4, help children learn the meaning of the equals sign (Carpenter, Franke, & Levi, 2003).
- Recent research explores how meaning is derived from various representations, the role of properties within symbol manipulation, and the use of technology (Kieran, 2007).
- Students need to understand the equals sign as a relational symbol of equivalence in order to succeed in algebra. Many middle school students do not have this view of the equals sign (Knuth et al., 2006; Kilpatrick, Swafford, & Findell, 2001).
- Students have difficulty dealing with equations used in various ways—as a formula (A = lw), an equation to solve (4x = 20), an identity (sinx/cosx = tanx), a property (a + b = b + a), or a function (y = 3x) (Chazan & Yerushalmy, 2003; Usiskin, 1988).
- Learning rule-based procedures without creating meaning for the rules or learning when to use them leads to choosing inappropriate strategies (Kilpatrick, Swafford, & Findell, 2001; Wenger, 1989).
- Language plays a crucial role in algebra (RAND, 2003; Wheeler, 1996).
- Connecting algebra with arithmetic, geometry, and statistics can help build student understanding (RAND, 2003).
- Word problems are easier for students than solving the corresponding equations without context, contrary to what most teachers believe (Nathan & Koedinger, 2000). In word problems, students tend to revert to arithmetic problem-solving methods instead of using algebra (Stacy & MacGregor, 1999; Kieran, 2007).

Functions

- A *function* pairs each x value with a corresponding y value, for example 2 and 8. This is called the *correspondence* approach. Alternately, the values in the x column increase by 1, while values in the y column increase by 3. This is called a *covariational perspective.*
- Teaching functions should build on students' prior knowledge, starting with a simple familiar context and having students express concepts in their own language; develop conceptual understanding, procedural fluency, and connected knowledge; and help students become self-regulating problem solvers, using multiple strategies and representations (Kalchman & Koedinger, 2005).

- Children in elementary grades can understand and express simple functions, that is, change in one quantity related to change in another quantity. Along with patterns, elementary level mathematics should include functional thinking (Blanton & Kaput, 2004).
- When students begin using function tables, they typically begin by noticing covariation, rather than ordered pairs (Smith, 2003; Confrey & Smith, 1995). The covariation approach is useful in modeling (Smith, 2003).
- Students have difficulties distinguishing functions from other relations and interpreting graphical representations of functions (RAND, 2003; (Leinhardt, Zaslavsky, & Stein, 1990).
- Student understanding is increased when functions are defined as relationships between variables, introduced earlier, used to model concrete situations, and expressed in various representational systems (O'Callaghan, 1998).
- Students who work with multiple representations of functions develop a more comprehensive understanding of them (Leinhardt, Zaslavsky, & Stein, 1990).

In the *Florida Math Connects* Grades K–2 Curriculum

- Beginning in Kindergarten, children recognize and extend patterns that involve shapes and numbers of objects. They explain the patterns.
- Children show which set of objects is more or less or equal to another set, before they count the objects.
- Number lines are introduced in Kindergarten and used in each grade to order numbers and to add and subtract.
- Skip counting involves using number lines and finding patterns.
- Number sentences use equals signs and operation symbols. Children write what +, -, and = mean. The equals sign means "is equal to."
- Number sentences in which children write the missing numbers show operations on both sides of the equals sign and lead to solving equations.
- Arithmetic properties, such as commutative property of addition, are introduced with concrete objects. Children explain why you can "add in any order."
- Fact families relate addition and subtraction as inverse operations.
- Multiplication as repeated addition is explored with concrete objects.

In the *Florida Math Connects* Grades 3–5 Curriculum

- Students describe and use properties of addition and multiplication. Fourth and fifth graders work with the distributive property.
- Students tell whether number sentences are true or false and compare expressions. Number sentences have two numbers on both sides of the equals sign.
- Third graders work with numerical expressions and equations. They translate from words to expressions.
- Fourth graders work with variables in expressions and equations, using counters and cups.
- Students solve equations using models (counters and cups), mentally, and using symbols.
- Students use function tables to find and extend a pattern or rule. Students use input and output to describe functions.
- Integers are introduced using number lines.
- Students graph points on a grid.
- Students graph functions on a grid.
- Students use equations (area formulas) to model area of rectangles.

In the *Florida Math Connects* Courses 1–3 Curriculum

- The equals sign and solutions of equations are defined and explained.
- Properties, like the distributive property, are emphasized to help students develop the meaning behind symbol manipulation.
- Number lines aid in ordering and adding integers.
- *Vocabulary Link* features provide both the everyday and the mathematical meaning of key terminology.
- Symbolic fluency is promoted through a variety of problems, including motivating real-world situations.
- Algebra is well-connected to geometry.
- In *Algebra Lab,* students use cups and counters to model solving equations.
- Students write equations from word problems and then solve the equations.
- Function tables are introduced using the correspondence approach. Later, in studying slope, the covariational approach is used to show the constant rate of change.
- Students write in every lesson, often explaining processes or defining terms in their own words.
- In the *Find the Error* exercises, students learn to monitor their understanding by finding errors in other students' work.
- Students use functions to model concrete situations.
- Students work with multiple representations of functions. They make conjectures about the result of changing a constant in an equation.

Geometry

Geometry is the study of spatial objects, relationships, and transformations (Clements, 2003). Spatial reasoning—the ability to inspect and reflect on objects and relationships—underlies geometry (Battista, 2007). School geometry begins with concrete exploration of characteristics and progresses to more abstract, mathematical analysis. It includes the development of reasoning. Geometric measurement is interwoven throughout geometry (Battista, 2007). Geometry provides a way to interpret our physical environment. It is an essential tool in other areas of mathematics (data graphing, function graphing) and in science, engineering, geography, art, and architecture.

Numerous research studies in geometry have been done, beginning with the influential work of Piaget and Inhelder (1967) that found that children's representation of space is constructed from active manipulation of their environment (Clements, 2003). People form two kinds of concepts, "fuzzy" and formal. Students already have fuzzy concepts for geometric objects. Formal, mathematical definitions may differ from these fuzzy concepts. For example, students may not agree that a square is a rectangle; the fuzzy concept of rectangle does not match the mathematical definition (Battista, 2007).

- The van Hiele theory (van Hiele, 1986; van Hiele, 1959/1985/2004), which defines sequential levels of reasoning in geometry (briefly summarized here for quick reference), is valid and useful in describing K-12 students' geometric concept development (Clements & Battista, 1992; Battista, 2007).
  - Level 1 Visual—recognize shapes but not think about properties
  - Level 2 Descriptive/analytic—characterize shapes by their properties
  - Level 3 Abstract/relational—form definitions; understand and provide arguments (also called *informal deduction*)
  - Level 4 Deduction—construct proofs; use definitions, axioms, theorems.
- Instruction should help students move from their current level to higher levels. In grades K–2, most students are at Level 1 and moving to Level 2 by Grade 2, where they are working with

properties of shapes. In grades 3–5, most students are at Level 2 and are making progress toward Level 3.

- Research indicates that the van Hiele levels may not be discrete; students often use more than one level of reasoning (Gutiérrez et al., 1991). The levels may develop simultaneously and at different rates, depending on maturation and instruction (Battista, 2007).
- Careful use of geometry vocabulary is important. Make clear the distinctions between common usage and mathematical usage (Clements, 2003; Fuys et al., 1988).
- Manipulatives can help students represent and comprehend geometric concepts, but they must be used thoughtfully (Clements, 2003; Clements, 1999).
- The concept of angle develops in stages: situated (such as hills or a folded straw), contextual (hills, roofs, and slope), and abstract (two lines that meet at a point) (Battista, 2007; Mitchelmore & White, 2000).
- Many students have misconceptions about angles, for example, that orientation or length of rays affects angle size, and have difficulties seeing angles in real-world objects (Clements, 2003).
- Students often confuse drawings with the theoretical geometric objects that the drawings represent (Battista, 2007). For example, they assume that lines that look parallel are parallel or that right triangles must have one horizontal side (Clements & Battista, 1992).
- Student difficulties in understanding the connection between linear equations and their graphs could be due to lack of understanding of how coordinates are related to length. Students may see coordinates as simply markers, like margin letters on a map (Battista, 200l; Knuth, 2000). Students who are not operating at Level 3 in high school geometry may have difficulty in learning the basics of mathematical proof.
- Students who receive training and practice in specific spatial skills (e.g., mental rotation, spatial perspective) improve their performance on mathematics tests (Halpern et al., 2007).

In the *Florida Math Connects* Grades K–2 Curriculum

- Children differentiate between rectangles and squares. Teachers are alerted to the common error—that students do not understand that a square is a special kind of rectangle.
- Children recognize both 3D and 2D figures, which is van Hiele Level 1 reasoning.
- Children classify plane shapes by the number of sides and vertices, which is van Hiele Level 2 reasoning.
- Children begin to reason logically about shapes; for example, explaining that all squares are also rectangles. This is van Hiele Level 3 reasoning.
- Vocabulary words are highlighted at the beginning of each lesson, included in the *Math at Home* letter, and reviewed in the *Chapter Review/Test.* The Teacher Editions offer additional tips for helping students master vocabulary, including Foldables®, vocabulary cards, and small-group interviews.

In the *Florida Math Connects* Grades 3–5 Curriculum

- In *H.O.T. Problems*, students decide whether geometric statements are always, never, or sometimes true and explain their reasoning. These promote Level 3 reasoning.
- Students explore the properties of polygons, working at van Hiele Level 2.
- Polygons are presented in non-standard position to help students learn mathematical definitions.
- Key vocabulary words and phrases are listed at the start of each lesson, highlighted in *Key Concept* boxes, and used in each lesson's exercises. The chapter *Study Guide and Review* includes a list of *Key Vocabulary* and a *Vocabulary Check* exercise. Students use *Foldables®* to record vocabulary.

- Students differentiate between right angles and angles that are greater or less than right angles, using drawings of real objects and triangles in various orientations.
- Students use informal deductive reasoning, such as determining the length of the third side of an equilateral triangle when the length of the other sides is known. They use informal if/then thinking, such as if a rectangle has a greater perimeter than another, then does it also have greater area?

In the *Florida Math Connects* Courses 1–3 Curriculum

- Students focus on the properties of shapes, strengthening their van Hiele Level 2 development.
- Key vocabulary words and phrases are listed at the start of each lesson, highlighted in *KeyConcept* boxes, and used in each lesson's exercises. The chapter *Study Guide and Review* includes a list of *Key Vocabulary* and a *Vocabulary Check* exercise.
- Points, lines, and planes are defined in formal terms, in contrast to drawn objects and everyday language.
- Angle concepts are introduced with concrete examples and then move to formal definitions.
- Students categorize polygons based on mathematical definitions.
- Students work with various manipulatives, including nets of 3D objects.
- Working with transformations on the coordinate plane strengthens students' understanding of the coordinate system and lengths.
- In *H.O.T. Problems*, students determine whether statements are true or false and provide evidence to support their answer and explain their reasoning, to help students move to van Hiele Level 3.
- *Writing in Math* asks students to explain geometric relationships, such as when two similar polygons are also congruent. These are van Hiele Level 3 activities.
- *Problem-Solving Investigations* include inductive and deductive reasoning, helping students move toward van Hiele Level 4.
- *Geometry Labs* involve students in using concrete objects to make conjectures about geometric objects.

Measurement

Measurement in grades K–2 includes length, area, capacity, weight, time, and temperature. It includes conceptual foundations, such as *unit*, *standardization*, *tiling*, and *precision*, and the use of simple measurement tools. Measurement in Grades 3–5 includes length, perimeter, area, capacity, volume, weight, time, and temperature. It includes conceptual foundations, such as *unit*, *standardization*, *tiling*, and *precision*, and the use of customary and metric measurement. Measurement in Courses 1–3 includes length, perimeter, area, volume, surface area, angle measure, capacity, weight, time, and temperature. It includes conceptual foundations, such as *unit*, *standardization*, *tiling*, and *precision*, and the use of customary and metric measurement. Measurement occurs in students' and adults' everyday experiences. It connects mathematics and science. Within mathematics, it integrates geometry and number and operations. It is central to many real-world applications (Clements, 2003).

- Through measurement activities, children can learn to think about spatial extent, time, approximation, and number (Clements, 2003).
- Measurement has roots in children's everyday experiences and can develop in the earliest grades (Clements, 2003).
- Conceptual foundations of measurement include the following:
  - Units correspond to what is being measured. (Unit-attribute relations)

        - Units can be reused. (Iteration)
        - Units can fill (or tile) lines, planes, volumes, and angles. (Tiling)
        - Units must be identical, so that you can count them.
        - Standard units facilitate communication.
        - Different-sized units lead to different quantities that represent the same measure. (Proportionality)
        - The sum of the measures of two parts that make up the whole equals the measure of the whole. (Additivity)
        - Scales have an origin (zero point) and have intervals that are equal in length (Lehrer, 2003).
- Researchers have identified levels of reasoning about length. At Level 1, students use visual comparison and often guess measurements. At Level 2, they connect numbers to iterative movements, but make errors in the process. At Level 3, they correctly iterate units and can reason about measurements without the objects (Battista, 2007).
- Learning measurement concepts and skills requires hands-on activities (Preston, 2004).
- Understanding perimeter, area, and volume measure requires knowing
    - what length/area/volume is and how it is conserved,
    - how to measure it by iterating units of length/area/volume,
    - numerical processes (formulas) to determine measures for special shapes, and
    - how these processes (formulas) are expressed in words and symbols (Battista, 2007).
- Drawing tilings of 2D shapes and checking the drawings using physical squares (or in 3D using cubes) promotes student understanding of arrays, which are fundamental to area and volume concepts (Battista, 2007).

In the *Florida Math Connects* PreK–2 Curriculum

- Children use everyday activities and objects to measure time, length, area, capacity, weight, and temperature.
- Children use both nonstandard units (plastic cubes, paper clips) and standard units to estimate and measure length.
- Teachers are alerted to common errors, such as incorrectly aligning the ruler when measuring length.

In the *Florida Math Connects* Grades 3–5 Curriculum

- Through hands-on activities, students explore length measurement in both customary and metric systems.
- Concepts of perimeter, area, and volume are expressed in words, symbols, and models.
- In *Explore* activities, students use grid paper and drawing to study area and cubes to study volume.

In the *Florida Math Connects* Courses 1–3 Curriculum

- Students explore length, capacity, and weight measurements in both customary and metric systems.
- *Measurement Labs* move students from drawings (models) to making conjectures about formulas for area.
- Students use centimeter cubes to explore volume and express volume in words and symbols.

Data Analysis & Probability

In grades K–2, data analysis includes organizing and displaying data and making and interpreting graphs. The main concepts are graphs, centers (mode), and variation (Shaughnessy, 2007). In Grades 3–5 data analysis includes organizing and displaying data, using mode and median to analyze data, and making predictions based on data (Schaeffer, 2000). The main concepts are centers (mode, median, mean), variation, graphs, and comparing data sets (Shaughnessy, 2007). Probability is the study of chance—the chance that certain events will occur. It includes both theoretical and experimental probability. In Courses 1–3, data analysis includes organizing and displaying data, using statistical methods to analyze data, and developing inferences and predictions based on data (Schaeffer, 2000). Probability in Courses 1–3 includes both theoretical and experimental probability.

Statistics, data, analysis, and statistical reasoning are important skills in the workplace as well as in everyday life. Many career fields in science and industry require the use of statistical methods. Working with data helps students understand variability and uncertainty. Statistics and data analysis have connections to arithmetic and number sense, algebra, measurement, and geometry. Probability is increasingly important in applications to science, business, and industry. Informed citizens need an understanding of probability to interpret information on politics, medicine, consumer products, and environmental issues.

Data Analysis

- Understanding and using data graphs involves six behaviors within three levels
  - *Read the data*
    - Recognize components of graphs
    - Speak the language of graphs
  - *Read between the data*
    - Relate tables, graphs, and data
    - Make sense of a graph and avoid personalization
  - *Read beyond the data*
    - Interpret information in a graph
    - Select appropriate graphs (Shaughnessy, 2007; Friel, Curcio, & Bright, 2001)
- No one data display—table or graph—is inherently better; each representation is useful for some purposes and for some data sets. Some graphs are harder to learn (Bright & Friel, 1998).
- Picture graphs help students connect data and real events (Konold & Higgins, 2003).
- Students have more difficulty with frequency bar graphs than with case-value plots (bar graphs of individual items) (Konold & Higgins, 2003; Bright & Friel, 1998; Cobb, 1999).
- Young children find it difficult to move from individual data values to considering the graphed data as a whole, that is; thinking about mode or range (Konold & Higgins, 2003; Russell et al., 2002).
- Teachers should
  - emphasize variability,
  - build on students' intuitive notions of centers (average) and variation,
  - introduce the comparison of two data sets, and
  - keep in mind that statistics is different from mathematics; the context of the data is critical (Shaughnessy, 2007).
- The concept of *mean* develops over many years. Introduce the mean as a *typical value* or a *fair share*, since they build on students' intuitions (Shaughnessy, 2007; Watson & Moritz, 2000; Mokros & Russell, 1995).

- Many students use the mode when the median or mean is more appropriate. Few students understand anything about the median and mean other than how to compute them (Konold & Higgins, 2003; Cai, 1998; Russell et al., 2002).

Probability

- Students show wide variations in understanding and many misconceptions in the concept of chance—possible, impossible, 50-50, likely, unlikely, fair, and unfair (Jones, Langrall, & Mooney, 2007; Watson & Moritz, 2003).
- Concepts of chance and randomness challenge learners of all ages. Students fail to interpret uncertainty in patterns that emerge from repetitions, believe that a person or device (e.g., spinner) can control outcomes, and believe that some sort of order or reason underlies events (Jones et al., 2007).
- Students do not consider sample spaces when determining probabilities, even if they can list the outcomes in the sample space (Jones et al., 2007).
- Probability instruction should
  - begin at an young age and continue throughout school years,
  - emphasize building sample spaces (all possible outcomes),
  - connect probability and statistics,
  - introduce probability through data, and
  - use a problem-solving approach (Shaughnessy, 2003).
- Teachers should
  - take into account student preconceptions and misconceptions,
  - provide instruction in probability reasoning to challenge misconceptions, and
  - use an experimental approach (Jones et al., 2007; Castro 1998).
- The curriculum should include
  - the concepts of chance variation, randomness, and independence,
  - calculating probabilities,
  - terminology and language use,
  - critical reasoning and questioning, and
  - interpretation of the context—the situation (Jones et al., 2007; Gal, 2005).

In the *Florida Math Connects* Grades PreK–2 Curriculum

- Children create real graphs (using actual objects) and picture graphs and collect their own data.
- Children make picture graphs and tally charts. They interpret and then create bar graphs and collect their own data.
- Children begin to explore range and mode, take a survey, use tally marks, and picture graphs. They interpret bar graphs with scales. They compare these ways to show data.

In the *Florida Math Connects* Grades 3–5 Curriculum

- Students make and interpret bar graphs and line plots. They explain how tally charts and lines plots are alike and different.
- Students use graphs to make predictions on more or less likely events.
- Students begin to explore probability by discussing likely, unlikely, and impossible events. They use colored wristbands in a bag to experiment.
- Students collect data, create tally charts, and create frequency tables.
- Students find and compare mode and median. They identify outliers.
- Students compare two data sets using double bar graphs.

- Students learn about outcomes, making an organized list and tree diagrams as ways of showing sample spaces.
- Students select appropriate displays for data.
- Students extend their understanding of probability through hands-on activities.
- They interpret and create line graphs showing change over time.

In the *Florida Math Connects* Courses 1–3 Curriculum

- Students use line plots, stem-and-leaf plots, bar graphs, circle graphs, multiple line graphs, and histograms.
- Students use data and sampling to predict.
- Students describe the similarities and differences among several types of graphs of a data set and explain which representations they prefer.
- Scaffolding questions help teachers focus on students' current understanding of "average." The concept of mean is introduces as "fair share" in a hands-on lab.
- Students use graphing calculators to find means and medians of data sets.
- Ratios, proportions, and percents help students use samples to predict information about a population.
- Students compare data sets using multiple-line graphs and multiple-bar graphs.
- Students apply critical thinking to situations that involve misleading statistics and graphs.
- Students use tree diagrams and tables to investigate sample spaces in a variety of situations. They also use the Fundamental Counting Principle.
- Students use simulations and compare theoretical and experimental probability of simple and compound events.
- Students use graphing calculators and spreadsheets to make histograms, line graphs, bar graphs, circle graphs, and box-and-whisker plots. They learn to select an appropriate display.
- Students work with measures of central tendency (mean, median, mode) and range. They also explore measures of variation—quartiles.

Ensuring Mathematical Proficiency for All Learners

Differentiated Instruction

Ample research has concluded that students find more success and satisfaction in school if they are taught in ways that are responsive to their readiness levels (e.g., Vygotsky, 1986), interests (e.g., Csikszentmihalyi, 1997), and learning profiles (e.g., Sternberg, Torff, & Grigorenko, 1998) (as cited by Tomlinson, 2000). Differentiated instruction is how this translates into classroom practice.

> Every elementary classroom holds a wide range of learners: In most elementary classrooms, some students struggle with learning, others perform well beyond grade-level expectations, and the rest fit somewhere in between. Within each of these categories of students, individuals also learn in a variety of ways and have different interests. To meet the needs of a diverse student population, many teachers differentiate instruction (Tomlinson, 2000).

Differentiated instruction involves varying ones' teaching according to each learner in either (1) content, (2) instructional process, (3) students' products (e.g., papers, projects, computer models), and/or (4) learning environment (e.g., cooperative learning in small groups, grouped by ability) (Tomlinson, 2000). By definition, differentiated instruction always involves ongoing assessment linked to instructional decisions and planning (Tomlinson, 2000). Because differentiated instruction focuses on each learner's varying needs, it is especially well suited for special-needs students.

The quality of the curriculum and instruction used during differentiation is crucial. Teachers need to know how children think in mathematics in order to make appropriate instructional decisions based on what each child knows and can do (Carey, Fennema, Carpenter, & Franke, 1995). High quality curriculum focuses on what experts deem the most essential mathematical concepts and skills. High quality instruction incorporates lessons, tasks, and materials designed to ensure that students (1) grapple with essential concepts and skills, (2) find the learning experiences relevant and interesting, and (3) are engaged in active learning experiences (Tomlinson, 2000). Research demonstrates that participation—what students do and become within the classroom community—and identity—students' perception of themselves and others' perception of them as doers of mathematics—are two critical aspects of classroom practices (Diversity…, 2007). There are specific strategies that encourage girls in mathematics, including:

- "Teach students that academic abilities are expandable and improvable.
- Provide prescriptive, informational feedback
- Expose girls to female role models who have succeeded in math and science.
- Create a classroom environment that sparks initial curiosity and fosters long-term interest in math and science.
- Provide spatial-skills training" (Halpern et al., 2007).

Research has also shown that flexible groupings can improve the mathematical achievement of special-needs students (Slavin, Madden, & Leavey, 1984, as cited by Mastropieri et al., 1991; Mastropieri et al., 1991; Secada, 1992; Slavin, Madden, Karweit, Livermon, & Dolan, 1990, as cited by Secada, 1992). Teachers can use flexible grouping to deliver a variety of differentiated learning environments in their classrooms, including small workgroups, cooperative learning groups, cross-grade groups, between-grade groups, grouping by ability for guided or independent practice, as well as whole class, and individual practice settings (Tomlinson, 2000).

Furthermore, Burris, Heubert, and Levin (2006) found that completers of advanced math courses increase in all heterogeneous grouped students including minority and low SES students. The same conclusion was reached for all students at whatever initial achievement level. Initial high achievers performed the same as counterparts in homogeneous groups. Rates of participation and test scores improved in all groups.

In yet another piece (Zehr, 2006), the changes brought by NCLB are debated as to their positive versus negative effects. The major point made is that many schools may be responding to NCLB in math and reading by narrowing the curriculum to focus only on test scores. With respect to the No Child Left Behind legislation, Kim and Sunderman (2005) suggest that use of the mean proficiency score may have a disparate effect on schools with low income children. Accountability rules for the sub groups can over identify racially diverse schools as failing to meet proficiency levels. The use of multiple indicators of performance may produce a more accurate assessment of student proficiency. Multiple measures could include a measure of improvement.

English Language Learners (ELL)

In his review of the research on how race, ethnicity, social class and language might affect student achievement in mathematics, Secada (1992) found a relationship between the amount of proficiency in a given language and mathematics achievement (Fernandez & Nielson, 1986, Duran, 1988, Secada, 1991b, as cited by Secada, 1992). To support academic achievement for non-native speakers of English and other diverse learners, Secada recommended:

- Intervening early;
- Providing ongoing extra support materials and strategies;
- Using a student's native language for instruction;
- Using a structured curriculum or focus teaching on basic skills;
- Using small-group instruction, preferably in cooperative learning settings; and
- Carefully grouping students by specific ability, if necessary (Secada, 1992).

According to an article by McElroy (2005) teachers need to expand their teaching tools to assist ELL students in content areas such as math. The article describes a website that teachers can use to work with ELL students. Material specific to ELL are presented along with teaching tips. While focused more on the language learning of ELL, several recommendations are made by Goldenberg (2006). The instructional practices seen as having a positive impact specific to math include:

- clear instructions and expectations
- additional opportunities for practice
- extended explanations

Abedi (2004) reports on the difficulty of assessment of math and reading for ELL, especially as related to the No Child Left Behind (NCLB) act. Factors are presented as issues including the sparse ELL populations in some states, subgroup lack of stability, linguistic complexity of assessment tools and lower ELL baselines requiring greater gains. The implications are that unless these issues are considered, schools with large ELL populations face unfair and undue pressure under NCLB. In a similar vein, the American Federation of Teachers (AFT, 2006) point out that NCLB challenges faced by ELL students in math and reading include defining ELL subgroups and the paucity of natural language assessment. The AFT identified four changes needed:

- appropriate tests for ELL students
- relevant and valid testing of ELL students in English
- clarifying assessment of proficiency versus math and reading skills
- clarifying existing policies regarding ELL immigrant and non-immigrant groups

Intervention

Before examining the results of studies themselves, it is useful to to note that in the research regarding math intervention, Seethaler and Fuchs (2005) analyze the literature in terms of the efficacy of studies completed. They found that randomized, controlled designs were clearly underrepresented in the literature. They conclude that to truly assess efficacy, study methodology might need to improve. In a related article, Augustyniak, Murphy and Phillips (2005) argue that the research on the definition of a math disability is lacking with respect to identification of core deficits. They identify the core areas needing further explanation as numerical skills, visual/spatial deficits, cognitive skill development (memory retrieval, working memory, speed of processing, attention regulation, problem solving) and social cognition. Mazzocco (2005) reviewed research regarding practices of early identification and intervention for students with math diffciulties. The commentary discusses the criteria and nature of math difficulties and notes the need for additional research.

The above being said, Butler, Beckingham and Lauscher (2005) report on three case studies regarding the support of students with math learning challenges. Three eighth grade students were given assistance in self-regulating their learning. General strategies found to be successful included:

- engaging the students in constructive conversation;
- supporting students reflection on their learning; and,
- the need for teachers to engage in dynamic, curriculum based forms of assessment

Fuchs, Fuchs, and Hamlett (2006) report on the validation of an intervention to improve math problems solving at the third grade. The intervention (Hot Math) involved explicit instructions, self-regulation strategies and tutoring. Results indicated positive, short term results for problem solving skills.

Stinson (2006) suggested that a focus on the discourse of achievment in mathematics rather than the discourses of deficiency and rejection could prove beneficial in reducing the well known achievment gap between white and Black students. He suggest that the limited amount of research shows that enrichment activities, mentoring competent teachers and Black students identifying with the 'good kids group' (p. 496) may enhance math achievement in Afican American males.

Research also suggests a variety of instructional strategies that are effective to meet the needs of students with special needs—including those with physical disabilities, mental impairments, and/or learning disabilities; English Language Learners (ELL); and low-performing students who require some special attention to bring out the best of their capabilities. Effective instruction for special-needs students, the research has found, includes:

- Setting clear goals for students (Bray and Turner, 1986, Cherkes-Julkowski and Gertner, 1989, Ferritti, 1989, Ferritti and Cavalier 1991, as cited by Baroody, 1996; Schunk, 1985, as cited by Mastropieri, Scraggs, and Shinh, 1991);
- Using a "big ideas" structure for concepts (Kameenui and Carnine, 1998, as cited by Fuson,2003, p. 88);
- Teaching content that is not too difficult (Bray and Turner, 1986, Cherkes-Julkowski and Gertner, 1989, Ferritti, 1989, Ferritti and Cavalier 1991, as cited by Baroody, 1996; Baroody, 1996) and presented within meaningful contexts (Miller and Mercer, 1997, as cited by Allsopp, Lovin, Green, and Savage-Davis, 2003);
- Laying ample groundwork by providing background knowledge (Bray and Turner, 1986, Cherkes-Julkowski and Gertner, 1989, Ferritti, 1989, Ferritti and Cavalier 1991, as cited by Baroody, 1996; Kameenui and Carnine, 1998, as cited by Fuson, 2003);
- Modeling by teachers (Allsopp et al., 2003; Baroody, 1996; Blankenship, 1978, as cited by Mastropieri et al., 1991);
- Sequencing instruction to go from the concrete to the abstract (Miller and Mercer, 1997, as cited by Allsopp et al., 2003);
- Using mediated scaffolding (e.g., visual supports with cues, teachers' feedback on thinking, peer tutoring) (Kameenui and Carnine, 1998, as cited by Fuson, 2003);
- Discussing mathematics using language (Miller and Mercer, 1997, as cited by Allsopp et al., 2003);
- Building in multiple practice opportunities (Miller and Mercer, 1997, as cited by Allsopp et al., 2003) and time for review by students (Kameenui and Carnine, 1998, as cited by Fuson, 2003);
- Using reinforcement (e.g., earning verbal praise) (Mastropieri et al., 1991); and,

- Providing continual feedback (Miller and Mercer, 1997, as cited by Allsopp et al., 2003; Fuson, 2003; Blankenship, 1978, Schunk and Cox, 1986, as cited by Mastropieri et al., 1991).

Three of these elements of effective special needs instruction—modeling, mediated scaffolding, and feedback—are discussed in further detail below.

Modeling Directly modeling both general problem-solving strategies and specific learning strategies using multisensory techniques has been shown to be useful with students having attention problems, cognitive-processing problems, memory problems, and metacognitive deficits, notes a summary of relevant research (Allsopp et al., 2003). A comparative study of 30 students suggests that direct modeling may be advantageous for students with slight mental retardation as well. One group of students who received direct modeling help (e.g., used blocks as physical manipulatives) and extra opportunities for purposeful practice employed "substantially fewer" inappropriate learning strategies than another group who didn't receive such support (Baroody, 1996, pp. 81-82). Furthermore, it was found that one or two direct-modeling demonstrations enabled such students to correct basic arithmetic procedural strategies and improve their proficiency (Baroody, 1996).

In addition, a review and synthesis of 30 studies on mathematics instruction for learning-disabled students found that modeling and demonstration with corrective feedback improved problem-solving accuracy and generalization skills by the students (Blankenship, 1978, as cited by Mastropieri et al., 1991). For example, an instructional model in which teachers solved a problem, verbalized how they did it, and left the problem as a reference model improved learning-disabled students' computational skills in seven different experiments (Smith and Lovitt, 1975, Rivera and Smith, 1987, 1988, as cited by Mastropieri et al., 1991).

Mediated Scaffolding A review of 30 studies found several types of scaffolding strategies to be effective in improving learning-disabled students' mathematical achievement in grades K-6 (Mastropieri et al., 1991):

- Use of manipulatives teamed with pictorial representations (Peterson, Mercer, and O'Shea, 1988, as cited by Mastropieri et al., 1991);
- Multisensory approaches—mixing visual, auditory, and/or kinesthetic methods. Verbalization (a specific practice in which teachers and students repeat aloud problems, instructions, and solution steps) also improved students' mathematical performance, found a comparative study of 90 such students in grades 6 through 8 (Schunk and Cox, 1986, as cited by Mastropieri et al., 1991); and,
- Use of preorganizers (e.g., read problem, underline numbers, decide on the operation sign and problem type) and/or postorganizers (e.g., read problem, check operation, check math statement, check calculations, write labels) to support students when solving word problems (Mastropieri et al., 1991).

Feedback Ongoing feedback is crucial with special-needs students. Such students require continual monitoring and feedback on their efforts to be successful, several studies and meta-analyses have found (Miller and Mercer, 1997, as cited by Allsopp et al., 2003; Fuson, 2003; Schunk and Cox, 1986, as cited by Mastropieri et al., 1991).

Addressing Specific Mathematics Disabilities

A synopsis of relevant research noted that four different kinds of mathematics disability have been identified (Geary, 1994, as cited by Fuson, 2003). They, and what the research suggested as useful strategies to address them, are as follows:

- Semantic memory disabilities: Students experience trouble with verbal and phonetic memory but may have normal visuospatial skills. Instruction that employs visual clues is most effective for these learners (Fuson, 2003).
- Procedural deficits: Students use less advanced methods overall. Conceptually based instruction is especially helpful for these students (Fuson, 2003);
- Visuospatial disabilities: Students struggle with concepts that use spatial relations, (e.g., place value). Instruction most helpful for these students includes extra cues to support visual processing, and focuses on methods that can be carried out in either direction (Fuson, 2003); and,
- Problem-solving deficits: Such students benefit from problem-drawing supports, including visual representations and manipulatives (Fuson, 2003).

Effective strategies for students with learning disabilities include using advance organizers to clarify the lesson's purpose; reviewing major concepts frequently; teaching generalization and application; modeling procedures at a slow pace with extra clues; presenting new skills using concrete materials, then pictures, and finally abstract explanations; providing extra practice in small steps with guidance; giving clear directions before practice; and checking for error patterns (Bishop & Forgasz, 2007). Studies show that students with learning disabilities benefit from activities-based, hands-on learning, as well as reasoning and problem-solving experiences. Strategies may need to be modified by providing explicit instruction, structuring activities, and stressing vocabulary acquisition (Goldman et al., 1998; Miller & Mercer, 1998).

Effective practices for teaching students with specific difficulties in mathematics (special education and low-achieving students) include systematic and explicit instruction, student think-alouds, visual and graphic depictions of problems, structured peer-assisted learning in heterogeneous ability groupings, and formative assessment data provided to teachers and to students (Gersten et al., 2006).

Research-Based Instructional Strategies

Throughout the research there are a number of instructional strategies which promote successful learning for all students. A few of the strongest examples are presented here.

Balanced Instruction Research shows that teachers cannot simply transfer knowledge to students by lecturing. Students have to take an active role in their own learning, and to accomplish this, mathematics programs must include ample opportunity to explore, question, discuss, and discover. The learning experience should include a balance of discovery learning and explicit instruction. Naylor (2006a, b, c, d, e, and f) provides numerous interactive ideas and activities to gain student interest and assist in instruction. The goals of these activities include promoting conceptual development, challenging students, identifying patterns, sharpening visualization of logical relationships and assisting with numerical and spatial awareness. Students need to learn how to grapple with problems and construct conceptual knowledge (Pressley, Harris, & Marks, 1992; Shulman & Keislar, 1996). Explicit instruction occurs when teachers and textbooks clearly explain problem-solving strategies to students (Duffy, 2002). In geometry instruction, a combination of expository and discovery methods has been found to be effective, especially when manipulatives are used to help students represent and comprehend geometric concepts (Klausmeier, 1992).

Use of Prior Knowledge Prior-knowledge strategies help students retrieve information stored in their long-term memories to learn new, related information. These strategies include recalling information, asking questions, elaborating on textbook and teacher information, and referring students to other meaningful information. Asking students to use prior knowledge may remind them of information already in their long-term memory that, for some reason, is not easily remembered (Bransford, 1979; Pressley & McCormick, 1995).

Practice Providing students with practice on important tasks has long been considered a successful strategy to improve understanding and memory. Giving students individual feedback on their practice helps them monitor and improve their mathematical learning. Practicing helps students acquire additional information as they work towards understanding and applying mathematical concepts. Research shows that mastering a skill requires focused practice. During practice, students adapt and shape what they have learned. In doing so, they increase their conceptual understanding (Clement, Lockhead, & Mink, 1979; Davis, 1984; Mathematical Science Education Board, 1990; Romberg & Carpenter, 1986). Mastering a skill requires considerable focused practice. To reach a level of 80% of mastery usually takes about 24 practice sessions. However, the first four practice sessions lead to an approximately 48% level of proficiency (Marzano et al., 2001).

Note Taking In the process of note taking, students identify important items from reading and write them in an organized format. As they write and draw notes, students see relationships within the information. Notes should not be verbatim; note-taking is most valuable when students learn to select important points (Bretzing & Kulhary, 1979; Marzano et al., 2001). Students benefit from learning various note-taking formats, since there is no single correct way to take notes (Marzano et al., 2001). Studies show that using notes as a study guide for tests is an effective form of review (Marzano et al., 2001). When study skills, such as note-taking, are taught within the teaching of content, they promote learner activity and improve metacognition (Hattie et al., 1996; Robinson & Kiewra, 1995).

Cooperative Learning Cooperative learning occurs when students work in pairs or groups of three or four to complete tasks. Research shows that cooperative learning provides practice at valuable skills, such as positive interdependence, face-to-face interactions, individual and group accountability, interpersonal skills, and group processing (Johnson & Johnson, 1999). Cooperative learning has a highly positive effect when compared with strategies in which students compete with each other and strategies in which students work on tasks individually (Marzano et al., 2001; Johnson, Maruyama, Johnson, Nelson, & Skon, 1981). A balance of cooperative learning and individual learning is needed, however, because students need time to practice skills independently (Anderson, Keder, & Simon, 1997).

Identifying Similarities and Differences Comparison and classification skills are vital in mathematics and science. Students identify similarities and differences to determine how the current problem and previously-solved problems are alike and different. Research tells us that comparing and classifying are effective ways to identify similarities and differences (Chen, 1996; English, 1997; Newby et al., 1995). The most effective methods of working with similarities and differences are to have students identify similarities and differences on their own (Chen, 1996; Mason & Sorzio, 1996), use graphic and symbolic representations as well as words (Mason, 1994), and begin with concrete examples and then move towards abstract knowledge (Reeves & Weisberg, 1993).

Use of Visuals Visuals used in conjunction with verbal description increase student understanding and improve memory of relationships and properties. Visuals are often the only way to effectively communicate ideas that explain central concepts needed to understand algebra and geometry. Research shows that students are better able to organize and group concepts when visuals illustrate different and common characteristics (Hegarty, Carpenter, & Just, 1991). Also, the mental images that high-quality visuals encourage are an indispensable tool for recalling information, especially compared to information presented with only text or lower-quality visuals (Willows & Houghton, 1987).

Graphic Novels Graphic novels create an increased interest in and positive attitudes about reading (Lee, 2004), increase literacy and visual literacy (Association for Library Service to Children [ALSC], 2006; Lavin, 1998; Lee, 2004; Schwarz, 2002), and engage and motivate reluctant readers (ALSC, 2006; Lavin, 1998; Leckbee, 2005; Mooney, 2005; Snowball, 2005; Young, 2007). Graphic novels are extremely popular with teenagers (Schwarz, 2004; Lavin, 1998), especially boys (ALSC, 2006). One study found them to be the most popular reading choice of sixth-graders, regardless of gender or reading ability (Snowball, 2005). Today's students are exposed to television, video games, music videos, Internet, etc.—graphic novels meet their expectation that print media present the same visual impact (Bickers, 2007; Bucher & Manning, 2004; Lavin, 1998; Snowball, 2005). They can be useful as a selected tool to teach key concepts, increase student participation, grab student attention, and stimulate critical thinking (Chessman, 2006; Ruiz-Loyola, 1996). Graphic novels have a direct connection to linguistic, spatial, and interpersonal intelligence (Heaney, 2007). The illustrations provide valuable contextual clues to the meaning of the written narrative and support text comprehension, improving language and literary development (Young, 2007; Snowball, 2005). They are effective for ELL populations and students with language and learning disabilities (ALSC, 2006; Christensen, 2006).

How **Florida Math Connects** *Reflects the Research on Ensuring Mathematical Proficiency for All Learners*

McGraw-Hill's *Florida Math Connects* series is designed to promote math proficiency for all learners. While high quality teaching practice promotes every student's learning, quality of instruction assumes essential importance for the learner with special needs.

The Teacher's Editions of *Florida Math Connects*, using an approach of differentiated instruction and extra supports, provides teachers with the tools they need for addressing the distinct needs of the diverse learners one finds in K-8 classrooms. This approach is extended in the Student Edition and ancillary materials, which support many different learning styles and ability levels to suit a range of learners. Thus, the program can meet the special needs of students who are low performing for reasons of specific disabilities (e.g., mental, physical, behavioral, learning, and/or lack of language skills), as well as gifted and advanced learners.

Specific research-based characteristics of McGraw-Hill's *Florida Math Connects* that especially benefit special-needs students include the following:

- Sets clear goals for students in the Student Editions
- Teaches the "big idea" conceptual base before moving to specific details of procedures
- Begins with the concrete, then progresses through the pictorial to the abstract.
- Models concepts and procedures
- Provides mediated scaffolding, including the following:

    - Uses manipulative materials and pictorial representations wherever appropriate
    - Uses multi-sensory instructional approaches
    - Teaches procedures in small steps, with correctly worked problems made available to students or modeled
    - Guides student use of pre- and post-organizers
  - Builds on previous knowledge and uses appropriate context for more meaningful learning
  - Uses verbal and written language to describe ideas and procedures, because symbolic representation by itself is usually insufficient
  - Provides meaningful practice, student review, and teacher feedback
  - Guides teachers to differentiate instruction by offering
    - Multiple teaching strategies and modalities
    - Extra support for struggling students
    - Multiple levels of practice and problem solving challenges
  - Provides targeted resources for English language learners, including multilevel language support and grouping suggestions to teachers.
  - Provides a graphic novel lesson for every chapter.

As the research states, it is important that all students understand instructional goals. To this end, McGraw-Hill's *Florida Math Connects* provides unit, chapter, and lesson goals throughout the program. Chapter objectives are noted in the Chapter Planner page in the Teacher Edition. Once clear learning objectives are set forth, teachers are guided to reach all learners at their individual levels throughout the program, as the research purports. Differentiated instruction is built into the Teacher's Edition, offering multiple strategies and modalities to reach all learners. Alternate activities and teaching strategies are integrated to give teachers the tools to make content accessible for students at all levels. The Reaching All Learners section of the Chapter Planner prompts in the Teacher's Edition supply further resources for students approaching, on, or beyond level and those needing language support (ELL). Scaffolded instruction is incorporated, breaking down problems into small steps so that students at all levels can become independent problem solvers.

This instructional approach, aimed at meeting the needs of all learners, is continued in the Student Edition. Instruction and practice in the Student Edition ensures that mathematical content is not too difficult for students, as asserted in the research. McGraw-Hill's *Florida Math Connects* is in line with the research that math instruction should build upon previous knowledge and use appropriate context for more meaningful learning. The multi-part lessons build on the objectives presented through out the program and connect to real world math ina deep and meaningful way. In the Skills Trace section of the Teacher's Editon, vertical alignment of skills is clearly shown.

The *Florida Math Connects* curriculum offers a balanced approach of real-world applications, hands-on labs, direct instruction, writing exercises, and practice that enables students to develop both conceptual understanding and procedural knowledge. Multi-part lessons include real-world problems to solve, and then students use multiple representations to explore new concepts.

The *Florida Math Connects* series provides all the resources necessary to prepare English language learners for success in math. Language support is integrated throughout the program with multilevel strategies to engage non-English speakers at three levels of acquisition – Preproduction,Early Production and Speech Emergence, and Intermediate and Advanced Fluency. Teachers receive grouping suggestions at the beginning of every chapter and with diagnostic tests to ensure language learners are placed appropriately and get ample opportunity for small group work, as suggested by the research. Intervention prompts provide strategies for teachers to ensure these students are attaining adequate mathematical proficiency, Daily lessons in the Teacher's Edition include Check Points that ensure early intervention is made available for English language learners needing extra support.

McGraw-Hill's *Florida Math Connects* series evidences an array of instructional characteristics that research suggests are beneficial to special-needs students. Teachers have the resources and guidance in the Teacher's Edition to effectively reach all learners. The Student Edition gives students many opportunities for practice at their individual levels. Other key ancillary materials ensure that all students are geared for success in the math classroom. The program follows a strategy of differentiated instruction and practice to address the needs of the diverse learners in grades K–8.

Educational Principles for Mathematical Proficiency

Reading and Writing in Mathematics

Reading and writing are two of the major activities through which students acquire mathematical understanding. Their importance increases as grade level increases. Reading mathematics requires unique knowledge and skills, since mathematical writing is dense, terse, and comprised of symbols as well as words (Barton & Heidema, 2002). Reading helps students understand concepts, work problems, organize ideas, extend their thinking, and view mathematics as a valuable subject (Martinez & Martinez, 2001). Teachers should model for students how to monitor and reflect on their reading. Modeling by thinking aloud is effective (Barton & Heidema, 2002). Teachers can model reading strategies by reading a problem aloud, paraphrasing it, and talking through how to figure out word meanings, by asking if everyone is clear on a word's meaning, and by asking questions about the meaning of the problem (Kenney et al., 2005). Reading word problems requires special skills: understanding the problem situation, identifying the main idea, and extracting relevant information (Musthafa, 1996).

Mastering vocabulary is fundamental to mathematical understanding. Students struggle with mathematics vocabulary because of the overlap between mathematical and everyday English usage, the use of mathematical symbols (Fuentes, 1998), and concepts embedded within other concepts (Barton & Heidema, 2002). Direct instruction on vocabulary words increases student achievement (Stahl & Fairbanks, 1986). Graphic organizers, such as concept maps, help students learn vocabulary (Monroe, 1997). Research has shown that repetition and multiple exposures to vocabulary words are effective (National Reading Panel, 2000).

Textbook features—organization, presentation, coherence—have a direct impact on reading comprehension (Barton & Heidema, 2002). Characteristics that assist reading comprehension include a page layout that makes the organization of the content evident, a consistent pattern within each lesson or chapter, and explicit instruction on the text structure (Barton & Heidema, 2002; Dickson, Simmons, & Kameenui, 1995). Teachers should introduce students to the visual cues used in their textbook—colors, fonts, graphic symbols, text boxes—and model how to use features of their textbook (Barton & Heidema, 2002).

Writing in mathematics includes writing study notes, vocabulary definitions, explanations, descriptions, predictions, and justifications, along with attitudes, confusions, and questions. Writing helps students make sense of mathematics (Countryman, 1992). The thinking skills involved in writing are major aspects of doing mathematics. Through writing, students clarify and organize their thinking. Writing reinforces what students have learned, helping to retain it in long-term memory.

Research shows that frequent, regular writing in mathematics class leads to improved quality of writing, improved student attitudes toward mathematics, and teachers' increased insight into their teaching (Miller & England, 1989). Writing is effective in helping learn mathematical concepts. Writing by students gives teachers valuable information on student learning (Powell, 1997). When students write explanations about how to solve numerical problems, they are doing much more than acquiring content and demonstrating mastery of a benchmark—they are also communicating and problem solving (Urquhart & McIver, 2005).

An argument can be made that good practices in the teaching of reading can be applied to the teaching of math (Center for the Education and Study of Diverse Populations, 2006). Making the comparison to reading, the article notes:

- mathematics should be enjoyable, limited to the real world with a focus on ideas
- research supports explicit instruction and applied problem solving
- students need concrete as well as abstract learning experiences
- starting with number sense can lead to presentation of big ideas
- students need to communicate their math ideas as a component of understanding
- questioning is relevant in the revealing of a student's math thinking
- fluency and flexibility with math facts and figures leads to conceptual understanding
- connections between new material and prior knowledge is important
- a variety of strategies are needed for computation and problem solving
- problem solving should be relevant to the students life and interests
- vocabulary in math needs to be in a conceptual context
- reflecting on patterns and applications are critical
- checklists and rubrics are useful in assessment

Hall (2004) adds that the teaching of math often involves reading. Hall reviews the research with respect to teacher's beliefs about the teaching of reading within a content area. The results of the study suggest that the importance of teaching reading in the content area needs to be stressed during preservice teacher training, then followed with emphasis during the inservice training years.

How* Florida Math Connects *Relates to Research on Reading and Writing in Mathematics

Consistent with research, many activities in the series provide opportunities for students to read and speak mathematics. Building Math Vocabulary gives teachers additional activities to reinforce new vocabulary terms, such as half and equal parts. Circle Time includes activities in which teachers read books and do activities related to the lesson content, which helps students value and enjoy reading. Learning Stations and Cross Curricular Links in each chapter include reading stations and language arts stations. The Math At Home letter includes the key vocabulary for that lesson, as well as suggested books to read.

Vocabulary is emphasized in each Chapter Opener, in each multi-part lesson, and in the Vocabulary Check in Study Guide and Review at the end of each Chapter. The Foldables® that students make for each chapter include key vocabulary. For each chapter, suggestions are given for six Learning Stations that provide Cross-Curricular Links. Most of these involve reading; some focus on writing.

The Student Editions of Glencoe's mathematics series offer a consistent book layout and a clear, consistent lesson structure. The consistent page layout makes effective use of color, graphics, and fonts. Concepts are highlighted in shaded *Key Concept* and *Theorem* boxes, in words, symbols, and models and are summarized in the *Study Guide and Review,* with examples and exercises. *Writing in Math* questions in every lesson require students to use critical thinking skills.

<u>Technology & Digital Tools</u>

[*Note: The research base for the use of technology and digital tools in mathematics instruction continues to expand. Please see the Digital Learning white paper for more information.*]

Several articles note the use of technology as important to bring application more closely to school curriculum. Ross (2004) highlights a locally developed curriculum project called the Math-Science Investigation (MSI). The article outlines technology and other reforms implemented with community participation. The reform integrates a more application based approach to math and science instruction. In a related approach, Chen, Benton, Cicatelli, and Yee (2004), present a generic case for collaborations among teachers, schools communities, and professionals to enhance learning. Again, through technology, a more hands-on approach with an applied curriculum is the key. Research shows that technology use increases *representation fluency*—the ability to move between representations while carrying the meaning from one representation to another—by providing multiple representations that are linked and interactive (Zbiek et al., 2007).

Research on computer use indicates that application software, such as spreadsheets and graphing tools, can promote learning (Grouws & Cebulla, 2000). The most effective use of instructional software for mathematics occurs when meaning and understanding are emphasized, mathematical skills are embedded in context, and connections are made between subject areas and real-life experiences (Reyes et al., 1999). Software like *Geometer's Sketchpad,* with its interactive, dynamic representations, can help students draw meaning about a mathematical entity if the students reflect on the connection between their actions and the change in representations (Zbiek et al., 2007).

Pogrow (2004) claims to have developed a math instructional program that increases basic skills, enhances problem solving, and increases student interest in math. In addition to using technology to grab student interest and to some degree automate instruction, Pogrow's method attempts to build new approaches to teaching through simulations and exploratory learning. While no data is presented, Pogrow suggests that his approach produces outcomes that lead to gains in achievement as well as comprehension skills and self-concept.

Ertmer (2005) notes that while all conditions for high level of technology integration in the classroom are present, technology use has been lower than expected. Ertmer points to ready access to technology, appropriate training for teachers and an environment that is technology friendly. However, she notes that barriers still exist. One specific barrier identified, specifically in math, reading, and science is the pedagogical beliefs of teachers. The article lays out a case and components for further research on teacher beliefs as related to technology integration.

Students' behaviors vary when using technological tools. Most students can be educated to work judiciously with these tools (Zbiek et al., 2007; Ball & Stacey, 2005). Research results on calculator use are generally positive (Apthorp et al., 2001). Research on graphing calculator use also shows positive effects on graphing ability, conceptual understanding of graphs, and relating graphs to other representations (Grouws & Cebulla, 2000). Calculator use enhances learning arithmetical concepts and skills, problem solving, and attitudes of students (Hiebert, 2003). Most graphing calculator studies have not found negative effects on basic computation skills or factual knowledge (Grouws & Cebulla, 1999). With calculators, teachers tend to ask more high-level questions; students are more actively involved in asking questions and conjecturing (Grouws & Cebulla, 1999).

How* Florida Math Connects *Relates to Research on Technology in Mathematics Instruction

Florida Math Connects offers fully integrated, state-of-the-art technology resources for students, parents, and teachers, providing support for differentiated instruction, alternate teaching approaches, additional assessment opportunities, and motivating activities. For each grade level, the ConnectEd portal offers access to the entire eBook editions of the Student and Teacher Edition, all print ancillaries, and a wealth of online resources—audio readings of the text, Self-Check Quizzes, Personal Tutor, Math in Motion animations, Math Adventures with Dot and Ray (grades K–5), eGlossary (14 languages), Math Tool Chest (activities from shapes and sorting to data and graphs for grades K–5), and numerous other interactive whiteboard-ready (IWB) manipulatives and activities. ExamView® Assessment Suite enables teachers to create and customize assessments and assignments. Advance Tracker Learner Management System helps teachers track progress and differentiate instruction.

Assessment

Research suggests that assessments greatly impact what and how teachers teach, and what their students learn. A large-scale study of grades 4-12 found that tests—both standardized and those in textbooks—have an "extensive and pervasive influence on math instruction" (Madaus, West, Harmon, Lomax, & Viator, 1992, p. 1, as cited by Mathematical Sciences Education Board [MSEB], National Research Council, 1993). As well, Ryan, Ryan, Arbuthnot, and Samuels (2007) suggest that motivational patterns of students may lead to less effective test-taking strategies.

Research has demonstrated that collecting assessment data from students, both formally and informally, is a key to shaping effective instruction (Brimijoin et al., 2003). This is called *data-driven assessment*. Given the key role that assessment plays in instruction, the National Council of Teachers of Mathematics' Principles and Standards for School Mathematics (2000) has recommended that "assessment should support the learning of mathematics and furnish useful information to both teachers and students" (NCTM, 2000, p. 22). It continues:

> Assessment should be more than merely a test at the end of instruction to see how students perform under special conditions; rather, it should be an integral part of instruction that informs and guides teachers as they make instructional decisions. Assessment should not merely be done *to* students; rather, it should also be done *for* students, to guide and enhance their learning (NCTM, 2000, p. 22).

The Mathematics Sciences Education Board (MSEB) of the National Research Council's (NRC) Center for Education summarized the key role of assessment even more succinctly: "Assessment is the guidance system of education just as standards are the guidance system of reform" (MSEB, NRC, 1993, p. 2).

Principles and Standards for Assessing Students' Mathematical Proficiency

Based on its synthesis of relevant research, the NCTM published Assessment Standards for School Mathematics (1995), which identified six standards for "exemplary mathematical assessment" that supported its standards for teaching and learning mathematics in kindergarten through grade 12 (NCTM, 1995, as cited by Wilson & Kenney, 2003, p. 53). Three of these standards are elaborations of core principles of the MSEB, based on its own review and synthesis of research on national efforts to reform the teaching and learning of preK-12 mathematics (MSEB, NRC, 1993). As summarized in Principles and Standards for School Mathematics (NCTM, 2000), assessment should:

1. "Reflect the mathematics that all students need to know and be able to do" (NCTM, 2000, p. 22). This is similar to the MSEB's "Content Principle," which stressed assessment that reflects "the mathematics that is most important for students to learn" (MSEB, NRC, 1993, p. 1).
2. "Enhance mathematics learning" (NCTM, 2000, p. 22). This is similar to the MSEB's "Learning Principle," which also noted that assessment should "support good instructional practice" (MSEB, NRC, 1993, p. 1).
3. "Promote equity" (NCTM, 2000, p. 22). This is similar to the MSEB's "Equity Principle," which explained, "Assessment should include opportunities to allow every student to demonstrate the extent of their mathematical knowledge." (MSEB, NRC, 1993, p. 1). By "every student," the MSEB meant each child, regardless of prior knowledge or societal background.
4. "Be an open process" (NCTM, 2000, p. 22).
5. "Promote valid inferences about mathematics learning" (NCTM, 2000, p. 22).
6. "Be a coherent process" (NCTM, 2000, p. 22).

Characteristics of Effective Assessment
Research has identified some key characteristics of effective approaches to assessing students' mathematical proficiency and understanding. In general, such assessments need to record "genuine accomplishments in reasoning and in formulating and solving mathematical problems," not just the ability to memorize or recognize correct answers, noted a retrospective analysis of the items in the 1990 National Assessment of Educational Progress (NAEP) test (Silver, Kenney, & Salmon-Cox, 1991, as cited by MSEB, NRC, 1993, p. 31-32).

To achieve coherence, a recent synthesis of research on assessing mathematical learning concluded, assessment must be aligned with both the curriculum and the instruction, it must match the instructional purpose, and the phases of assessment must fit together in a meaningful progression (Wilson & Kenney, 2003). No single form of assessment can adequately measure the breadth and depth of students' knowledge. In general, the richer in quantity and variety of evidence used for assessment, the more valid a teacher's inferences of a student's understanding and proficiency are likely to be, concluded the NCTM's Assessment Standards Working Groups and other researchers (NCTM, 1995; Ames, 1992, as cited by Wilson & Kenney, 2003; Wilson & Kenney, 2003). Multiple types of assessments enable teachers to look for "convergence" and "common conclusions" (Linn, Baker, & Dunbar, 1991, Baxter, Shavelson, Herman, Brown, & Valadez, 1993, as cited by MSEB, NRC, 1993, p. 133; Wilson & Kenney, 2003). In textbook-based learning, adjunct questions—when used to focus students' attention on the concepts presented in the text—were found to enhance learning (Crooks, 1988, as cited by Black & Wiliam, 1998).

Additionally, a recent synthesis of research on assessing mathematical proficiency found that classroom assessment should include adequate dialogue between teachers and students. Moreover, teachers need to ask higher-level questions, which "has not been the norm," the synthesis emphasized (Wilson & Kenney, 2003, p. 55).

Self-Assessment
"Assessment is to be seen as a moment of learning, and students have to be active in their own assessment and to picture their own learning in light of what it means to get better," noted Zessoules and

Gardner in their review of research (Zessoules & Gardner, 1991, as cited by Black & Wiliam, 1998, p. 30). Engaging students in measuring their own and their peers' mathematical knowledge is effective, research has found. For example, two quantitative studies involving more than 700 students aged 8-14 years indicated that student self assessment and peer assessment of their mathematical understanding did help enhance their learning—as long as they understood the learning goals and assessment criteria, and had an opportunity to reflect on their work (Fontana & Fernandes, 1994, Frederiksen & White, 1997, as cited by Black & Wiliam, 1998). The NCTM's Assessment Standards Working Groups concurred with this finding (NCTM, 1995). Similarly, higher academic achievement resulted "from combinations of self-assessment and peer assessment" by students, found a study of the methods' effectiveness when used by young children (Higgins, Harris, & Kuehn, 1994, as cited by Wilson & Kenney, 2003, p. 57). Zessoules and Gardner also concluded that assessment that helps students "develop reflective habits of mind" contributes to their learning (Zessoules & Gardner, 1991, as cited by Black & Wiliam, 1998, p. 29).

Training students in aspects of self assessment may give them a learning advantage. For example, one study found that fifth and sixth graders, after being trained to monitor their own performance on mathematical problems, were more successful than those who had no such training. They could solve more complex problems and also succeed more quickly. They seemed to do better "not because they could use particular strategies more effectively, but because they started by considering the possibilities of using different strategies before proceeding" (Delclos & Harrington, 1991, as cited by Black & Wiliam, 1998, p. 27). Reviews of the literature on assessment have also concluded that an assessment must reflect the instructional setting(s) in which the content was taught. For example, if students' work was done in group settings, the assessment should reflect the value of group interaction (Davidson, 1985, Good, Mulryan, & McCaslin, 1992, as cited by MSEB, NRC, 1993).

A meta-analysis of 40 studies on the effects of in-class testing showed that frequent testing—but no more than one or two per week—improved student achievement. Several short tests proved more effective in improving student achievement than fewer long tests (Bangert-Drowns, Kulik, & Kulik, 1991, as cited by Black & Wiliam, 1998; Wilson & Kenney, 2003). In addition, student self assessment has been demonstrated as effective when they are made a part of everyday class instruction. Significant student achievement was attained by having "students self-assess on a regular (mainly daily) basis," found a study of 356 students aged 8-14 years (Fernandes & Fontana, 1996, as cited by Wilson & Kenney, 2003, p. 57).

Formative Assessment

Formative assessment includes determining where learners are in their learning, where they are going, and how they will get there (Wiliam & Thompson, 2007). The use of day-to-day formative assessment is one of the most powerful ways of improving learning in mathematics (Wiliam, 2007). Many studies confirm that formative assessments—measures of students' learning of mathematical concepts and procedures that teachers use to guide ongoing instruction in the classroom—significantly enhance student achievement. For example, a meta-analysis of 250 studies found that "the research shows conclusively that formative assessment does improve learning," (Black & Wiliam, 1998, p. 61.) Furthermore, "the gains in achievement appear to be quite considerable, amongst the largest ever reported for educational interventions" (Black & Wiliam, 1998, p. 61). A mean effect size of 0.7, as reported by Fuchs and Fuchs' (1986) analysis of 21 studies on assessment of preK-12 children, is cited as representative (Black & Wiliam, 1998).

To illustrate such a large gain attributed to formative assessment, if accomplished nationwide, it would be the equivalent of raising students' mathematics-attainment score on the Third International Mathematics and Science Study (TIMSS) from "average" (where the U.S. was ranked) up into the "top five" (after Singapore, Korea, Japan and Hong Kong) (Beaton et al., 1996, as cited by Black and Wiliam, 1998, p. 61).

Research also shows that students have higher achievement if their learning goals are "dynamic," that is, if the goals are adjusted or amended with corresponding changes in instruction in light of measured progress (Fuchs, 1993, as cited by Black and Wiliam, 1998, p. 44).

Formative assessment by teachers offers benefits beyond raising test scores, noted the NCTM's Assessment Standards Working Groups: "Continuous assessment of students' work not only facilitates their learning of mathematics but also enhances their confidence in what they understand and can communicate" (NCTM, 1995, p. 14).

Furthermore, several studies and syntheses of research on assessment in grades K-12 have found that in order to effectively measure mathematical knowledge, tasks used for formative assessment should not only be varied but should also be nonroutine (i.e., different from what students have previously experienced), offer reasonable challenge, and focus on meaningful aspects of learning (Ames, 1992, McLeod, 1992, MSEB, NRC, 1993, as cited by Wilson & Kenney, 2003). Useful instruments for formative assessment include oral comments; written papers, including drafts and polished work; journal entries; drawings; computer-generated models; out-of-class long-term projects; and different types of small-group work, researchers have concluded (NCTM, 1995; NCTM, 2000; Wilson & Kenney, 2003). To effectively use formative assessment, teachers should clarify, share, and understand goals and criteria for success; use classroom discussions, questions, activities, and tasks that elicit evidence of students' learning; provide feedback that tells students how to improve; help students learn to help each other; and help students become responsible for their own learning (Wiliam, 2007).

Teachers generally believe that what they see and hear as their students engage in mathematical learning and practice, along with other documentation of students' performances, are the most useful components of formative assessment. For example, a study of 269 teachers of grades 1-4 mathematics rated their own observations and students' performances as the two most important elements of assessment. Similarly they rated teacher observations and students' performances as their most often-used assessment practices (Adams and Hsu, 1998). However, the same study also showed that grades 1-4 mathematics teachers believe that using a variety of assessment methods is valuable as well, and reported using more than half a dozen methods either "very often" or "almost always" in their formative assessments. These additional methods included watching how students applied mathematics, how they explored problem solving, use of direct questioning, and class discourse/discussion (Adams & Hsu, 1998, p. 177).

Larger pieces of work—such as performance tasks, projects, and portfolios—allow students to demonstrate how they have grown in their mathematical power (NCTM, 1995). Portfolios are considered valuable for summative assessments precisely because they measure different constructs than standardized tests (Reckase, 1997, as cited by Wilson & Kenney, 2003). Having students participate in choosing which work samples to include in their portfolios prompts them to reflect on and monitor their own learning, and stimulates teachers to reflect on their own practice as well, several studies and research reviews have noted (Columba & Dolgos, 1995, Daro, 1996, Mills, 1996, Yancey, 1996, as cited by Wilson & Kenney, 2003).

Feedback

Teachers need interpretive frameworks to help them coordinate the many separate bits of assessment information into a usable result, revealed a study of elementary teachers' practices. The researchers found that currently, such frameworks are generally absent (Bachor & Anderson, 1994, as cited by Black & Wiliam, 1998). Results from Fuchs and Fuchs's meta-analysis of 21 studies on assessment of preK-12 students suggest that interpretive frameworks would improve students' mathematical achievement. It was found that teachers who produced graphs of individual children's progress and used them to guide further action reported greater student gains than teachers who did not make or use progress graphs, with mean effect sizes of 0.70 and 0.26, respectively (Fuchs & Fuchs, 1986, as cited by Black & Wiliam, 1998).

According to a meta-analysis of 58 experiments of feedback's effect on instruction, it is the quality of feedback from assessments that has the most influence on students' performance gains (Bangert-Drowns, Kulik, & Kulik, 1991, as cited by Black & Wiliam, 1998). Feedback from teachers proved to be most effective when it provided specific help on correcting students' errors and improving their strategies, yet kept the overall focus on deep conceptual understanding, found a related meta-analysis (Bangert-Drowns, Kulik, Kulik, & Morgan, 1991, as cited by Wilson & Kenney, 2003).

Furthermore, feedback appears to be most effective when it focuses on the task and not on the student. Studies have found that students who received feedback that was focused on learning goals (task-oriented) had higher achievement, sought help more often when they needed it, and generally had a more positive attitude about themselves and towards learning than peers who received feedback that was focused on performance goals (ego-oriented) (Butler, 1988, Butler & Neuman, 1995, as cited by Black & Wiliam, 1998). A meta-analysis of relevant studies reached similar conclusions (Bangert-Drowns, Kulik, Kulik, & Morgan, 1991, as cited by Wilson & Kenney, 2003). Studies and syntheses of relevant research have also concluded that teachers should routinely perform instructional follow-up to assessments in the form of classroom discourse that "revisits, extends, and generalizes" interesting and/or key types of mathematical problems (MSEB, NRC, 1993, pp. 11, 78).

Feedback to students after assessment must be timely. Frequent short tests are better than infrequent long ones. New learning should first be tested within about a week of its first encounter (Black & Wiliam, 1998b). Performance improves with frequent testing up to a certain point but additional tests have little extra impact (Wilson & Kenney, 2003; Bangert-Drowns, Kulik, & Kulik, 1991). As part of their meta-analysis, Bangert-Drowns, Kulik, and Kulik found that immediate feedback was more effective than feedback that was delayed by even a day after the assessment (Bangert-Drowns, Kulik, & Kulik, 1991, as cited by MSEB, NRC, 1993). Effective feedback is focused on the mastery of the task, given soon after the task, and specific and related to students' needs (Marzano et al., 2001; Crooks, 1988).

Equity

In the process of gauging students' knowledge, educators must be careful that their methods are equitable and don't contain hidden biases or assumptions concerning students' capabilities. For an assessment to be equitable, it must provide each and every student with the opportunities to demonstrate his or her level and/or range of mathematical knowledge, found a recent synthesis of research on assessing mathematical learning (Wilson & Kenney, 2003). This may mean, for example, that students receive partial credit for successfully completing parts of a mathematical task (Wilson & Kenney, 2003).

In addition, to ensure equity, assessment tasks must utilize "a context that is familiar to all students," regardless of their prior knowledge or social background, found another review of relevant research on mathematical learning (MSEB, NRC, 1993, p. 50). One way to achieve this, developed by the NRC's Learning Research and Development Center, is to first introduce the context in a separate lesson prior to the assessment (Learning Research and Development Center, University of Pittsburgh, and National Center on Education and the Economy, 1993, as cited by MSEB, NRC, 1993).

How* Florida Math Connects *Reflects the Research on Assessing Mathematical Proficiency

Consistent with the research, McGraw-Hill's *Florida Math Connects* series integrates a comprehensive, coherent assessment plan, featuring multiple, frequent opportunities and diverse methods to assess students' mathematical learning. The program expressly addresses diagnostic assessment (for placement or readiness), formative assessment (to inform ongoing instruction), and summative assessment (evaluations of performance for a grade). These different types of assessments are clearly identified in the materials and guidance found in the Teacher's Editions of McGraw-Hill's *Math Connects* series.

In each unit in the Teacher's Edition is a section titled "Assessment Options" . This lists all of the assessments for the unit, categorizing them as diagnostic, formative assessment, or summative assessment. It also suggests resources suitable for each assessment, correlated to that unit's particular content. Thus, teachers receive varied assessment resources for instructional units in advance, as well as chapter by chapter within units. Teachers can also create and customize their own assessments using *ExamView Assessment Suite* software.

In *Florida Math Connects*, assessment begins with a pre-chapter diagnostic readiness section of information on options for diagnostic assessment. *Florida Math Connects* provides teachers with strategies for reaching all learners – including those in need of language support – as they observe students, engage them in assessment dialogues, and have them read and write about math. Using these assessment techniques, teachers can ensure an equal opportunity for all students to demonstrate their mathematical proficiency.

Florida Math Connects' comprehensive, coherent assessment plan reflects the research on assessment by deliberately emphasizing the use of formative assessment to inform and improve ongoing classroom instruction and by providing both the assessment instruments and guidance. Frequent, multiple assessment opportunities for students and related resources for teachers are present in every lesson of every unit at every grade level. The program's Teacher's Edition, Assessment Guide, Intervention Handbook, and ESL Activity Guide promote frequent use of these assessment options by teachers, providing the extra resources and guidance to do so with confidence. The Advance Tracker Learner Management System helps teachers track progress on assessments and differentiate instruction.
In short, McGraw-Hill's *Florida Math Connects* series provides a wealth of material for teachers and students to ensure valid, dynamic and comprehensive assessment to support ongoing instruction for all students.

Professional Development

Teachers are central to students' success in mathematics, especially in the elementary grades. Teachers serve in a variety of critical roles—including classroom manager, content presenter and demonstrator, activity coordinator, project director, and learning guide—that help define students' educational experiences with mathematics. "Students' understanding of mathematics, their ability to use it to solve problems, and their confidence in, and disposition toward, mathematics are all shaped by the teaching they encounter in school" (NCTM, 2000, pp. 16-17). Successful mathematics learning by students, therefore, depends on the skills, knowledge, and practices of their teachers.

Research shows that to be effective, teachers need to fully understand the concepts that make up mathematics and underlie mathematical procedures. They need to understand the interconnections among topics (Mewborn, 2003; Swafford, Jones, & Thornton, 1997). Many teachers lack a conceptual understanding of mathematics, although they have strong procedural knowledge (Mewborn, 2003; Even & Tirosh, 1995). Teachers need to be actively engaged in learning new ideas and to translate what they learn into classroom practice (Mewborn, 2003; Shifter, 1998). Teachers must listen to students' reasoning and be able to make sense of students' mathematical thinking (Mewborn, 2003; Thompson & Thompson, 1994, 1996).

Professional development is the primary vehicle for improving teachers' skills, knowledge, and practices in mathematics and instruction. For teachers to "understand deeply the mathematics they are teaching and be able to draw on that knowledge with flexibility in their teaching tasks," notes the NCTM in Principles and Standards for School Mathematics, they "must have frequent and ample opportunities and resources to enhance and refresh their knowledge" (NCTM, 2000, p. 17). In support, Hill, Rowan, and Ball (2005) found that teacher's math content knowledge related to student achievement in grades 1-3.

This view of the importance of professional development is supported by research on its cost-effectiveness. In their review of the research, Greenwald, Hedges, and Laine (1996) concluded, "Increased spending on teacher education has a greater positive effect on student learning than increased spending on other school variables" (as cited by Stigler & Hiebert, 1999, p. 146). Studies show that collegial support for teachers working to improve instructional practices is valuable, but it must be accompanied by a strong focus on mathematical content, students' thinking, and curriculum (Mewborn, 2003).

With the NCTM's vision of effective mathematics teaching and learning in mind, Mewborn (2003) reviewed more than 20 years of research on professional development in mathematics instruction. Based on her synthesis of that research, she recommended that such professional development give teachers opportunities to learn mathematics in the ways they are now expected to teach it to students, that is, for conceptual understanding as well as procedural proficiency. She further concluded that professional development should help teachers gain insight into the conceptual underpinnings of topics and the interconnections among topics of the mathematics they teach. The body of evidence "overwhelmingly suggests," summarized Mewborn (2003, p. 49), that enhancing teachers' mathematical knowledge is "a crucial part of learning to teach differently" (Campbell & White, 1997; Heaton, 1994, 2000; Kilpatrick, Hancock, Mewborn, & Stallings, 1996, as cited by Mewborn, 2003). Teachers need to learn and use three key classroom practices for doing and learning mathematics:

- Supporting discourse,
- Establishing norms, such as what counts as a mathematical explanation,

- Building relationships, which includes understanding students' identity and culture (Franke et al., 2007).

For instance, a two-year study of five middle-school teachers teaching rational numbers, quantity, and proportional reasoning found teachers' practices changed as their content knowledge increased and deepened (Sowder, Philipp, Armstrong, & Schappelle, 1998, as cited by Mewborn, 2003). Also, a comparative study of 21 upper elementary-grade classrooms in California (following its adoption in 1992 of a new problem-solving Mathematics Framework), found that professional development designed to enhance teachers' understanding of mathematics contributed to greater opportunities for students to engage with numeric representations in ways that helped students build conceptual understanding (Gearhart, Saxe, Seltzer, Schlackman, Ching, Nasir, Fall, Bennett, Rhine, & Sloan, 1999).

Schifter (1998) found that effective professional development has teachers "struggle with important mathematical ideas, justify their thinking to peers, investigate alternative solutions proposed by others, and reconsider their conceptions of what it means to do mathematics" (as cited by Mewborn, 2003, p. 49). Schifter's research explored the ability of teachers to readily translate what they learned during inservice into classroom practice. Professional development in which teachers actively learned new ideas (as would be expected of their students) better enabled those teachers to turn the training into improved instructional practice, compared to teachers who were simply told the new ideas, Schifter reported.

Research also suggests that professional development should help teachers encourage their students to explore multiple ways to accomplish mathematical tasks. For example, a review of research found that "richer, more conceptual" learning opportunities become available when teachers encourage students to explore the relative advantages of different mathematical approaches to solving a problem (Hiebert, Fennema, Carpenter, Fuson, & Wearne, 1997, as cited by Stigler & Hiebert, 1999). And findings by Ball and Bass (2000) suggest that teachers must be able to "decompose" a math task, such as working backwards to show students its many possible ways of enactment. However, such decomposition works only if it is sufficiently granular—"if its logical steps are small enough to make sense for a particular learner or class," based on what they already know (Ball & Bass, 2000, pp. 98-99). Professional development can provide that granularity of understanding to teachers.

To implement the kind of math instruction recommended by the National Council of Teachers of Mathematics effectively, a review of research concluded, changes in both beliefs and practice are necessary—and they are iterative (Goldsmith & Schifter, 1997, as cited by Nelson, 1997). It is not clear yet, however, which is more efficient: changing beliefs first or changing practices first. It seems reasonable that changes in belief may need to precede changes in practice (Schifter & Fosnot, 1993, as cited by Nelson, 1997). However, another review of the research reveals that in some cases, changes in practice have "jump started" the process for teachers (Franke, Fennema, & Carpenter, 1997, as cited by Nelson, 1997, p. 11).

Professional development focused on students' mathematical thinking can have a positive impact on student achievement, found a comparative study in California of 21 upper elementary-grade classrooms covering the topic of fractions (Gearhart et al., 1999). In the study, one group of teachers received inservice during which they added to their own knowledge base of mathematics and assessment, and also engaged in peer dialogue and support. A second group of elementary teachers only had a collegial support group for dialogue and reflection, while a third of teachers group had no inservice at all.

Students whose teachers were in the first group (receiving inservice and support) had more opportunities to conduct conceptual discussions and build their understanding of fractions and were more likely to experiece gains in performance on problem-solving tasks than students of the other teachers. Specifically, the study found that for each 1-unit increase on a 4-point Conceptual/Assessment Opportunity scale, there was an increase of .087 units on a 13-point Problem-Solving scale (Gearhart et al., 1999, following procedures developed by Stein & Lane [1996] and Stein, Grover, & Henningsen [1996] for coding mathematical tasks and relating them to student outcomes). Thus, concluded the researchers, professional development for teachers that is expressly designed to give students more opportunities to engage and explore mathematical concepts and problem solving results in learning gains (Gearhart et al., 1999).

Another study of California's mathematics reform initiative noted that gains in student achievement were maximized when the teacher professional development, curriculum, and student assessment measures were all aligned with the prevailing policies of reform (Cohen & Hill, 1998, as cited by Mewborn, 2003).

Among the best-supported conclusions in the literature on educational improvement is this: "Almost all successful attempts to improve teaching have involved teachers working together to improve students' learning" (Stigler & Hiebert, 1999, p. 135). A comprehensive review of the research on professional development summarizes the relevant studies and their findings (Darling-Hammond, 1998, as cited by Stigler & Hiebert, 1999). For example, a comparative study of teachers at an urban elementary school found that professional development was more effective in helping teachers change their practice when it included collegial support (Kazemi & Franke, 2000, as cited by Mewborn, 2003).

However, receiving collegial support alone is not enough, found a comparative study of the effects of different types of inservice on 21 elementary-grade classrooms. Professional development programs that only supplied collegial support were not as effective as those that also concomitantly focused on curriculum, mathematics, and/or children's mathematical thinking (Gearhart et al., 1999).

Davis and Krajcik (2005) provide nine design heuristics related to professional development from a NSF study to promote student learning (science related but non grade specific):

1. engage students in the topic specific phenomena (physical experiences, etc.)
2. utilize scientific instructional representations (analogies, models, diagrams, etc.)
3. anticipate understanding and dealing with student ideas about science (age specific) – incorporate into teaching
4. engage students in questioning – provides focus for teaching
5. engage students to collect and analyze data and observe
6. engage students to design their own inquiry
7. explain using evidence
8. promote scientific communication
9. develop subject matter expertise of teachers

The conclusion that teaching is clearly a system of unique features that have to work together to support learning goals was stated by Hiebert, Stigler, Jacobs, Givvin, Garnier, Smith, Hollingsworth, Manaster, Wearne, and Gallimore (2005). They add that <u>all</u> of the features must be implemented effectively with students and that no one single feature added could be said to be individually effective.

Japanese students have academically outperformed U.S. students in science and mathematics, according to Third International Mathematics and Science Study (TIMSS) results in 1995, 2000, and 2003. Stigler and Hiebert's (1999) analysis of TIMSS videotapes of classrooms in Japan, Germany, and the U.S., as well as Fernández' (2002) exploration of Japan's "lesson study" form of professional development, reveal likely explanations for this learning advantage.

First and foremost, U.S. elementary mathematics teaching was very limited and focused mainly on a narrow band of procedural skills, showed the TIMMS' videotapes of classrooms. Comparatively, Japanese teachers taught math "in a deeper way," for conceptual understanding. Thus, U.S. students spent most of their time "acquiring isolated skills through repeated practice" while Japanese students spent as much time "solving challenging problems and discussing math concepts as they [did] practicing procedures" (Stigler & Hiebert, 1999, p. 11). Furthermore, the U.S. students typically practiced previously demonstrated procedures on many simple problems. Conversely, Japanese students typically developed a variety of advanced techniques by themselves to use for solving a few challenging problems (Stigler & Hiebert, 1999).

"Lesson study" is a method of collaborative professional development in Japan that focuses on a specific goal for students, which is addressed through instruction. Teachers meet regularly to discuss and design detailed lessons to meet an agreed-upon goal, then also come together to watch and take notes as the lessons unfold in real classrooms. When they next meet, the educators comment, critique and collaborate to improve each lesson. The process is iterative, and a lesson might be continually honed for months or years. Lesson study allows teachers to deepen their understanding of both mathematics and of creating lessons that help students to come, by multiple ways, to a similar deep understanding, explains an empirical study of the feasibility of a similar approach to professional development in the U.S. (Fernández, 2002). Fernández' empirical study, conducted over the course of three years with 33 K-8 teachers engaged in mathematics lesson study, concluded that this approach to professional development did promote teachers' exploration of their own understanding of both mathematics and pedagogy. According to Stigler and Hiebert (1999), the lesson study approach to professional development holds promise for improving teaching and learning in the U.S. These researchers point out that lesson study directly improves instruction in context by focusing "on the lesson as the unit to be analyzed and improved" (Stigler & Hiebert, 1999, pp. 122). Moreover, since improvements are curriculum-based, professional development based on Japan's lesson-study model "uniquely allows" teachers to devote their time to those improvements that best align with local and state standards and for which they are held accountable (Stigler & Hiebert, 1999, pp. 154).

The collaborative nature of lesson study helps counter-balance individual teachers' self-critiquing with "the idea that improved teaching is a joint process and not the responsibility of any individual" (Stigler & Hiebert, 1999, p. 124-125). And finally, the TIMMS study emphasizes that lesson study is a long-term professional development model of gradual, but continuous, improvement (Stigler & Hiebert, 1999).

In contrast, Wang and Lin (2005) state that there is not enough evidence in the existing literature (international comparative) to say conclusively that a strong positive relationship exists between professional development, implementing a curriculum or pedagogical features (standards, organization, teacher content knowledge, etc.), and math performance. The relationships are complex and interactive. All features working together may be necessary to directly influence performance.

The above perspective was confirmed by Desimone, Smith, Baker, and Ueno (2005). They identified perceived barriers to use of a conceptual approach in U.S. math education that have been used to explain why U.S. students do not do as well as international students on math tests:

1. too much individualism and autonomy of teachers (both style and implementation of known strategies to lead to expected reforms
2. more use of conceptual strategies will diminish use of computation – seen as a dichotomy as opposed to complementary
3. U.S. teachers - use conceptual only with high achievers, computation for low achievers
4. too large class sizes in US schools – keep conceptual approach at low use
5. only teachers with a lot of math content and math pedagogy have the ability to use conceptual approach

Their results failed to confirm any of the above as evidence of differences in U. S. versus international student performance.

How* Florida Math Connects *Reflects the Research on Professional Development in Mathematics Teaching

McGraw-Hill's *Florida Math Connects* series provides multiple avenues of assistance for the professional development of teachers. While it focuses primarily on teaching K–8 students and does not purport to be a comprehensive professional development program, the series does include several valuable professional development features in the Teacher's Edition and ancillary materials. These features help teachers develop greater content knowledge and deeper understanding of key math concepts as they teach their students, as recommended by research.

Research indicates the benefits of exposing teachers to different approaches to solve problems. In every problem-solving lesson an alternative strategy is given. This provides the teacher with instruction for solving the problem using another approach and enables the teacher to open the door for discussion of how there are always alternative strategies to solve a problem.

Consistent with the literature regarding the need for pedagogical practice, each chapter in the Teacher Editon series begins by describing the mathematical Big Idea and how it relates to the NGSSS for that grade level. A Skills Trace section (regarding the specific skills considered) is also provided, showing what students should have learned for the previous grade, what they will learn in the chapter being studied and what will be considered in the next grade. As well, there is a Chapter Planner showing objectives, vocabulary, lesson resources, technology, and ways to reach all learners. Detailed daily planning help is provided along with suggested questions to ask students. Scaffolding questions are included for the Teach step in the lesson. Each multi-part lesson includes Focus on Math Background, highlighting the most important aspects of the parts of the lesson. Teachers are directed to the Professional Development Video Library for further support.

The Teacher Reference Handbook includes numerous Professional Development articles on current issues in mathematics education geared to each grade level. These include developmental descriptions of students, providing equity, reaching English language learners, data-driven decision making in assessment, intervention strategies, cognitively guided instruction, literature and mathematics, and cooperative group strategies.

To further improve professional practices, mhpdonline.com combines video and interactive Web content to model instructional strategies and provide a framework for self-assessment by teachers and for assessment by mentors or peer coaches. There is also an opportunity for teachers to share ideas and experiences with each other, benefiting from the collegial support that research indicates improves educators' teaching.

Because *Florida Math Connects* provides a carefully structured sequence of lessons and ample support materials that model and guide teachers to exemplary instructional practice, the program can serve as a component of a district's own comprehensive program of professional development in mathematics instruction. Such a staff development program might include presentations and demonstrations by staff developers, readings for development of conceptual understanding and deeper content knowledge, coaching and mentoring, opportunities to try out new methods and approaches, collegial support groups, and/or lesson study similar to the Japanese model cited in the research.

Since *Florida Math Connects* provides a comprehensive, coherent curriculum, a rich variety of corresponding assessment measures, and related professional development features, using the program as the basis for the district's professional development offering will ensure that staff development, curriculum, and student assessment measures are aligned, as recommended by research. Over the long term, as the research suggests, such a program of professional development, rich in mathematical content knowledge and important mathematical ideas integrated in a logical sequence of lessons, is likely to improve teacher practice—with the ultimate goal of improving students' mathematical proficiency.

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