

## Lesson 9-5

### Example 1 Factor the Difference of Squares

Factor each binomial.

a.  $a^2 - 64$

$$\begin{aligned} a^2 - 64 &= a^2 - 8^2 \\ &= (a + 8)(a - 8) \end{aligned}$$

Write in the form  $a^2 - b^2$ .  
Factor the difference of squares.

b.  $16m^4 - n^2$

$$\begin{aligned} 16m^4 - n^2 &= (4m^2)^2 - (n)^2 \\ &= (4m^2 + n)(4m^2 - n) \end{aligned}$$

$16m^4 = 4m^2 \cdot 4m^2$  and  $n^2 = n \cdot n$   
Factor the difference of squares.

### Example 2 Factor Out a Common Factor

Factor  $5x^2y - 500y$ .

$$\begin{aligned} 5x^2y - 500y &= 5y(x^2 - 100) \\ &= 5y(x^2 - 10^2) \\ &= 5y(x + 10)(x - 10) \end{aligned}$$

The GCF of  $5x^2y$  and  $500y$  is  $5y$ .  
 $100 = 10^2$   
Factor the difference of squares.

### Example 3 Apply a Factoring Technique More than Once

Factor  $16x^5 - 625x$ .

$$\begin{aligned} 16x^5 - 625x &= x(16x^4 - 625) \\ &= x[(4x^2)^2 - 25^2] \\ &= x(4x^2 + 25)(4x^2 - 25) \\ &= x(4x^2 + 25)[(2x)^2 - 5^2] \\ &= x(4x^2 + 25)(2x + 5)(2x - 5) \end{aligned}$$

Original polynomial  
The GCF of  $16x^5$  and  $625x$  is  $x$ .  
 $16x^4 = 4x^2 \cdot 4x^2$  and  $625 = 25 \cdot 25$   
Factor the difference of squares.  
 $4x^2 = 2x \cdot 2x$  and  $25 = 5 \cdot 5$   
Factor the difference of squares.

### Example 4 Apply Several Different Factoring Techniques

Factor  $a^4 - 5a^3 - 4a^2 + 20a$ .

$$\begin{aligned} a^4 - 5a^3 - 4a^2 + 20a &= a(a^3 - 5a^2 - 4a + 20) \\ &= a[(a^3 - 5a^2) + (-4a + 20)] \\ &= a[a^2(a - 5) + 4(-a + 5)] \\ &= a[a^2(a - 5) + 4(-1)(a - 5)] \\ &= a[a^2(a - 5) - 4(a - 5)] \\ &= a[(a - 5)(a^2 - 4)] \\ &= a[(a - 5)(a^2 - 2^2)] \\ &= a(a - 5)(a + 2)(a - 2) \end{aligned}$$

Original polynomial  
Factor out the GCF.  
Group terms with common factors.  
Factor each grouping.  
 $(-a + 5) = -1(a - 5)$   
Simplify.  
 $a - 5$  is the common factor.  
 $a^2 = a \cdot a$  and  $4 = 2 \cdot 2$   
Factor the difference of squares.

**Example 5 Solve Equations by Factoring**  
**Solve each equation. Check your solutions.**

a.  $a^3 + 2a^2 - a = 2$

$$a^3 + 2a^2 - a = 2$$

Original equation

$$a^3 + 2a^2 - a - 2 = 0$$

Subtract 2 from each side.

$$(a^3 + 2a^2) + (-a - 2) = 0$$

Group terms with a common factor.

$$a^2(a+2) + -1(a+2) = 0$$

Factor each grouping.

$$(a+2)(a^2 - 1) = 0$$

$a + 2$  is the common factor.

$$(a+2)(a+1)(a-1) = 0$$

$a^2 = a \cdot a$  and  $1 = 1 \cdot 1$

Applying the Zero Product Property, set each factor equal to 0 and solve the resulting three equations.

$$a + 2 = 0 \quad \text{or} \quad a + 1 = 0 \quad \text{or} \quad a - 1 = 0$$

$$a = -2 \quad \quad \quad a = -1 \quad \quad \quad a = 1$$

The solution set is  $\{-2, -1, 1\}$ . Check each solution in the original equation.

b.  $4b^2 - 1 = 0$

$$4b^2 - 1 = 0$$

Original equation

$$(2b)^2 - 1^2 = 0$$

$4b^2 = 2b \cdot 2b$  and  $1 = 1 \cdot 1$

$$(2b+1)(2b-1) = 0$$

Factor the difference of squares.

$$2b+1=0 \quad \text{or} \quad 2b-1=0$$

Zero Product Property

$$2b = -1 \quad \quad \quad 2b = 1$$

Solve each equation.

$$b = -\frac{1}{2} \quad \quad \quad b = \frac{1}{2}$$

The solution set is  $\{-\frac{1}{2}, \frac{1}{2}\}$ . Check each solution in the original equation.

**Example 6 Use Differences of Two Squares****OPEN ENDED TEST ITEM**

A triangle is cut off the corner of a right triangle. The right triangle has legs that both measure 6 feet. The cut is a right triangle with both legs of measure  $x$  feet.



- Write an equation in terms of  $x$  that represents the area  $A$  of the paper after the corner is removed.
- What value of  $x$  will result in an area that is  $\frac{3}{4}$  the area of the original triangular piece of paper? Show how you arrived at your answer.

**Read the Test Item**

$A$  is the area of the triangle minus the area of the triangular corner to be removed.

**Solve the Test Item**

- The area of the triangle is  $\frac{1}{2} \cdot 6 \cdot 6$  or 18 square feet, and the area of the cut triangle is  $\frac{1}{2} \cdot x \cdot x$  or  $\frac{1}{2}x^2$  square feet. Thus  $A = 18 - \frac{1}{2}x^2$ .

- Find  $x$  so that  $A$  is  $\frac{3}{4}$  the area of the original triangular piece of paper.

$$A = \frac{3}{4} \text{ area of original triangle}$$

$$18 - \frac{1}{2}x^2 = \frac{3}{4}(18)$$

$$A = 18 - \frac{1}{2}x^2 \text{ and area of original triangle is 18.}$$

$$18 - \frac{1}{2}x^2 = \frac{27}{2}$$

Simplify.

$$18 - \frac{1}{2}x^2 - \frac{27}{2} = 0$$

Subtract  $\frac{27}{2}$  from each side.

$$\frac{9}{2} - \frac{1}{2}x^2 = 0$$

Simplify.

$$9 - x^2 = 0$$

Multiply each side by 2 to remove fractions.

$$(3 + x)(3 - x) = 0$$

Factor the difference of squares.

$$3 + x = 0 \quad \text{or} \quad 3 - x = 0$$

Zero Product Property

$$x = -3 \quad \quad x = 3$$

Solve each equation.

Since length cannot be negative, the only reasonable solution is 3.