

Lesson 10-3

Example 1 Irrational Roots

Solve $x^2 + 12x + 36 = 11$ by taking the square root of each side. Round to the nearest tenth if necessary.

$$x^2 + 12x + 36 = 11$$

$$(x + 6)^2 = 11$$

$$\sqrt{(x + 6)^2} = \sqrt{11}$$

$$|x + 6| = \sqrt{11}$$

$$x + 6 = \pm \sqrt{11}$$

$$x + 6 - 6 = \pm \sqrt{11} - 6$$

$$x = -6 \pm \sqrt{11}$$

Original equation

$x^2 + 12x + 36$ is a perfect square trinomial

Take the square root of each side.

Simplify.

Definition of absolute value

Subtract 6 from each side.

Simplify.

Use a calculator to evaluate each value of x .

$$x = -6 + \sqrt{11}$$

$$\approx -2.7$$

$$x = -6 - \sqrt{11}$$

$$\approx -9.3$$

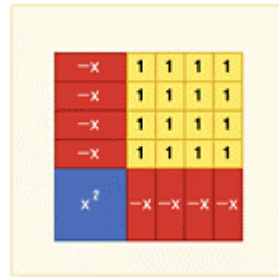
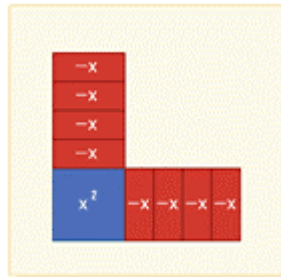
The solution set is $\{-9.3, -2.7\}$.

Example 2 Complete the Square

Find the value of c that makes $x^2 - 8x + c$ a perfect square.

Method 1 Use algebra tiles.

Arrange the tiles for $x^2 - 8x$ so that two sides of the figure are congruent.



To make the figure a square, add 16 positive 1-tiles.

$x^2 - 8x + 16$ is a perfect square.

Method 2 Complete the square.

Step 1 Find $\frac{1}{2}$ of -8.

$$\frac{-8}{2} = -4$$

Step 2 Square the result of Step 1.

$$(-4)^2 = 16$$

Step 3 Add the result of Step 2 to $x^2 - 8x$.

$$x^2 - 8x + 16$$

Thus, $c = 16$. Notice that $x^2 - 8x + 16 = (x - 4)^2$.

Example 3 Solve an Equation by Completing the Square

Solve $4x^2 + 16x + 7 = 0$ by completing the square.

Step 1 Isolate the x^2 and x terms.

$$4x^2 + 16x + 7 = 0 \quad \text{Original equation}$$

$$4x^2 + 16x + 7 - 7 = 0 - 7 \quad \text{Subtract 7 from each side.}$$

$$4x^2 + 16x = -7 \quad \text{Simplify.}$$

$$4(x^2 + 4x) = -7 \quad \text{Factor out 4.}$$

$$x^2 + 4x = \frac{-7}{4} \quad \text{Divide each side by 4.}$$

Step 2 Complete the square and solve.

$$x^2 + 4x + 4 = \frac{-7}{4} + 4 \quad \text{Since } \left(\frac{4}{2}\right)^2 = 4, \text{ add 4 to each side.}$$

$$(x + 2)^2 = \frac{9}{4} \quad \text{Factor } x^2 + 4x + 4.$$

$$x + 2 = \pm \frac{3}{2} \quad \text{Take the square root of each side.}$$

$$x + 2 - 2 = \pm \frac{3}{2} - 2 \quad \text{Subtract 2 from each side.}$$

$$x = -2 \pm \frac{3}{2} \quad \text{Simplify.}$$

$$x = -2 + \frac{3}{2} \quad \text{or} \quad x = -2 - \frac{3}{2}$$

$$= -\frac{1}{2} \quad \quad \quad = -\frac{7}{2}$$

CHECK Substitute each value for x in the original equation.

$$4x^2 + 16x + 7 = 0$$

$$4\left(-\frac{1}{2}\right)^2 + 16\left(-\frac{1}{2}\right) + 7 \stackrel{?}{=} 0$$

$$1 + -8 + 7 \stackrel{?}{=} 0$$

$$0 = 0 \quad \checkmark$$

$$4x^2 + 16x + 7 = 0$$

$$4\left(-\frac{7}{2}\right)^2 + 16\left(-\frac{7}{2}\right) + 7 \stackrel{?}{=} 0$$

$$49 + -56 + 7 \stackrel{?}{=} 0$$

$$0 = 0 \quad \checkmark$$

The solution set is $\left\{-\frac{7}{2}, -\frac{1}{2}\right\}$.

Example 4 Solve a Quadratic Equation in Which $a \neq 1$.

Solve $5x^2 - 80x - 5 = 0$.

$$5x^2 - 80x - 5 = 0$$

$$\frac{5x^2 - 80x - 5}{5} = \frac{0}{5}$$

$$x^2 - 16x - 1 = 0$$

$$x^2 - 16x - 1 + 1 = 0 + 1$$

$$x^2 - 16x = 1$$

$$x^2 - 16x + 64 = 1 + 64$$

$$(x - 8)^2 = 65$$

$$x - 8 = \pm \sqrt{65}$$

$$x - 8 + 8 = \pm \sqrt{65} + 8$$

$$x = 8 \pm \sqrt{65}$$

Use a calculator to evaluate each value of x .

$$x = 8 + \sqrt{65} \\ \approx 16.1$$

or

$$x = 8 - \sqrt{65} \\ \approx -0.1$$

Original equation

Divide each side by 5.

Simplify.

Add 1 to each side.

Simplify.

Since $\left(\frac{-16}{2}\right)^2 = 64$, add 64 to each side.

Factor $x^2 - 16x + 64$.

Take the square root of each side.

Add 8 to each side.

Simplify.