

Lesson 4-7

Example 1 Identify Arithmetic Sequences

Determine whether each sequence is arithmetic. Justify your answer.

a. .7, 0.5, 0.3, 0.1,...

$$\begin{array}{ccccccc} 0.7 & \rightarrow & 0.5 & \rightarrow & 0.3 & \rightarrow & 0.1 \\ & & -0.2 & & -0.2 & & -0.2 \end{array}$$

This is an arithmetic sequence because the difference between terms is constant.

b. $\frac{1}{3}, -\frac{1}{9}, -\frac{1}{27}, -\frac{1}{81}, \dots$

$$\begin{array}{ccccccc} \frac{1}{3} & \rightarrow & -\frac{1}{9} & \rightarrow & -\frac{1}{27} & \rightarrow & -\frac{1}{81} \\ & & +\frac{2}{9} & & +\frac{2}{27} & & +\frac{2}{81} \end{array}$$

This is not an arithmetic sequence because the difference between terms is not constant.

Example 2 Extend a Sequence

Find the next three terms of the arithmetic sequence 23, 29, 35, 41, ...

Find the common difference by subtracting successive terms.

$$\begin{array}{ccccccccccc} 23 & \rightarrow & 29 & \rightarrow & 35 & \rightarrow & 41 & \rightarrow & ? & \rightarrow & ? & \rightarrow & ? \\ & & 6 & & 6 & & 6 & & 6 & & 6 & & 6 \end{array}$$

The common difference is 6.

Add 6 to the last term of the sequence to get the next term in the sequence. Continue adding 6 until the next three terms are found.

$$\begin{array}{ccccccc} 41 & \rightarrow & 47 & \rightarrow & 53 & \rightarrow & 59 \\ & & 6 & & 6 & & 6 \end{array}$$

The next three terms are 47, 53, 59.

Example 3 Find a Specific Term

Find the 11th term in the arithmetic sequence -0.4, -0.1, 0.2, 0.5,...

In this sequence, the first term, a_1 , is -0.4. You want to find the 11th term, so $n = 11$. Find the common difference.

$$\begin{array}{ccccccc} -0.4 & \rightarrow & -0.1 & \rightarrow & 0.2 & \rightarrow & 0.5 \\ & & +0.3 & & +0.3 & & +0.3 \end{array}$$

The common difference is 0.3.

Use the formula for the n th term of an arithmetic sequence.

$$a_n = a_1 + (n-1)d \text{ Formula for the } n\text{th term}$$

$$a_{11} = -0.4 + (11-1)0.3 \quad a_1 = -0.4, n = 11, d = 0.3$$

$$a_{11} = -0.4 + 3 \text{ Simplify.}$$

$$a_{11} = 2.6$$

The 11th term in the sequence is 2.6.

Example 4 Write an Equation for a Sequence

Consider the arithmetic sequence 2, -2, -6, -10,...

a. Write an equation for the n th term of the sequence.In this sequence, the first term, a_1 , is 2. Find the common difference.

$$\begin{array}{ccccccc} 2 & \rightarrow & -2 & \rightarrow & -6 & \rightarrow & -10 \\ & & -4 & & -4 & & -4 \end{array}$$

The common difference is -4 .Use the formula for the n th term to write an equation.

$$a_n = a_1 + (n - 1)d \quad \text{Formula for the } n\text{th term}$$

$$a_n = 2 + (n - 1)(-4) \quad a_1 = 2, d = -4$$

$$a_n = 2 + -4n + 4 \quad \text{Distributive Property}$$

$$a_n = -4n + 6 \quad \text{Simplify.}$$

Check: For $n = 1$, $-4(1) + 6 = 2$.
 For $n = 2$, $-4(2) + 6 = -2$.
 For $n = 3$, $-4(3) + 6 = -6$, and so on.

b. Find the 25th term in the sequence.Replace n with 25 in the equation written in part a.

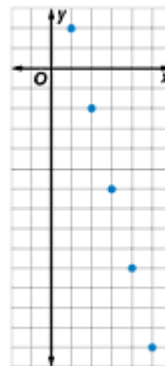
$$a_n = -4n + 6 \quad \text{Equation for the } n\text{th term}$$

$$a_{25} = -4(25) + 6 \quad \text{Replace } n \text{ with } 25.$$

$$a_{25} = -94 \quad \text{Simplify.}$$

c. Graph the first five terms of the sequence.

n	$-4n + 6$	a_n	(n, a_n)
1	$-4(1) + 6$	2	(1, 2)
2	$-4(2) + 6$	-2	(2, -2)
3	$-4(3) + 6$	-6	(3, -6)
4	$-4(4) + 6$	-10	(4, -10)
5	$-4(5) + 6$	-14	(5, -14)



The points fall on a line. The graph of an arithmetic sequence is linear.