Lesson 10-4

Example 1 Integral Roots

Use two methods to solve $x^2 + 12x = -20$.

First, rewrite the equation so one side is equal to zero.

$x^2 + 12x = -20$	Original equation
$x^2 + 12x + 20 = -20 + 20$	Add 20 to each side.
$x^2 + 12x + 20 = 0$	Simplify.

Method 1 Quadratic Formula

For this equation, $a = 1$, $b = 12$, and $c = 20$.

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	Quadratic Formula
$= \dfrac{-12 \pm \sqrt{12^2 - 4(1)(20)}}{2(1)}$	$a = 1$, $b = 12$, and $c = 20$
$= \dfrac{-12 \pm \sqrt{144 - 80}}{2}$	Multiply.
$= \dfrac{-12 \pm \sqrt{64}}{2}$	subtract.
$= \dfrac{-12 \pm 8}{2}$	Take the square root of 64.

$$x = \frac{-12 - 8}{2} \quad \text{or} \quad x = \frac{-12 + 8}{2}$$

$$= -10 \qquad\qquad = -2$$

Method 2 Factoring

$x^2 + 12x + 20 = 0$	Original equation
$(x + 10)(x + 2) = 0$	Factor $x^2 + 12x + 20$.
$x + 10 = 0$ or $x + 2 = 0$	Zero Product Property
$= -10$ $\quad$ $= -2$	Solve.

The solution set is {-10, -2}.

Example 2 Irrational Roots

Solve $-3x^2 + 5x + 9 = 0$ by using the Quadratic Formula. Round to the nearest tenth if necessary.

For this equation, $a = -3$, $b = 5$, and $c = 9$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ Quadratic Formula

$$= \frac{-5 \pm \sqrt{5^2 - 4(-3)(9)}}{2(-3)}$$ $a = -3$, $b = 5$, and $c = 9$

$$= \frac{-5 \pm \sqrt{25 + 108}}{-6}$$ Multiply.

$$= \frac{-5 \pm \sqrt{133}}{-6}$$ Add.

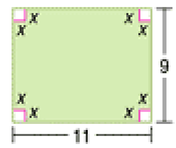
$$x = \frac{-5 - \sqrt{133}}{-6} \quad \text{or} \quad x = \frac{-5 + \sqrt{133}}{-6}$$

$$\approx 2.8 \qquad\qquad \approx -1.1$$

Check the solutions by using the CALC menu on a graphing calculator to determine the zeros of the related quadratic function.

The approximate solution set is {-1.1, 2.8}.

Example 3 Use the Quadratic Formula to Solve a Problem
A box is to be formed by cutting x inch by x inch squares from each corner of a piece of cardboard and then folding the sides. If the dimensions of the piece of cardboard are 9 inches by 11 inches and the area of the base of the box will be approximately 49 square inches, what will be the value of x?



The area of the base can be found using the formula $A = lw$, where the length in this case is $11–2x$ and the width is $9-2x$.

$A = lw$	Formula for area of the base
$49 = (11–2x)(9 – 2x)$	$A = 49$, $l = 11– 2x$, $w = 9 – 2x$
$49 = 99 – 40x + 4x^2$	Multiply.
$0 = 4x^2 – 40x + 50$	Set equal to 0.

This equation cannot be factored, and completing the square would involve a lot of computation. To find accurate solutions, use the Quadratic Formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad \text{Quadratic Formula}$$

$$= \frac{-(-40) \pm \sqrt{(-40)^2 - 4(4)(50)}}{2(4)} \qquad a = 4,\ b = -40,\ c = 50$$

$$= \frac{40 \pm \sqrt{800}}{8}$$

$x \approx 8.5$ or $x \approx 1.5$

Since 8.5 would result in a length of 11-2(8.5) or –6 inches and a width of –8 inches, use the value of 1.5 inches. For the box to have a base area of approximately 49 square inches, you will cut squares out of the corner of the cardboard that measure 1.5 inches by 1.5 inches.

Example 4 Use the Discriminant
State the value of the discriminant for each equation. Then determine the number of real roots of the equation.

a. $\mathbf{4x^2 + 1 = -4x}$

Step 1 Rewrite the equation in standard form.

$4x^2 + 1 = -4x$	Original equation
$4x^2 + 1 + 4x = -4x + 4x$	Add $4x$ to each side.
$4x^2 + 4x + 1 = 0$	Simplify.

Step 2 Find the discriminant.

$b^2 – 4ac = 4^2 - 4(4)(1)$	$a = 4$, $b = 4$, and $c = 1$
$= 0$	Simplify.

Since the discriminant is zero, the equation has 1 real root.

b. $\mathbf{2x^2 - 7x + 3 = 0}$

$b^2 - 4ac = (-7)^2 - 4(2)(3)$ — $a = 2$, $b = -7$, and $c = 3$

$= 25$ — Simplify.

Since the discriminant is positive, the equation has two real roots.