

Lesson 6-3

Example 1 Solve a Real-World Problem

The gravitational force on the moon is 0.17 of the force on Earth. This means, if you weigh 100 pounds on Earth, you would weigh 100×0.17 or 17 pounds on the moon. Shawn is an astronaut who will be doing a moon walk. He weighs 165 pounds and will carry a pack. His weight including the pack cannot exceed 34 pounds on the moon. If x represents the weight of the pack, the inequality $0.17(165 + x) \leq 34$ represents this situation. What are the possible weights of Shawn's pack?

$$\begin{aligned}
 0.17(165 + x) &\leq 34 && \text{Original inequality} \\
 28.05 + 0.17x &\leq 34 && \text{Distributive Property} \\
 28.05 + 0.17x - 28.05 &\leq 34 - 28.05 && \text{Subtract 28.05 from each side.} \\
 0.17x &\leq 5.95 && \text{Simplify.} \\
 \frac{0.17x}{0.17} &\leq \frac{5.95}{0.17} && \text{Divide each side by 0.17.} \\
 x &\leq 35 && \text{Simplify.}
 \end{aligned}$$

Shawn's pack cannot exceed 35 pounds on Earth.

Example 2 Inequality Involving a Negative Coefficient

Solve $-8 - 2t \geq -6$. Then check your solution.

$$\begin{aligned}
 -8 - 2t &\geq -6 && \text{Original inequality} \\
 -8 - 2t + 8 &\geq -6 + 8 && \text{Add 8 to each side.} \\
 -2t &\geq 2 && \text{Simplify.} \\
 \frac{-2t}{-2} &\leq \frac{2}{-2} && \text{Divide each side by } -2 \text{ and change } \geq \text{ to } \leq. \\
 t &\leq -1 && \text{Simplify.}
 \end{aligned}$$

Check: To check this solution, substitute -1 , a number less than -1 , and a number greater than -1 .

Let $t = -1$	Let $t = -2$	Let $t = 0$
$-8 - 2t \geq -6$	$-8 - 2t \geq -6$	$-8 - 2t \geq -6$
?	?	?
$-8 - 2(-1) \geq -6$	$-8 - 2(-2) \geq -6$	$-8 - 2(0) \geq -6$
?	?	?
$-8 + 2 \geq -6$	$-8 + 4 \geq -6$	$-8 - 0 \geq -6$
$-6 \geq -6$	$-4 \geq -6$	$-8 \not\geq -6$

The solution set is $\{t \mid t \leq -1\}$.

Example 3 Write and Solve an Inequality

Write an inequality for the sentence below. Then solve the inequality.

Sixteen minus three fifths of a number is at most two fifths of the number minus twelve.

$$\underbrace{\text{sixteen}}_{16} \quad \underbrace{\text{minus}}_{-} \quad \underbrace{\text{three}}_{\frac{3}{5}} \underbrace{\text{fifths}}_{\times} \underbrace{\text{of}}_{\times} \underbrace{\text{a number}}_{n} \quad \underbrace{\text{is at most}}_{\leq} \quad \underbrace{\text{two}}_{\frac{2}{5}} \underbrace{\text{fifths}}_{\times} \underbrace{\text{of}}_{\times} \underbrace{\text{the number}}_{n} \quad \underbrace{\text{minus}}_{-} \quad \underbrace{\text{twelve}}_{12}$$

$$16 - \frac{3}{5}n \leq \frac{2}{5}n - 12$$

Original inequality

$$16 - \frac{3}{5}n + \frac{3}{5}n \leq \frac{2}{5}n - 12 + \frac{3}{5}n$$

Add $\frac{3}{5}n$ to each side.

$$16 \leq n - 12$$

Simplify.

$$16 + 12 \leq n - 12 + 12$$

Add 12 to each side.

$$28 \leq n$$

Simplify.

The solution set is $\{n \mid n \geq 28\}$.

Example 4 Distributive Property

Solve $2(p - 3) - 3(2p + 6) > 4p$.

$$2(p - 3) - 3(2p + 6) > 4p$$

Original inequality

$$2p - 6 - 6p - 18 > 4p$$

Distributive Property

$$-4p - 24 > 4p$$

Combine like terms

$$-4p - 24 + 4p > 4p + 4p$$

Add $4p$ to each side.

$$-24 > 8p$$

Simplify

$$\frac{-24}{8} > \frac{8p}{8}$$

Divide each side by 8

$$-3 > p$$

Simplify.

Since $-3 > p$ is the same as $p < -3$, the solution set is $\{p \mid p < -3\}$.

Example 5 Empty Set

$$\text{Solve } 2(x + 3) \leq \frac{1}{2}(4x + 2) - 1.$$

$$2(x + 3) \leq \frac{1}{2}(4x + 2) - 1$$

Original inequality

$$2x + 6 \leq 2x + 1 - 1$$

Distributive Property

$$2x + 6 \leq 2x$$

Combine like terms.

$$2x + 6 - 2x \leq 2x - 2x$$

Subtract $2x$ from each side.

$$6 \leq 0$$

This statement is false.

Since the inequality results in a false statement, the solution set is the empty set \emptyset .