

## Lesson 10-2

### Example 1 Two Roots

Solve  $x^2 + 12 = -8x$  by graphing.

First rewrite the equation so one side is equal to zero.

$$x^2 + 12 = -8x \quad \text{Original equation}$$

$$x^2 + 12 + 8x = -8x + 8x \quad \text{Add } 8x \text{ to each side.}$$

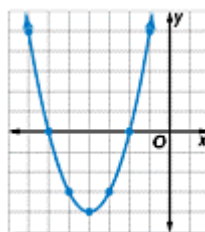
$$x^2 + 8x + 12 = 0 \quad \text{Simplify.}$$

Graph the related function  $f(x) = x^2 + 8x + 12$ . The equation of the axis of symmetry is  $x = -\frac{8}{2(1)}$  or

$x = -4$ . When  $x = -4$ ,  $f(x)$  equals  $(-4)^2 + 8(-4) + 12$  or  $-4$ . So, the coordinates of the vertex are  $(-4, -4)$ .

Make a table of values to find other points to sketch the graph.

$x$	$f(x)$
-7	5
-6	0
-5	-3
-4	-4
-3	-3
-2	0
-1	5



To solve  $x^2 + 8x + 12 = 0$ , you need to know where the value of  $f(x)$  is 0. This occurs at the  $x$ -intercepts. The  $x$ -intercepts of the parabola appear to be  $-6$  and  $-2$ .

**CHECK:** Solve by factoring.

$$x^2 + 8x + 12 = 0$$

$$(x + 6)(x + 2) = 0$$

$$x + 6 = 0 \quad \text{or} \quad x + 2 = 0$$

$$x = -6 \quad \quad \quad x = -2$$

Original equation

Factor.

Zero Product Property

Solve for  $x$ .

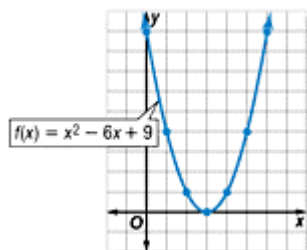
The solutions of the equation are  $-6$  and  $-2$ .

### Example 2 A Double Root

Solve  $x^2 - 6x + 9 = 0$  by graphing.

Graph the related function  $f(x) = x^2 - 6x + 9$ .

$x$	$f(x)$
0	9
1	4
2	1
3	0
4	1
5	4
6	9



Notice that the vertex of the parabola is the  $x$ -intercept. Thus, one solution is 3. What is the other solution?

Try solving the equation by factoring.

$$x^2 - 6x + 9 = 0$$

Original equation

$$(x - 3)(x - 3) = 0$$

Factor.

$$x - 3 = 0 \quad \text{or} \quad x - 3 = 0$$

Zero Product Property

$$x = 3 \quad x = 3$$

Solve for  $x$ .

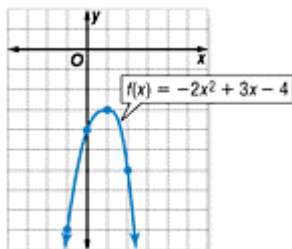
There are two identical factors for the quadratic function, so there is only one root, called a double root. The solution is 3.

### Example 3 No Real Roots

Solve  $-2x^2 + 3x - 4 = 0$  by graphing.

Graph the related function  $f(x) = -2x^2 + 3x - 4$ .

$x$	$f(x)$
-1	-9
0	-4
1	-3
2	-6



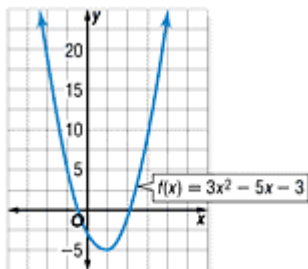
The graph has no  $x$ -intercept. Thus, there are no real number solutions for this equation. The symbol  $\emptyset$ , indicating an empty set, is often used to represent no real solutions.

#### Example 4 Rational Roots

Solve  $3x^2 - 5x - 3 = 0$  by graphing. If integral roots cannot be found, estimate the roots by stating the consecutive integers between which the roots lie.

Graph the related function  $f(x) = 3x^2 - 5x - 3$ .

$x$	$f(x)$
-2	19
-1	5
0	-3
1	-5
2	-1
3	9
4	25



Notice that the value of the function changes from positive to negative between the  $x$  values of  $-1$  and  $0$  and between  $2$  and  $3$ .

The  $x$ -intercepts of the graph are between  $-1$  and  $0$  and between  $2$  and  $3$ . So, one root is between  $-1$  and  $0$ , and the other root is between  $2$  and  $3$ .

#### Example 5 Estimate Solutions to Solve a Problem

Alex will catch a ball that is thrown to him from a tree house. If the ball is thrown from  $15$  feet above the ground with an initial velocity of  $24$  feet per second, the function  $y = -16t^2 + 24t + 10$  represents the height of the ball  $y$  in feet above Alex's hand after  $t$  seconds. (Alex will catch the ball when it is  $5$  feet above ground.) When will Alex catch the ball?

You need to find the solution of the equation  $0 = -16t^2 + 24t + 10$ . Use a graphing calculator to graph the related function

$y = -16t^2 + 24t + 10$ . The  $x$ -intercept is about  $1.84$ . Therefore, the ball will reach Alex's hand in  $1.84$  seconds.

