Lesson 9-3

Example 1 *b* and *c* Are Positive

Factor $x^2 + 16x + 63$.

In this trinomial, $b = 16$ and $c = 63$. You need to find two numbers whose sum is 16 and whose product is 63. Make an organized list of the factors of 63, and look for the pair of factors whose sum is 16.

Factors of 63	Sum of Factors	
1, 63	64	
3, 21	24	
7, 9	16	The correct factors are 7 and 9.

$x^2 + 16x + 63 = (x + m)(x + n)$ — Write the pattern.

$= (x + 7)(x + 9)$ — $m = 7$ and $n = 9$

Check: You can check this result by multiplying the two factors.

F O I L

$(x + 7)(x + 9) = x^2 + 9x + 7x + 63$ — FOIL method

$= x^2 + 16x + 63$ — Simplify.

Example 2 *b* Is Negative and *c* Is Positive

Factor $x^2 - 11x + 24$.

In this trinomial, $b = -11$ and $c = 24$. This means that $m + n$ is negative and mn is positive. So m and n must both be negative. Therefore, make a list of the negative factors of 24, and look for the pair of factors whose sum is –11.

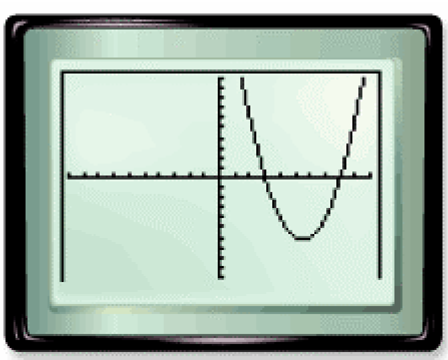
Factors of 24	Sum of Factors	
-1, -24	-25	
-2, -12	-14	
-3, -8	-11	The correct factors are –3 and –8.
-4, -6	-10	

$x^2 - 11x + 24 = (x + m)(x + n)$ — Write the pattern.

$= (x - 3)(x - 8)$ — $m = -3$ and $n = -8$

Check: You can check this result by using a graphing calculator.

Graph $y = x^2 - 11x + 24$ and $y = (x - 3)(x - 8)$ on the same screen.
Since only one graph appears, the two graphs must coincide.
Therefore, the trinomial has been factored correctly.



Example 3 *b* Is Positive and *c* Is Negative

Factor $x^2 + 2x - 15$.

In this trinomial, $b = 2$ and $c = -15$. This means that $m + n$ is positive and mn is negative. So either m or n is negative, but not both. Therefore, make a list of the factors of –15, where one factor of each pair is negative. Look for the pair of factors whose sum is 2.

Factors of –15	Sum of Factors	
-1, 15	14	
1, -15	-14	
-3, 5	2	The correct factors are –3 and 5.
3, -5	-2	

$x^2 + 2x - 15 = (x + m)(x + n)$ Write the pattern.

$= (x - 3)(x + 5)$ $m = -3$ and $n = 5$

Example 4 *b* Is Negative and *c* Is Negative

Factor $x^2 - 4x - 21$.

Since $b = -4$ and $c = -21$, $m + n$ is negative and mn is negative. So either m or n is negative, but not both.

Factors of –21	Sum of Factors	
-1, 21	20	
1, -21	-20	
-3, 7	4	
3, -7	-4	The correct factors are 3 and –7.

$x^2 - 4x - 21 = (x + m)(x + n)$ Write the pattern.

$= (x + 3)(x - 7)$ $m = 3$ and $n = -7$

Example 5 Solve an Equation by Factoring

Solve $x^2 - 8x + 7 = 0$. Check your solutions.

$x^2 - 8x + 7 = 0$ Original equation

$(x - 1)(x - 7) = 0$ Factor.

$x - 1 = 0$ or $x - 7 = 0$ Zero Product Property

$x = 1$ $x = 7$ Solve each equation.

The solution set is {1, 7}.

Check: Substitute 1 and 7 for x in the original equation.

$x^2 - 8x + 7 = 0$ $x^2 - 8x + 7 = 0$

$(1)^2 - 8(1) + 7 \stackrel{?}{=} 0$ $(7)^2 - 8(7) + 7 \stackrel{?}{=} 0$

$0 = 0$ ✓ $0 = 0$ ✓

Example 6 Solve a Real-World Problem by Factoring
Find 2 consecutive integers whose product is 56.

Explore You are to find two consecutive integers that have a product of 56.

Plan Let x = the first consecutive integer. Then $x + 1$ = the next consecutive integer.

$$\underbrace{\text{first integer}}_{x} \quad \underbrace{\text{times}}_{\cdot} \quad \underbrace{\text{consecutive integer}}_{(x+1)} \quad \underbrace{\text{equals}}_{=} \quad \underbrace{56}_{56}$$

Solve

$x(x + 1) = 56$	Write the equation.
$x^2 + x = 56$	Multiply.
$x^2 + x - 56 = 0$	Subtract 56 from each side.
$(x + 8)(x - 7) = 0$	Factor.
$x + 8 = 0$ or $x - 7 = 0$	Zero Product Property
$x = -8$ $\quad$ $x = 7$	Solve each equation.

Examine One possible set of consecutive integers is -8 and -8 + 1 or -7. The other is 7 and 7 + 1 or 8.