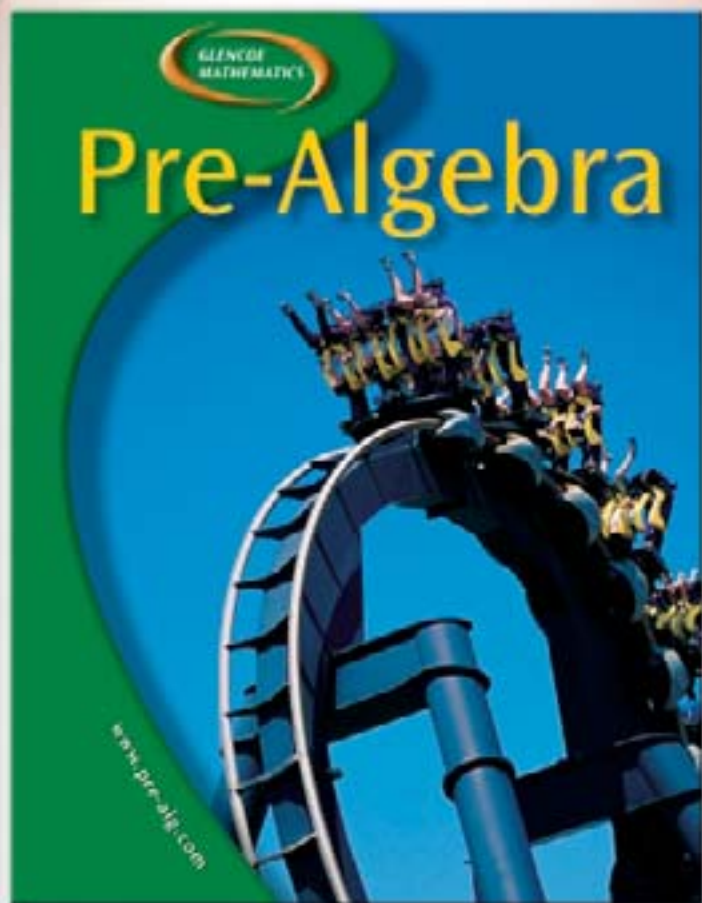


# Research- Based Strategies Used to Develop



*Glencoe Pre-Algebra*



# MATHEMATICS WITH UNDERSTANDING

## The NCTM *Principles and Standards for School Mathematics*

In today's technology-driven society, mathematical skills and understanding are more important than ever. The need to use mathematics with fluency and comfort occurs daily—not just for those in the scientific and technical community, but in the workplace and in everyday situations. Those who understand and can use mathematics will have significantly enhanced opportunities and options for shaping their careers and futures. The past two decades have seen an increased recognition of the importance of mathematics for every student and accompanying need for creating uniform national standards in mathematics education. The National Council of Teachers of Mathematics

(NCTM) has led this reform movement from its beginning.

NCTM is the world's largest mathematics education organization, with more than 100,000 members and 250 affiliates throughout the United States and Canada. Between 1989 and 1995, NCTM released a trio of publications on curriculum and evaluation, assessment, and professional standards to articulate goals for mathematics teachers and policy makers. Since the release of these publications, they have given focus, organization, and fresh ideas to efforts to improve mathematics education.

In 2000, NCTM released its most comprehensive project—the *Principles and Standards for School Mathematics*. The *Principles and*

### History of the Mathematics Education Reform Movement

- 1989 National Council of Teachers of Mathematics (NCTM) publishes *Curriculum and Evaluation Standards for School Mathematics*
- 1991 NCTM publishes *Professional Standards for Teaching Mathematics*
- 1995 NCTM publishes *Assessment Standards for School Mathematics*
- 1995 NCTM appoints the Commission on the Future of Standards to oversee the *Standards 2000* project
- 1997 The Commission on the Future of Standards appoints the Standards 2000 Writing Group and the Standards 2000 Electronic Format Group
- 1997 to 1999 The Standards 2000 Writing Group, with input from Association Review Groups, the NCTM Research Advisory Committee, the National Research Council, and more than 650 individuals and 70 groups, writes the *Principles and Standards for School Mathematics*
- 2000 NCTM publishes the *Principles and Standards for School Mathematics*

All *Standards* documents are available at [www.nctm.org](http://www.nctm.org).

*Standards for School Mathematics* represents the culmination of five years of development by the NCTM's Commission on the Future of the Standards and their Standards 2000 Writing Group. The Standards 2000 Writing Group included teachers, teacher educators, administrators, researchers, and mathematicians with a wide range of expertise. The first draft was released in 1998. Over 650 individuals and more than 70 groups, including a committee of experts from the National Research Council, provided assistance and feedback, and the final version of the *Principles and Standards for School Mathematics* was released in 2000.

The *Principles* set forth important overall characteristics of mathematics programs, and the *Standards* describe the mathematical content that students should learn. Together, the *Principles and Standards for School Mathematics* constitutes a vision to guide educators as they strive for the continual improvement of mathematics education in classrooms, schools, and educational systems. The *Principles and Standards for School Mathematics* are consistent with the best and most recent evidence on teaching and learning mathematics; they are chosen through a complex process that involves past practice, research findings, societal expectations, and the vision of the professional field (Heibert, 1999).

The vision for mathematics education described in the *Principles and Standards for School Mathematics* is highly ambitious.

Achieving this vision requires committed, competent, and knowledgeable teachers who can integrate instruction with assessment, administrative policies that support learning and access to technology, and solid mathematics curricula.

### **ACHIEVING THE NCTM PRINCIPLES AND STANDARDS FOR SCHOOL MATHEMATICS WITH GLENCOE PRE-ALGEBRA**

Realizing the *Principles and Standards for School Mathematics* requires raising expectations for students' learning, developing effective methods of supporting the learning of mathematics by all students, and providing students and teachers with the resources and curricula they need. A school's or district's choice of mathematics curriculum can be a strong determinant of what students have an opportunity to learn.

Glencoe/McGraw-Hill, one of the nation's largest textbook developers, has risen to the challenge set by the *Principles and Standards for School Mathematics* and developed *Glencoe Pre-Algebra*. This text was specifically designed with several key characteristics recommended by the *Principles and Standards for School Mathematics* for effective curricula.

- Different topical strands, such as algebra, geometry, and statistics, that are highly interconnected;

### **NCTM Principles for School Mathematics**

**Equity** Excellence in mathematics education requires equity—high expectations and strong support for all students.

**Curriculum** A curriculum is more than a collection of activities; it must be coherent, focused on important mathematics, and well articulated across the grades.

**Teaching** Effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well.

**Learning** Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge.

**Assessment** Assessment should support the learning of important mathematics and furnish useful information to both teachers and students.

**Technology** Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning.

*Principles and Standards for School Mathematics* is available at [www.nctm.org](http://www.nctm.org).

- Central mathematical ideas that are organized and integrated, so that students can see how the ideas build on, or connect with, other ideas;
- Foundational ideas such as integers, fractions, ratio, proportion, and percent;
- Activities to facilitate development of mathematical thinking and reasoning skills, including making conjectures and developing sound deductive arguments;
- Opportunities for experiences that demonstrate mathematics' usefulness in modeling and predicting real-world phenomena;
- Guidance for teachers on the depth of study warranted at particular times and when closure is expected for particular skills or concepts;
- Emphasis on the mathematics processes and skills that support the quantitative literacy of students, such as judging claims, finding fallacies, evaluating risks, and weighing evidence.

## PRINCIPLES

The *Glencoe Pre-Algebra* text was designed to meet all six of the Principles set forth in the *Principles and Standards for School Mathematics*.

- **Equity** *Glencoe Pre-Algebra* encourages high achievement at all levels. Numerous teacher support materials provide activities for differentiated instruction, promotion of reading and writing skills, pacing for individual levels of achievement, and daily intervention opportunities. In the Teacher Wraparound Edition of *Glencoe Pre-Algebra*, Differentiated Instruction strategies focus on eight learning styles: verbal/linguistic, logical, visual/spatial, auditory/musical, kinesthetic, interpersonal, intrapersonal, and naturalist. Daily Intervention features provide suggestions for reaching all learners and helping students who are having difficulty. Skills Practice Masters allows students who are progressing at a slower pace to practice the concept using easier problems, while the Practice Masters provides more challenging problems for students who are moving at a regular or faster pace.
- **Curriculum** *Glencoe Pre-Algebra* was developed with a philosophy, scope and sequence to ensure a continuum of mathematical learning that builds on prior knowledge and extends concepts toward more advanced mathematical thinking. Chapters are grouped into units to bring depth to algebraic concepts, and most lessons are divided into two related objectives to allow teachers flexibility in presenting the lesson. *Glencoe Pre-Algebra* provides students with a solid conceptual foundation for success in algebra 1, geometry, and algebra 2.
- **Teaching** The comprehensive Teacher Wraparound Edition for *Glencoe Pre-Algebra* provides mathematical background, teaching tips, and resource management guidelines. Tips for New Teachers provide helpful suggestions for classroom management, teaching techniques, and assessment. Suggestions are provided for average and advanced pacing for both year-long schedules and block schedules, so that educators can create a pre-algebra course that meets the needs of each class. Glencoe also offers a Staff Development Series to help educators implement effective research-based mathematics strategies and enhance classroom performance. Publications include *Using the Internet in the Mathematics Classroom*, *Reading and Writing in the Mathematics Classroom*, *Teaching Mathematics with Foldables™*, and *Teaching Pre-Algebra with Manipulatives*.
- **Learning** *Glencoe Pre-Algebra* offers extensive support to help all students achieve success in mathematics. Key Concept and Concept Summary boxes in the Student Edition help students identify main concepts, and Study Tips in the margins help students understand new material. The Teacher Wraparound Edition includes instruction on building from prior knowledge with materials in each interleaf and in Building On Prior Knowledge features. Find the Error and Unlocking Misconception teaching tips help to evaluate how students are thinking and learning.

■ **Assessment** *Glencoe Pre-Algebra* offers multiple strategies for the teacher to assess student learning, as well as student self-assessment. Standardized Test Practice questions and Standardized Test Practice Examples appear throughout the Student Edition and help students learn how to approach test questions. Two pages of Standardized Test Practice at the end of each chapter include multiple-choice, short-response, quantitative-comparison, and open-ended questions. The Fast File Chapter Resource Masters offer a variety of assessment resources for each chapter: six chapter tests, open-ended assessment with scoring rubric, vocabulary test and review, four quizzes, a mid-chapter test, and cumulative review. Online Study Tools, such as Self-Check Quizzes, offer a unique way for students to use Internet access to monitor their progress.

■ **Technology** The *Glencoe Pre-Algebra* Student Edition offers opportunities to utilize graphing calculators and spreadsheets in the exploration of pre-algebra concepts. The Teacher Wraparound Edition offers teaching tips on using technology, and the Graphing Calculator and Computer Masters offer additional activities. Glencoe's Web site is constantly updated to meet the needs of students and teachers in excelling in mathematics education. WebQuest Internet Projects also provide project ideas in which students do research on the Internet, gather data, and make presentations using word processing, graphing, or presentation software. *Using the Internet in the Mathematics Classroom*, a Glencoe Staff Development Series publication, provides guidelines for using the Internet, as well as a guide to additional mathematical resources on the Internet.

## STANDARDS

The *Glencoe Pre-Algebra* text was also designed to meet all of the *Principles and Standards for School Mathematics'* Content Standards. The Content Standards state that

instructional programs from pre-kindergarten through grade 12 should enable all students to master specific skills in ten content areas.

### How *Glencoe Pre-Algebra* Meets the Standards

<b>Content Area and Specific Standards from NCTM <i>Principles and Standards in School Mathematics</i></b>	<b>Examples from <i>Glencoe Pre-Algebra</i> (page numbers)</b>
<b>Numbers and Operations</b> <ul style="list-style-type: none"> <li>• Understand numbers, ways of representing numbers, relationships among numbers, and number systems</li> <li>• Understand the meaning of operations and how they relate to each other</li> <li>• Compute fluently and make reasonable estimates</li> </ul>	6–44, 56–84, 98–107, 148–190, 200–236, 238–242, 244–248, 264–315, 340–359, 436–445, 453–482, 506–511, 520–531, 533–543, 563–582, 584–594, 641–657, 674–686
<b>Algebra</b> <ul style="list-style-type: none"> <li>• Understand patterns, relations, and functions</li> <li>• Represent and analyze mathematical situations and structures using algebraic symbols</li> <li>• Use mathematical models to represent and understand quantitative relationships</li> <li>• Analyze change in various contexts</li> </ul>	17–21, 40–44, 56–61, 64–89, 98–137, 148–190, 210–231, 237, 244–253, 264–280, 288–292, 298–302, 304–315, 328–359, 368–391, 393–423, 441–445, 453–482, 492–497, 500–504, 513–517, 563–582, 584–588, 669–697
<b>Geometry</b> <ul style="list-style-type: none"> <li>• Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships</li> <li>• Specify locations and describe spatial relationships using coordinate geometry and other representational systems</li> <li>• Apply transformations and use symmetry to analyze mathematical situations</li> <li>• Use visualization, spatial reasoning, and geometric modeling to solve problems</li> </ul>	85–89, 131–137, 175–179, 210–219, 249–253, 286–287, 304–308, 334–338, 393–397, 447–482, 492–543, 554–594, 673, 682, 687–697
<b>Measurement</b> <ul style="list-style-type: none"> <li>• Understand measurable attributes of objects and the units, systems and processes of measurement</li> <li>• Apply appropriate techniques, tools and formulas to determine measurements</li> </ul>	23–27, 169–173, 186–190, 210–219, 238–242, 264–280, 286–287, 386, 392, 436–440, 447–482, 492–504, 512, 518–531, 533–543, 562–594, 673, 682

<p style="text-align: center;"><b>Content Area and Specific Standards from NCTM <i>Principles and Standards in School Mathematics</i></b></p>	<p style="text-align: center;"><b>Examples from <i>Glencoe Pre-Algebra</i> (page numbers)</b></p>
<p><b>Data Analysis and Probability</b></p> <ul style="list-style-type: none"> <li>• Formulate questions that can be addressed with data and collect, organize and display relevant data to answer them</li> <li>• Select and use appropriate statistical methods to analyze data</li> <li>• Develop and evaluate inferences and predictions that are based on data</li> <li>• Understand and apply basic concepts of probability</li> </ul>	<p>40–44, 210–219, 237–243, 275, 303, 309–315, 409–413, 583, 606–657</p>
<p><b>Problem Solving</b></p> <ul style="list-style-type: none"> <li>• Build new mathematical knowledge through problem solving</li> <li>• Solve problems that arise in mathematics and in other contexts</li> <li>• Apply and adapt a variety of appropriate strategies to solve problems</li> <li>• Monitor and reflect on the process of mathematical problem solving</li> </ul>	<p>6–10, 28–32, 56–89, 98–107, 110–137, 148–185, 200–230, 232–236, 238–253, 270–280, 288–309, 330–338, 345–359, 369–385, 387–391, 393–401, 404–422, 436–440, 447–482, 492–497, 500–504, 513–517, 520–531, 533–543, 556–594, 606–621, 623–628, 635–639, 641–657, 697</p>
<p><b>Reasoning and Proof</b></p> <ul style="list-style-type: none"> <li>• Recognize reasoning and proof as fundamental aspects of mathematics</li> <li>• Make and investigate mathematical conjectures</li> <li>• Develop and evaluate mathematical arguments and proofs</li> <li>• Select and use various types of reasoning and methods of proof</li> </ul>	<p>137, 148–152, 175–190, 205–209, 220–231, 249–253, 286–292, 368, 386, 392, 402–403, 458–464, 520–525, 533–538, 562, 583, 635–640, 650–657, 673, 697</p>
<p><b>Communication</b></p> <ul style="list-style-type: none"> <li>• Organize and consolidate their mathematical thinking through communication</li> <li>• Communicate their mathematical thinking coherently and clearly to peers, teachers, and others</li> <li>• Analyze and evaluate the mathematical thinking and strategies of others</li> <li>• Use the language of mathematics to express mathematical ideas precisely</li> </ul>	<p>148–190, 249–253, 275, 288–297, 303, 309–314, 340–349, 404–408, 436–440, 453–481, 492–497, 500–504, 506–517, 520–531, 533–543, 556–561, 563–594, 630–633, 656–657, 682</p>
<p><b>Connections</b></p> <ul style="list-style-type: none"> <li>• Recognize and use connections among mathematical ideas</li> <li>• Understand how mathematical ideas build on one another to produce a coherent whole</li> <li>• Recognize and apply mathematics in contexts outside of mathematics</li> </ul>	<p>98–107, 110–137, 148–190, 249–253, 264–315, 355–359, 392–397, 402–403, 453–457, 471–482, 492–497, 500–504, 506–511, 513–517, 520–531, 533–543, 556–582, 584–594, 612–621, 641–657, 669–697</p>
<p><b>Representation</b></p> <ul style="list-style-type: none"> <li>• Create and use representations to organize, record, and communicate mathematical ideas</li> <li>• Select, apply, and translate among mathematical representations to solve problems</li> <li>• Use representations to model and interpret physical, social, and mathematical phenomena</li> </ul>	<p>56–61, 85–89, 108–109, 137, 148–190, 210–214, 232–236, 264–292, 298–315, 328–329, 368, 374–385, 387–423, 471–481, 520–532, 539–543, 554–561, 606–611, 617–640, 669–697</p>

## RESEARCH-BASED INSTRUCTIONAL STRATEGIES USED IN *GLENCOE PRE-ALGEBRA*

In addition to responding to the goals set by the *Principles and Standards for School Mathematics*, extensive efforts were undertaken to ensure that the latest research on best practices in mathematics education was used in the development of *Glencoe Pre-Algebra*. Educational research serves as a basis for many of the assertions made throughout about what is possible for students to learn about certain content areas at certain levels and under certain pedagogical conditions.

*Glencoe Pre-Algebra* was specifically designed to utilize several important research-based instructional strategies that reinforce the *Principles and Standards for School Mathematics*.

### 1. Balancing implicit and explicit learning

Research shows us that teachers cannot simply transfer knowledge to students by lecturing. Students have to take an active role in their own learning, and to accomplish this, mathematics programs must include ample opportunity to explore, question, discuss, and discover. This is not to say that teachers are removed from the educational process. Rather, the learning experience should include a balance of implicit and explicit instruction. Implicit instruction occurs when students figure out for themselves how to grapple with problems and construct conceptual knowledge (Pressley, Harris, & Marks, 1992; Shulman & Keislar, 1996). Explicit instruction occurs when teachers and textbooks clearly explain problem-solving strategies to students in a direct, low-inference fashion (Duffy, 2002).

*Glencoe Pre-Algebra* offers a balanced approach of real-world applications, hands-on exercises, writing exercises, and practice that enables students to develop both conceptual understanding and procedural knowledge. Many lessons begin with real-world problems for students to solve, and then students use multiple representations to explore new concepts (see Figure 1). Differentiated Instruction strategies in the Teacher Wrap-

**3-7 Using Formulas**

**What You'll Learn**

- Solve problems by using formulas.
- Solve problems involving the perimeters and areas of rectangles.

**Vocabulary**

- formula
- perimeter
- area

**Why are formulas important in math and science?**

The top recorded speed of a mallard duck in level flight is 65 miles per hour. You can make a table to record the distances that a mallard could fly at that rate.

a. Write an expression for the distance traveled by a duck in  $t$  hours.

b. What disadvantage is there in showing the data in a table?

c. Describe an easier way to summarize the relationship between the speed, time, and distance.

Speed (mph)	Time (hr)	Distance (mi)
65	1	65
65	2	130
65	3	195
65	$t$	?

**FORMULAS** A **formula** is an equation that shows a relationship among certain quantities. A formula usually contains two or more variables. One of the most commonly-used formulas shows the relationship between distance, rate (or speed), and time.

**Words** Distance equals the rate multiplied by the time.

**Variables** Let  $d$  = distance,  $r$  = rate, and  $t$  = time.

**Equation**  $d = rt$

**Example 1 Use the Distance Formula**

**SCIENCE** What is the rate in miles per hour of a dolphin that travels 120 miles in 4 hours?

$$d = rt \quad \text{Write the formula.}$$

$$120 = r \cdot 4 \quad \text{Replace } d \text{ with 120 and } t \text{ with 4.}$$

$$\frac{120}{4} = \frac{r \cdot 4}{4} \quad \text{Divide each side by 4.}$$

$$30 = r \quad \text{Simplify.}$$

The dolphin travels at a rate of 30 miles per hour.

**Concept Check** Name an advantage of using a formula to show a relationship among quantities.

Lesson 3-7 Using Formulas 131

Figure 1

*Glencoe Pre-Algebra*, page 131

around Edition provide suggestions in eight different learning styles: verbal/linguistic, logical, visual/spatial, auditory/musical, kinesthetic, interpersonal, intrapersonal, and naturalist. Calculator and Spreadsheet Investigations use technology to promote discovery of patterns and relationships.

### 2. Using prior knowledge to learn new information

Prior knowledge strategies help students retrieve information stored in their long-term memories to learn new, related information. These strategies include recalling remembered information, asking questions, and elaborating on textbook and teacher information, and referring students to the textbook (including use of analogies) and other meaningful information. Asking students to use prior knowledge located in a text may remind them of information already in their long-term memory that, for some reason, is not easily remembered (Bransford, 1979; Pressley & McCormick, 1995).

*Glencoe Pre-Algebra* intertwines concepts and continuously refers to material in previous chapters and in students' personal experiences to make mathematics more relevant. Prerequisite Skills at the beginning of each chapter and in each lesson assess student readiness. In the Teacher Wraparound Edition, Building on Prior Knowledge links what students have previously learned to the content of the current chapter. Mathematical Connections and Background provides an overview of the mathematics in the chapter and links to prior knowledge and future concepts (see Figure 2).

### 3. Practicing important tasks and skills

Providing students with practice on important tasks has long been considered a successful strategy to improve understanding and memory. Giving students individual feedback on their practice helps in monitoring and fostering their mathematical learning. Practicing helps students acquire additional information as they search and productively struggle, with teacher guidance, for understanding and application of mathematical information. Research shows that mastering

a skill requires focused practice. During practice, students adapt and shape what they have learned. In doing so, they increase their conceptual understanding of the skill (Clement, Lockhead, & Mink, 1979; Davis, R.B., 1984; Mathematical Science Education Board, 1990; Romberg & Carpenter, 1986).

Check for Understanding in each lesson of *Glencoe Pre-Algebra* ensures that students understand the concepts and skills of the lesson. After students work through the Guided Practice as a class, they are ready to work through practice exercises on their own, either in class or as homework. Practice and Apply exercises can be assigned, and higher-difficulty exercises are marked in the Teacher Wraparound Edition to provide further challenges and practice for students. The Mixed Review section in every lesson includes spiraled, cumulative exercises from the two previous lessons as well as earlier lessons. Here students practice the important skills learned earlier, building toward mastery. Each lesson also contains at least one Standardized Test problem. (See Figure 3 on the next page.)

Each chapter provides two pages of Standardized Test Practice problems correlated to prerequisite skills and to the content in previous chapters. This enables students to practice skills in a different form and to gain experience in taking standardized tests.

### 4. Note-Taking

In the process of note-taking, students identify the important items from reading and write that information in an organized format. While writing and drawing notes, students see relationships within the information. Notes need not be verbatim; note-taking is most valuable when students learn to analyze information and select the important points (Bretzing & Kulhary, 1979). When study skills, such as note-taking, are taught within the teaching of content, they promote learner activity and improve metacognition (Hattie et al., 1996; Robinson & Kiewra, 1995).

*Glencoe Pre-Algebra* includes instructions for study organizers, called Foldables™, created by Dinah Zike (see Figure 4). Foldables™ are handmade paper booklets, folded and

Figure 2

**Chapter 9 Mathematical Connections and Background**

**Continuity of Instruction**

**Prior Knowledge**  
In Chapter 7, students learned to solve equations with variables on each side of the equation. They also learned to write and graph inequalities, and to solve multi-step inequalities. In Chapter 8, students wrote and graphed equations, found slopes of lines, and used slope to describe rates of change. Students also learned to use best-fit lines to make predictions.

**This Chapter**  
Students learn to use squares and square roots, classify angles and triangles, and find missing angle measurements in triangles. They use the Pythagorean Theorem to explore right triangles, and study the Distance and Midpoint Formulas. Students also identify and use properties of similar figures. In addition, they learn to use trigonometric ratios to solve problems.

**Future Connections**  
The properties of right triangles are the basis for the study of trigonometry, which is related to geometry. Trigonometry was developed to calculate distances and angles in various fields such as cartography, astronomy, and surveying. The Distance Formula and Midpoint Formula will be used in Calculus.

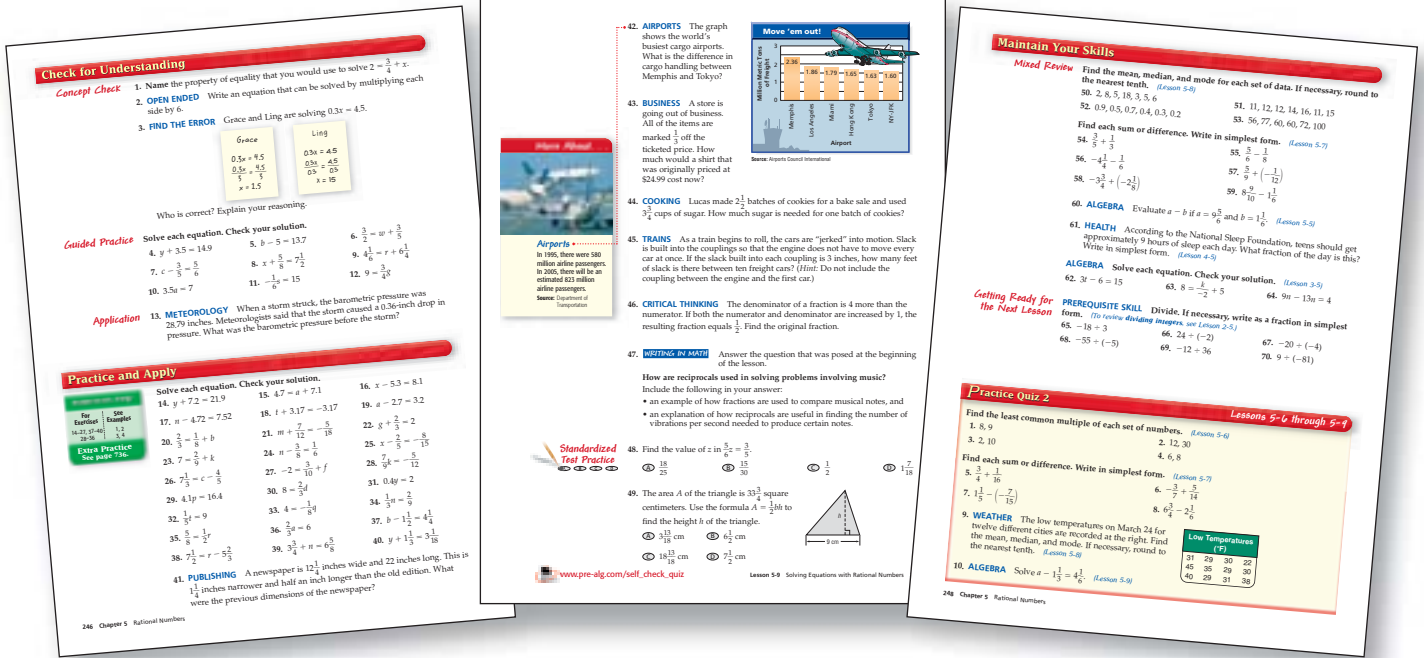
**9-1 Squares and Square Roots**  
The square root of a number is one of two equal factors of a number. Every positive number has a positive and a negative square root. For example, since  $8 \cdot 8 = 64$ , 8 is a square root of 64. Since  $(-8) \cdot (-8) = 64$ , -8 is a square root of 64. The notation  $\sqrt{\quad}$  indicates the positive square root,  $-\sqrt{\quad}$  indicates the negative square root, and  $\pm\sqrt{\quad}$  indicates both square roots. For example,  $\sqrt{81} = 9$ ,  $-\sqrt{81} = -9$ , and  $\pm\sqrt{4} = \pm 2$ . A negative number has no real square root because the square root of a number is never negative. That is,  $\sqrt{-32}$  is not possible.

**9-2 The Real Number System**  
The real number system is made up of the sets of rational and irrational numbers. Rational numbers are numbers that can be written in the form  $\frac{a}{b}$  where  $a$  and  $b$  are integers and  $b \neq 0$ . Examples are  $0.45$ ,  $\frac{1}{2}$ , and  $\sqrt{0.36}$ . Irrational numbers are non-repeating or non-terminating decimals. Examples are  $\sqrt{71}$ ,  $\pi$ , and  $0.02002002\dots$

**9-3 Angles**  
Two rays that have the same endpoint form an angle. The common endpoint is called the vertex, and the two rays that make up the angle are called the sides of the angle. The most common unit of measure for angles is the degree. Protractors can be used to measure angles or to draw an angle of a given measure.  
Angles can be classified by their degree measure. Acute angles have measures less than  $90^\circ$  but greater than  $0^\circ$ . Obtuse angles have measures greater than  $90^\circ$  but less than  $180^\circ$ . Right angles have measures of  $90^\circ$ .

**9-4 Triangles**  
A triangle is a figure formed by three line segments that intersect only at their endpoints. The sum of the measures of the angles of a triangle is  $180^\circ$ .  
Triangles can be classified by their angles. An acute triangle contains all acute angles. An obtuse triangle has one obtuse angle. A right triangle has one right angle.  
Triangles also can be classified by their sides. A scalene triangle has no congruent sides. An isosceles triangle has at least two sides congruent. An equilateral triangle has all sides congruent.

434C Chapter 9 Real Numbers and Right Triangles



Glencoe Pre-Algebra, pages 246–248

Figure 3

cut into tabs. Designed to fit each chapter's content, the Foldable™ guides students in choosing the important concepts and recording them in an organized format. Since students make their own three-dimensional Foldables as well as enter the notes, they feel a sense of ownership. In addition, the Study Guide and Review feature at the end of each chapter consists of concept summaries plus worked examples and practice exercises. The Lesson-by-Lesson Review presents a clear picture of the important concepts in each lesson and provides students with a model for how they might take notes.

### 5. Cooperative learning

Cooperative learning occurs when students work in pairs or groups of three or four to complete tasks. Research shows that cooperative learning provides practice at valuable skills, such as positive interdependence, face-to-face interactions, individual and group accountability, interpersonal skills, and group processing (Johnson & Johnson, 1999). Cooperative learning has a highly positive effect when compared with strategies in which students compete with each other and strategies in which students work on tasks individually (Johnson, Maruyama,

Johnson, Nelson & Skon, 1981). A balance of cooperative learning and individual learning is needed, however, because students need time to practice skills independently (Anderson, Keder, & Simon, 1997).

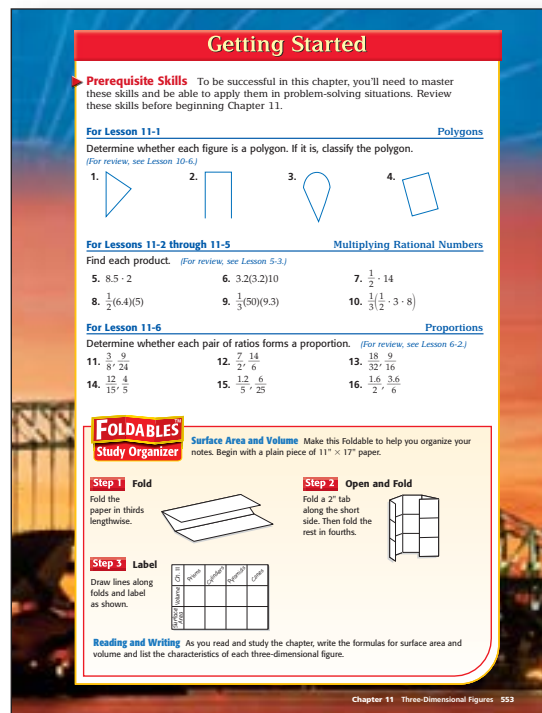


Figure 4

Glencoe Pre-Algebra, page 553

Figure 5

**Algebra Activity**  
A Preview of Lesson 1-7

### Scatter Plots

Sometimes, it is difficult to determine whether a relationship exists between two sets of data by simply looking at them. To determine whether a relationship exists, we can write the data as a set of ordered pairs and then graph the ordered pairs on a coordinate system.

**Collect the Data**  
Let's investigate whether a relationship exists between height and arm span.

**Step 1** Work with a partner. Use a centimeter ruler to measure the length of your partner's height and arm span to the nearest centimeter. Record the data in a table like the one shown.

Name	Height (cm)	Arm Span (cm)

**Step 2** Extend the table. Combine your data with that of your classmates.

**Step 3** Make a list of ordered pairs in which the  $x$ -coordinate represents height and the  $y$ -coordinate represents arm span.

**Step 4** Draw a coordinate grid like the one shown and graph the ordered pairs (height, arm span).

**Analyze the Data**

- Does there appear to be a trend in the data? If so, describe the trend.

**Make a Conjecture**

- Estimate the arm span of a person whose height is 60 inches. 72 inches.
- How does a person's arm span compare to his or her height?
- Suppose the variable  $x$  represents height, and the variable  $y$  represents arm span. Write an expression for arm span.

**Extend the Activity**

- Collect and graph data to determine whether a relationship exists between height and shoe length. Explain your results.

Algebra Activity Scatter Plots 39

Glencoe Pre-Algebra, page 39

Glencoe Pre-Algebra was designed to provide a mix of individual and cooperative learning opportunities. In Algebra Activities (see Figure 5) and Geometry Activities in the Student Edition, students work together

in small groups to collect data, analyze data, and make conjectures. Differentiated Instruction suggestions, located in the Teacher Wraparound Edition, offer ideas for interpersonal techniques utilizing pairs and groups. MindJogger Videoquizzes offer practice in an interactive, game-show format, with students working in teams to earn points. WebQuest Internet Projects also offer the opportunity for team projects, in which students do research on the Internet, gather data, and make presentations.

## 6. Emphasis on reading and writing in mathematics

Reading and writing are two of the major activities through which students acquire mathematical knowledge, and the reliance on both increases as the grade level increases. Becoming fluent in the language of mathematics requires unique knowledge and skills, since mathematical writing may be dense, terse, and comprised of symbols as well as words (Barton & Heidema, 2002). Reading helps students understand concepts, work problems, organize ideas, extend their thinking, and view mathematics as a valuable subject (Martinez & Martinez, 2001). Mastering mathematical vocabulary is key and has been shown to increase achievement in arithmetic and problem-solving (Earp & Tanner, 1980; Helwig, Rozek-Tedesco, Tindal, Heath & Almond, 1999). Writing is also effective in helping students learn mathematical concepts (Able & Able, 1988). Frequent, regular writing in mathematics class has been linked to improved quality of writing, improved student attitudes toward mathematics, and teachers' increased insight into their teaching (Miller & England, 1989).

Glencoe Pre-Algebra uses a variety of techniques and features to incorporate reading and writing in mathematical learning. Reading Mathematics pages (see Figure 6) and Reading Math Tips help students master new mathematical vocabulary words and develop technical reading skills. Writing in Math questions in every lesson require students to summarize what they have learned in the lesson, using critical thinking skills to develop their answers. Vocabulary terms are listed at the beginning of each lesson, and the Vocabulary and Concept Check in each Study Guide

Figure 6

### Reading Mathematics

### Meanings of At Most and At Least

The phrases *at most* and *at least* are used in mathematics. In order to use them correctly, you need to understand their meanings.

Phrase	Meaning	Mathematical Symbol
at most	<ul style="list-style-type: none"> <li>no more than</li> <li>less than or equal to</li> </ul>	$\leq$
at least	<ul style="list-style-type: none"> <li>no less than</li> <li>greater than or equal to</li> </ul>	$\geq$

Here is an example of one common use of each phrase, its meaning, and a mathematical expression for the situation.

<b>Verbal Expression</b>	You can spend <i>at most</i> \$20.
<b>Meaning</b>	You can spend \$20 or any amount less than \$20.
<b>Mathematical Expression</b>	$s \leq 20$ , where $s$ represents the amount you spend.

<b>Verbal Expression</b>	A person must be <i>at least</i> 18 to vote.
<b>Meaning</b>	A person who is 18 years old or any age older than 18 may vote.
<b>Mathematical Expression</b>	$a \geq 18$ , where $a$ represents age.

Notice that the word *or* is part of the meaning in each case.

**Reading to Learn**

- Write your own rule for remembering the meanings of *at most* and *at least*.

For each expression, write the meaning. Then write a mathematical expression using  $\leq$  or  $\geq$ .

- You need to earn at least \$50 to help pay for a class trip.
- The sum of two numbers is at most 6.
- You want to drive at least 250 miles each day.
- You want to hike 4 hours each day at most.

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checks students' understanding of the key concepts of the chapter. *Reading and Writing in the Mathematics Classroom*, a Glencoe Staff Development Series publication, explains the importance of reading and writing in mathematics and provides specific activities to help students, including English-language learners, master crucial skills.

## 7. Use of high-quality visuals to communicate, organize, and reinforce mathematical learning

Visuals—such as complex diagrams and elaborate drawings—used in conjunction with verbal description increase students' chances of learning, understanding, and remembering relationships and properties of mathematics concepts. Visuals are often the only way to effectively communicate ideas that explain central concepts needed to understand algebra and geometry. Research shows that students are better able to organize and group ideas when visuals illustrate different and common characteristics (Hegarty, Carpenter, & Just, 1991). Also, the mental images that high-quality visuals encourage are an indispensable tool for recalling information, especially compared to information presented with only text or lower-quality visuals (Willows & Houghton, 1987).

*Glencoe Pre-Algebra* includes high-quality charts, tables, graphs, art, and photographs throughout the Student Edition (see Figure 7). Visuals are often accompanied by questions and ideas for effective use of models and manipulatives. In an exclusive partnership with Glencoe/McGraw-Hill, USA TODAY® Education has provided USA TODAY Snapshots® that make *Glencoe Pre-Algebra* come alive with current, relevant data in eye-catching graphs, charts, and tables.

13-4


### Multiplying a Polynomial by a Monomial

**What You'll Learn**

- Multiply a polynomial by a monomial.

**How** is the Distributive Property used to multiply a polynomial by a monomial?

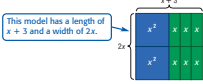
The Grande Arche office building in Paris, France, looks like a hollowed-out prism, as shown in the photo at the right.



$2x - 6z$

- Write an expression that represents the area of the rectangular region outlined on the photo.
- Recall that  $2(4 + 1) = 2(4) + 2(1)$  by the Distributive Property. Use this property to simplify the expression you wrote in part a.
- The Grande Arche is approximately 40 feet deep. Explain how you can write a polynomial to represent the volume of the hollowed-out region of the building. Then write the polynomial.

**MULTIPLY A POLYNOMIAL AND A MONOMIAL** You can model the multiplication of a polynomial and a monomial by using algebra tiles.



This model has a length of  $x + 3$  and a width of  $2x$ .

The model shows the product of  $2x$  and  $x + 3$ . The rectangular arrangement contains  $2x^2$  tiles and  $6x$  tiles. So, the product of  $2x$  and  $x + 3$  is  $2x^2 + 6x$ . In general, the Distributive Property can be used to multiply a polynomial and a monomial.

**Study Tip**  
Look Back  
To review the Distributive Property, see Lesson 3-1.

**Example 1 Products of a Monomial and a Polynomial**

Find each product.

- $$4(5x + 1) = 4(5x) + 4(1) \quad \text{Distributive Property}$$

$$= 20x + 4 \quad \text{Simplify.}$$
- $$(2x - 6)(3x) = 2x(3x) - 6(3x) \quad \text{Distributive Property}$$

$$= 6x^2 - 18x \quad \text{Simplify.}$$

[www.pre-alg.com/extra\\_examples](http://www.pre-alg.com/extra_examples)

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Figure 7

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## **SUMMARY**

Glencoe/McGraw-Hill is committed to the idea that curricula should strive to reach all of the *Principles and Standards for School Mathematics*, thereby providing road maps that help teachers guide students to increasing levels of sophistication and depths of knowledge. The NCTM *Principles and Standards for School Mathematics* were developed to accomplish several goals, including guiding the development of curriculum frameworks, assessments, and other instructional materials. Attaining the vision of the *Principles and Standards for School Mathematics* will require the talents, energy, and attention of many individuals, including students, teachers, school administrators, policy makers, teacher educators, parents, mathematicians, local communities, and curriculum developers. Glencoe is proud to provide the *Glencoe Pre-Algebra* text as an informed road map to excellence in mathematics education in the 21st century.

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