

Lesson 5-4

Example 1 Write an Equation Given Slope and One Point

Write an equation of a line that passes through $(-1, 2)$ with a slope of $\frac{1}{4}$.

Step 1 The line has slope $\frac{1}{4}$. To find the y-intercept, replace m with $\frac{1}{4}$ and (x, y) with $(-1, 2)$ in the slope-intercept form. Then, solve for b .

$$y = mx + b \quad \text{Slope-intercept form}$$

$$2 = \frac{1}{4}(-1) + b \quad \text{Replace } m \text{ with } \frac{1}{4}, y \text{ with } 2, \text{ and } x \text{ with } -1.$$

$$2 = -\frac{1}{4} + b \quad \text{Multiply.}$$

$$2 + \frac{1}{4} = -\frac{1}{4} + b + \frac{1}{4} \quad \text{Add } \frac{1}{4} \text{ to each side.}$$

$$2\frac{1}{4} = b \quad \text{Simplify.}$$

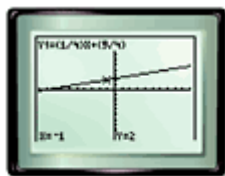
Step 2 Write the slope-intercept form using $m = \frac{1}{4}$ and $b = 2\frac{1}{4}$ or $\frac{9}{4}$.

$$y = mx + b \quad \text{Slope-intercept form}$$

$$y = \frac{1}{4}x + \frac{9}{4} \quad \text{Replace } m \text{ with } \frac{1}{4} \text{ and } b \text{ with } \frac{9}{4}.$$

$$\text{Therefore, the equation is } y = \frac{1}{4}x + \frac{9}{4}.$$

Check: You can check your result by graphing $y = \frac{1}{4}x + \frac{9}{4}$ on a graphing calculator. Use the CALC menu to verify that it passes through $(-1, 2)$.



Example 2 Write an Equation Given Two Points

Multiple-Choice Test Item

The table of ordered pairs shows the coordinates of two points on the graph of a function. Which equation describes the function?

x	y
-2	-7
1	5

- A. $y = -4x + 1$
- B. $y = \frac{1}{4}x + 1$
- C. $y = 4x + 1$
- D. $y = -\frac{1}{4}x - 1$

Read the Test Item

The table represents the ordered pairs (-2, -7) and (1, 5).

Solve the Test Item

Step 1 Find the slope of the line containing the points. Let $(x_1, y_1) = (-2, -7)$ and $(x_2, y_2) = (1, 5)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{Slope formula}$$

$$m = \frac{5 - (-7)}{1 - (-2)} \quad x_1 = -2, x_2 = 1, y_1 = -7, \text{ and } y_2 = 5$$

$$m = \frac{12}{3} \text{ or } 4 \quad \text{Simplify.}$$

Step 2 You know the slope and two points. Choose one point and find the y-intercept.

$$\begin{aligned} y &= mx + b && \text{Slope-intercept form} \\ 5 &= 4(1) + b && \text{Let } (x, y) = (1, 5); \text{ replace } m \text{ with } 4. \\ 5 &= 4 + b && \text{Multiply.} \\ 5 - 4 &= 4 + b - 4 && \text{Subtract 4 from each side.} \\ 1 &= b && \text{Simplify.} \end{aligned}$$

Step 3 Write the slope-intercept form using $m = 4$ and $b = 1$.

$$\begin{aligned} y &= mx + b && \text{Slope-intercept form} \\ y &= 4x + 1 && \text{Replace } m \text{ with } 4 \text{ and } b \text{ with } 1. \end{aligned}$$

Therefore, the equation is $y = 4x + 1$. The answer is C.

Example 3 Write an Equation to Solve a Problem

In 1975, the World record for the one-mile run was 3 minutes 51 seconds. In 1999, the world record was approximately 3 minutes 43 seconds. Write a linear equation to estimate the world record time, in seconds, for any year after 1975.

Explore You know the time in 1975 is 3 minutes 51 seconds or 231 seconds. You know the time in 1999 is 3 minutes 43 seconds or 223 seconds.

Plan Let x represent the year after 1975. Let y represent the time in seconds. Write an equation of the line that passes through $(0, 231)$ and $(24, 223)$

Solve Find the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{Slope formula}$$

$$m = \frac{223 - 231}{24 - 0} \quad \text{Let } (x_1, y_1) = (0, 231) \text{ and } (x_2, y_2) = (24, 223).$$

$$m = \frac{-8}{24} \text{ or about } -0.33 \quad \text{Simplify.}$$

Choose $(0, 231)$ and find the y -intercept of the line.

$$\begin{aligned} y &= mx + b && \text{Slope-intercept form} \\ 231 &= -0.33(0) + b && \text{Replace } m \text{ with } -0.33, x \text{ with } 0, \text{ and } y \text{ with } 231. \\ 231 &= 0 + b && \text{Multiply.} \\ 231 &= b && \text{Simplify.} \end{aligned}$$

Write the slope-intercept form using $m = -0.33$, and $b = 231$.

$$\begin{aligned} y &= mx + b && \text{Slope-intercept form} \\ y &= -0.33x + 231 && \text{Replace } m \text{ with } -0.33 \text{ and } b \text{ with } 231. \end{aligned}$$

Therefore, the equation is $y = -0.33x + 231$.

Examine Check your result by substituting the coordinates of the point not chosen, $(24, 223)$, into the equation.

$$\begin{aligned} y &= -0.33x + 231 && \text{Original equation} \\ 223 &\stackrel{?}{=} -0.33(24) + 231 && \text{Replace } y \text{ with } 223 \text{ and } x \text{ with } 24. \\ 223 &\stackrel{?}{=} -7.92 + 231 && \text{Multiply.} \\ 223 &\approx 223.08 && \text{The slope was rounded so the answers vary slightly.} \end{aligned}$$

Example 4 Linear Extrapolation

Predict the world record time for the one-mile run in 2002.

$$\begin{aligned} y &= -0.33x + 231 && \text{Original equation} \\ y &= -0.33(27) + 231 && \text{Replace } x \text{ with } 2002 - 1975 \text{ or } 27. \\ y &= 222.09 && \text{Simplify.} \end{aligned}$$

You would predict the world record to be about 222 seconds or about 3 minutes 42 seconds in 2002.