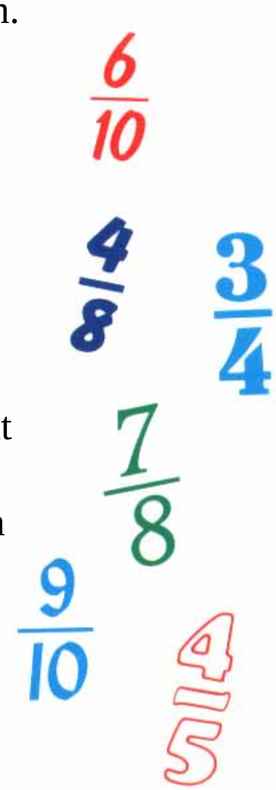
A Disturbing Distribution

Problem-of-the-Week

The Problem

There is a distributive property for multiplication over addition. Do you think there is a distributive property for addition over multiplication? Such a property would be represented by the algebraic sentence, $a + (b \cdot c) = (a + b) \cdot (a + c)$.



Does this property hold for whole numbers? Try it by letting $a = 2$, $b = 3$, and $c = 4$.

Does the property hold for fractions? Try some and see. Now test the property for $a = \frac{1}{6}$, $b = \frac{1}{4}$, and $c = \frac{7}{12}$. Find other combinations of fractions for which the statement is true. What relationship must the fractions have for them to satisfy the statement? Explain why the statement is true for fractions with this relationship.

Strategies and Hints

1. Express the fractions you use with common denominators.
2. What is the sum of each set of fractions that you can use in the statement?
3. To explain why this happens, use fractions such as $\frac{a}{b}$, $\frac{c}{d}$, and $\frac{e}{f}$ to do the arithmetic.

Extensions

1. Do these fractions also satisfy a possible distributive property of subtraction over multiplication?
2. Do these fractions satisfy possible distributive properties of addition or subtraction over division?