

Lesson 9-6

Example 1 Factor Perfect Square Trinomials

Determine whether each trinomial is a perfect square trinomial. If so, factor it.

a. $4x^2 + 12x + 9$

1. Is the first term a perfect square? Yes, $4x^2 = (2x)^2$.

2. Is the last term a perfect square? Yes, $9 = (3)^2$.

3. Is the middle term equal to $2(2x)(3)$? Yes, $12x = 2(2x)(3)$.

$4x^2 + 12x + 9$ is a perfect square trinomial.

$$\begin{aligned} 4x^2 + 12x + 9 &= (2x)^2 + 2(2x)(3) + (3)^2 && \text{Write as } a^2 + 2ab + b^2. \\ &= (2x + 3)^2 && \text{Factor using pattern.} \end{aligned}$$

b. $4a^2 - 8a + 16$

1. Is the first term a perfect square? Yes, $4a^2 = (2a)^2$.

2. Is the last term a perfect square? Yes, $16 = (4)^2$.

3. Is the middle term equal to $2(2a)(4)$? No, $8a \neq 2(2a)(4)$.

$4a^2 - 8a + 16$ is not a perfect square trinomial.

Example 2 Factor Completely

Factor each polynomial.

a. $3x^2 - 12x + 12$

This polynomial has a GCF of 3. First, factor out the GCF and you are left with $3(x^2 - 4x + 4)$. The resulting trinomial has the first term as a perfect square $x^2 = (x)^2$, the last term is also a perfect square $4 = 2^2$, and the middle term is equal to $2(x)(2)$ or $4x$. Therefore, the polynomial is a perfect square trinomial.

$$\begin{aligned} 3x^2 - 12x + 12 &= 3(x^2 - 4x + 4) && 3 \text{ is the GCF.} \\ &= 3[(x)^2 - 2(x)(2) + (2)^2] && \text{Write as } a^2 - 2ab + b^2. \\ &= 3(x - 2)^2 && a = x \text{ and } b = 2. \end{aligned}$$

b. $2x^3 - x^2 - 15x$

This polynomial has three terms that have a GCF of x . The resulting trinomial is then in the form $ax^2 + bx + c$. Are there two numbers m and n whose product is $2 \cdot -15$ or -30 and whose sum is -1 ? Yes, the product of 5 and -6 is -30 and their sum is -1 .

$$\begin{aligned} 2x^3 - x^2 - 15x &= x(2x^2 - x - 15) && x \text{ is the GCF.} \\ &= x(2x^2 + mx + nx - 15) && \text{Write the pattern.} \\ &= x(2x^2 + 5x + -6x - 15) && m = 5 \text{ and } n = -6 \\ &= x[(2x^2 + 5x) + (-6x - 15)] && \text{Group terms with common factors.} \\ &= x[x(2x + 5) + -3(2x + 5)] && \text{Factor out the GCF from each grouping.} \\ &= x(2x + 5)(x - 3) && 2x + 5 \text{ is the common factor.} \end{aligned}$$

Example 3 Solve Equations with Repeated Factors**Solve $16x^2 + 8x + 1 = 0$.**

$$\begin{aligned}
 16x^2 + 8x + 1 &= 0 \\
 (4x)^2 + 2(4x)(1) + (1)^2 &= 0 \\
 (4x + 1)^2 &= 0 \\
 4x + 1 &= 0 \\
 4x &= -1 \\
 x &= -\frac{1}{4}
 \end{aligned}$$

Original equation

Recognize $16x^2 - 8x + 1$ as a perfect square trinomial.

Factor the perfect square trinomial.

Set repeated factor equal to zero.

Solve for x .

Thus, the solution set is $\{-\frac{1}{4}\}$. Check this solution in the original equation.

Example 4 Use the Square Root Property to Solve Equations**Solve each equation. Check your solutions.**

a. $(x - 2)^2 = \frac{4}{9}$

$$(x - 2)^2 = \frac{4}{9}$$

Original equation

$$x - 2 = \pm \sqrt{\frac{4}{9}}$$

Square Root Property

$$x - 2 = \pm \frac{2}{3}$$

$$\frac{4}{9} = \frac{2}{3} \cdot \frac{2}{3}$$

$$x = 2 \pm \frac{2}{3}$$

Add 2 to each side.

$$x = 2 + \frac{2}{3} \quad \text{or} \quad x = 2 - \frac{2}{3}$$

Separate into two equations.

$$= \frac{8}{3} \qquad = \frac{4}{3}$$

Simplify.

The solution set is $\{\frac{4}{3}, \frac{8}{3}\}$. Check each solution in the original equation.

b. $x^2 + \frac{1}{2}x + \frac{1}{16} = 36$

$$x^2 + \frac{1}{2}x + \frac{1}{16} = 36$$

Original equation

$$(x)^2 + 2(x)\left(\frac{1}{4}\right) + \left(\frac{1}{4}\right)^2 = 36$$

Recognize perfect square trinomial.

$$\left(x + \frac{1}{4}\right)^2 = 36$$

Factor perfect square trinomial.

$$x + \frac{1}{4} = \pm \sqrt{36}$$

Square Root Property

$$x + \frac{1}{4} = \pm 6$$

$$36 = 6 \cdot 6$$

$$x = -\frac{1}{4} \pm 6$$

Subtract $\frac{1}{4}$ from each side.

$$x = -\frac{1}{4} + 6 \quad \text{or} \quad x = -\frac{1}{4} - 6$$

Separate into two equations.

$$= \frac{23}{4}$$

$$= -\frac{25}{4}$$

Simplify.

The solution set is $\left\{-\frac{25}{4}, \frac{23}{4}\right\}$. Check each solution in the original equation.

c. $(x + 1)^2 = 10$

$$(x + 1)^2 = 10$$

Original equation

$$x + 1 = \pm \sqrt{10}$$

Square Root Property

$$x = -1 \pm \sqrt{10}$$

Subtract 1 from each side.

Since 10 is not a perfect square, the solution set is $\{-1 \pm \sqrt{10}\}$. Using a calculator, the approximate solutions are $-1 + \sqrt{10}$ or about 2.16 and $-1 - \sqrt{10}$ or about -4.16.

Check: You can check your answer using a graphing calculator. Graph $y = (x + 1)^2$ and $y = 10$. Using the INTERSECT feature of your graphing calculator, find where $(x + 1)^2 = 10$. The check of 2.16 as one of the approximate solutions is shown at the right.

