11-6 Adding and Subtracting Rational Expressions

Then

You added and subtracted polynomials. (Lesson 7-5)

Now

- Add and subtract rational expressions with like denominators.
- Add and subtract rational expressions with unlike denominators.

New Vocabulary

least common multiple (LCM)
least common denominator (LCD)

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- Extra Examples
- Personal Tutor
- Self-Check Quiz
- Homework Help

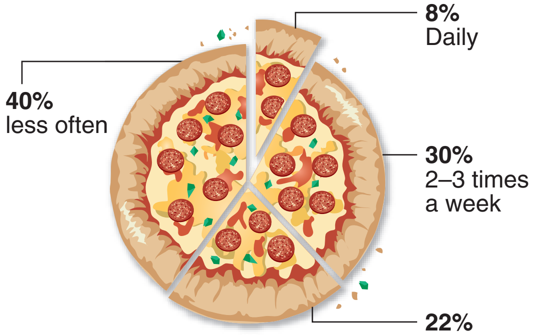
Why?

A survey asked families how often they eat takeout. To determine the fraction of those surveyed who eat takeout more than once a week, you can add. Remember that percents can be written as fractions with denominators of 100.

2–3 times a week	plus	daily	equals	more than once a week
$\frac{30}{100}$	$+$	$\frac{8}{100}$	$=$	$\frac{38}{100}$

Thus, $\frac{38}{100}$ or 38% eat takeout more than once a week.

How Many Times a Week Families Eat Takeout



Source: Reader's Digest

Add and Subtract Rational Expressions with Like Denominators

To add or subtract rational expressions, add or subtract the numerators and write the sum or difference over the common denominator.

Key Concept — For Your Foldable

Add or Subtract Rational Expressions with Like Denominators

Let a, b, and c be polynomials with $c \neq 0$.

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c} \qquad \frac{a}{c} - \frac{b}{c} = \frac{a-b}{c}$$

EXAMPLE 1 Add Rational Expressions with Like Denominators

Find $\frac{5n}{n+3} + \frac{15}{n+3}$.

$\frac{5n}{n+3} + \frac{15}{n+3} = \frac{5n+15}{n+3}$ — The common denominator is $n + 3$.

$= \frac{5(n+3)}{n+3}$ — Factor the numerator.

$= \frac{5\cancel{(n+3)}^{1}}{\cancel{n+3}_{1}}$ — Divide by the common factor, $n + 3$.

$= \frac{5}{1}$ or 5 — Simplify.

Check Your Progress

Find each sum.

1A. $\frac{8c}{6} + \frac{5c}{6}$ **1B.** $\frac{4t}{5xy} + \frac{7}{5xy}$ **1C.** $\frac{3y}{3+y} + \frac{y^2}{3+y}$

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EXAMPLE 2 Subtract Rational Expressions with Like Denominators

Watch Out!

Common Terms Remember that every term of the numerator and the denominator must be multiplied or divided by the same number for the fraction to remain equivalent.

Find $\frac{3m-5}{m+4} - \frac{4m+2}{m+4}$.

$\frac{3m-5}{m+4} - \frac{4m+2}{m+4} = \frac{(3m-5)-(4m+2)}{m+4}$ The common denominator is $m + 4$.

$= \frac{(3m-5)+[-(4m+2)]}{m+4}$ The additive inverse of $(4m + 2)$ is $-(4m + 2)$.

$= \frac{3m-5-4m-2}{m+4}$ Distributive Property

$= \frac{-m-7}{m+4}$ Simplify.

Check Your Progress

Find each difference.

2A. $\frac{2h+4}{h+1} - \frac{5+h}{h+1}$

2B. $\frac{17h+4}{15h-5} - \frac{2h-6}{15h-5}$

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You can use additive inverses to form like denominators.

EXAMPLE 3 Inverse Denominators

Find $\frac{3n}{n-4} + \frac{6n}{4-n}$.

$\frac{3n}{n-4} + \frac{6n}{4-n} = \frac{3n}{n-4} + \frac{6n}{-(n-4)}$ Rewrite $4 - n$ as $-(n - 4)$.

$= \frac{3n}{n-4} - \frac{6n}{n-4}$ Rewrite so the denominators are the same.

$= \frac{3n-6n}{n-4}$ or $-\frac{3n}{n-4}$ Subtract the numerators and simplify.

Check Your Progress

Find each sum or difference.

3A. $\frac{t^2}{t-3} + \frac{3}{3-t}$

3B. $\frac{2p}{p-1} - \frac{2p}{1-p}$

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Add and Subtract Rational Expressions with Unlike Denominators The **least common multiple (LCM)** is the least number that is a multiple of two or more numbers or polynomials.

EXAMPLE 4 LCMs of Polynomials

Find the LCM of each pair of polynomials.

a. $6x$ and $4x^3$

Step 1 Find the prime factors of each expression.

$6x = 2 \cdot 3 \cdot x$ $4x^3 = 2 \cdot 2 \cdot x \cdot x \cdot x$

Step 2 Use each prime factor, 2, 3, and x, the greatest number of times it appears in either of the factorizations.

$6x = 2 \cdot 3 \cdot x$ $4x^3 = 2 \cdot 2 \cdot x \cdot x \cdot x$

LCM $= 2 \cdot 2 \cdot 3 \cdot x \cdot x \cdot x$ or $12x^3$

Review Vocabulary

Factored Form A monomial is in factored form when it is expressed as the product of prime numbers and variables, and no variable has an exponent greater than 1. (Lesson 8-1)

b. $n^2 + 5n + 4$ and $(n + 1)^2$

$n^2 + 5n + 4 = (n + 1)(n + 4)$ — Factor each expression.

$(n + 1)^2 = (n + 1)(n + 1)$

$(n + 1)$ is a factor twice in the second expression. $(n + 4)$ is a factor once.

LCM $= (n + 1)(n + 1)(n + 4)$ or $(n + 1)^2(n + 4)$

Check Your Progress

4A. $8m^2t$ and $12m^2t^3$

4B. $x^2 - 2x - 8$ and $x^2 - 5x - 14$

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To add or subtract fractions with unlike denominators, you need to rename the fractions using the least common multiple of the denominators, called the **least common denominator (LCD)**.

Key Concept

For Your FOLDABLE

Add or Subtract Rational Expressions with Unlike Denominators

Step 1 Find the LCD.

Step 2 Write each rational expression as an equivalent expression with the LCD as the denominator.

Step 3 Add or subtract the numerators and write the result over the common denominator.

Step 4 Simplify if necessary.

StudyTip

Checking Answers You can check whether you have simplified rational expressions by substituting values, but this does not guarantee that the expressions are always equal. If the results are different, check for an error in arithmetic.

EXAMPLE 5 Add Rational Expressions with Unlike Denominators

Find $\frac{3t + 2}{t^2 - 2t - 3} + \frac{t + 1}{t - 3}$.

Find the LCD. Since $t^2 - 2t - 3 = (t - 3)(t + 1)$, the LCD is $(t - 3)(t + 1)$.

$$\frac{3t + 2}{t^2 - 2t - 3} + \frac{t + 1}{t - 3} = \frac{3t + 2}{(t - 3)(t + 1)} + \frac{t + 1}{t - 3}$$ Factor $t^2 - 2t - 3$.

$$= \frac{3t + 2}{(t - 3)(t + 1)} + \frac{t + 1}{t - 3}\left(\frac{t + 1}{t + 1}\right)$$ Write $\frac{t + 1}{t - 3}$ using the LCD.

$$= \frac{3t + 2}{(t - 3)(t + 1)} + \frac{t^2 + 2t + 1}{(t - 3)(t + 1)}$$ Simplify.

$$= \frac{3t + 2 + t^2 + 2t + 1}{(t - 3)(t + 1)}$$ Add the numerators.

$$= \frac{t^2 + 5t + 3}{(t - 3)(t + 1)}$$ Simplify.

Check Your Progress Find each sum.

5A. $\frac{4d^2}{d} + \frac{d + 2}{d^2}$

5B. $\frac{b + 3}{b} + \frac{b - 5}{b + 1}$

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The formula time $= \frac{\text{distance}}{\text{rate}}$ is helpful in solving real-world applications.

Real-World Link

The distance a hang glider can travel is determined by its *glide ratio*, or the ratio of the forward distance traveled to the vertical distance dropped.

Source: HowStuffWorks

Real-World EXAMPLE 6 Add Rational Expressions

HANG GLIDING **For the first 5000 meters, a hang glider travels at a rate of x meters per minute. Then, due to a stronger wind, it travels 6000 meters at a speed that is 3 times as fast.**

a. Write an expression to represent how much time the hang glider is flying.

Understand For the first 5000 meters, the hang glider's speed is x. For the last 6000 meters, the hang glider's speed is $3x$.

Plan Use the formula $d = r \times t$ or $t = \frac{d}{r}$ to represent the time t of each section of the hang glider's trip, with rate r and distance d.

Solve Time to fly 5000 meters: $\frac{d}{r} = \frac{5000}{x}$ — $d = 5000$ m, $r = x$

Time to fly 6000 meters: $\frac{d}{r} = \frac{6000}{3x}$ — $d = 6000$ m, $r = 3x$

Total flying time: $\frac{5000}{x} + \frac{6000}{3x}$

$\frac{5000}{x} + \frac{6000}{3x} = \frac{5000}{x}\left(\frac{3}{3}\right) + \frac{6000}{3x}$ — The LCD is $3x$.

$= \frac{15{,}000}{3x} + \frac{6000}{3x}$ — Multiply.

$= \frac{\overset{7000}{\cancel{21{,}000}}}{\underset{1}{\cancel{3}}x}$ or $\frac{7000}{x}$ — Simplify.

Check $\frac{5000}{x} + \frac{6000}{3x} = \frac{5000}{1} + \frac{6000}{3(1)}$ — Let $x = 1$ in the original expression.

$= 5000 + 2000$ or 7000 — Simplify.

$\frac{7000}{x} = \frac{7000}{1}$ or 7000 — Let $x = 1$ in the answer expression. Simplify.

Since the expressions have the same value for $x = 1$, they are equivalent. So, the answer is reasonable. ✓

b. If the hang glider is flying at a rate of 600 meters per minute for the first 5000 meters, find the total amount of time that the hang glider is flying.

$\frac{7000}{x} = \frac{7000}{600}$ — Substitute 600 for x in the expression.

≈ 11.7 — Simplify.

So, the hang glider is flying for approximately 11.7 minutes.

c. If the hang glider flew for approximately 15 minutes, find the rate the hang glider flew for the first 5000 meters.

$\frac{7000}{x} = 15$ — Set the expression equal to 15.

$7000 = 15x$ — Multiply each side by x.

$446.7 \approx x$ — Divide each side by 15 and simplify.

The hang glider was flying at a rate of 466.7 meters per minute.

Check Your Progress

6. TRAINS A train travels 5 miles from Lynbrook to Long Beach and then back. The train travels about 1.2 times as fast returning from Long Beach. If r is the train's speed from Lynbrook to Long Beach, write and simplify an expression for the total time of the round trip.

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To subtract rational expressions with unlike denominators, rename the expressions using the LCD. Then subtract the numerators.

StudyTip

Simplifying Answers When simplifying a rational expression, you can leave the denominator in factored form, or multiply the terms. In Example 4b, both solutions below are acceptable.

- $\frac{-m^2 - 4m - 6}{(m - 4)(m + 2)}$
- $\frac{-m^2 - 4m - 6}{m^2 - 2m - 8}$

EXAMPLE 7 Subtract Rational Expressions with Unlike Denominators

Find $\frac{5}{x} - \frac{2x + 1}{4x}$.

$\frac{5}{x} - \frac{2x + 1}{3x} = \frac{5}{x}\left(\frac{4}{4}\right) - \frac{2x + 1}{4x}$ — Write $\frac{5}{x}$ using the LCD, $4x$.

$= \frac{20}{4x} - \frac{2x + 1}{4x}$ — Simplify.

$= \frac{20 - (2x + 1)}{4x}$ — Subtract the numerators.

$= \frac{20 - 2x - 1}{4x}$ or $\frac{19 - 2x}{4x}$ — Simplify.

Check Your Progress

Find each difference.

7A. $\frac{6}{t + 3} - \frac{7}{t}$

7B. $\frac{y}{y - 3} - \frac{2}{y^2 + y - 12}$

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Check Your Understanding

Examples 1–3 pp. 706–707

Find each sum or difference.

1. $\frac{3}{7n} + \frac{2}{7n}$

2. $\frac{x + 8}{2} + \frac{x}{2}$

3. $\frac{14r}{9 - r} - \frac{2r}{r - 9}$

4. $\frac{7}{5t} - \frac{3 + t}{5t}$

Example 4 pp. 707–708

Find the LCM of each pair of polynomials.

5. $3t, 8t^2$

6. $5m + 15, 2m + 6$

7. $(x^2 - 8x + 7), (x^2 + x - 2)$

Examples 5 and 7 pp. 708 and 710

Find each sum or difference.

8. $\frac{6}{n^4} + \frac{2}{n^2}$

9. $\frac{3}{4x} + \frac{2}{5y}$

10. $\frac{4}{5n} - \frac{1}{10n^3}$

11. $\frac{8}{3c} - \frac{-5}{6d}$

12. $\frac{a}{a + 4} + \frac{6}{a + 2}$

13. $\frac{x}{x - 3} - \frac{3}{x + 2}$

Example 6 p. 709

14. **EXERCISE** Joseph walks 10 times around the track at a rate of x laps per hour. He runs 8 times around the track at a rate of $3x$ laps per hour. Write and simplify an expression for the total time it takes him to go around the track 18 times.

Practice and Problem Solving

= **Step-by-Step Solutions** begin on page R12.
Extra Practice begins on page 815.

Examples 1–3 pp. 706–707

Find each sum or difference.

15. $\frac{a}{4} + \frac{3a}{4}$

16. $\frac{1}{6m} + \frac{5m}{6m}$

17. $\frac{5y}{6} - \frac{y}{6}$

18. $\frac{11}{4r} - \frac{-1}{4r}$

19. $\frac{8b}{ab} + \frac{3a}{ab}$

20. $\frac{t + 2}{3} + \frac{t + 5}{3}$

21. $\frac{3c - 7}{2c - 1} + \frac{2c + 1}{1 - 2c}$

22. $\frac{15x}{33x - 9} + \frac{3}{9 - 33x}$

23. $\frac{n + 6}{10} - \frac{n + 1}{10}$

24. $\frac{5x + 2}{2x + 5} - \frac{x - 8}{2x + 5}$

25. $\frac{w + 2}{8w} - \frac{2w - 3}{8w}$

26. $\frac{3a + 1}{a - 1} - \frac{a + 4}{a - 1}$

Example 4
pp. 707–708

Find the LCM of each pair of polynomials.

27. x^3y, x^2y^2 **28.** $5ab, 10b$ **29.** $(3r - 1), (r + 2)$

30. $2n - 10, 4n - 20$ **31.** $(x^2 + 9x + 18), x + 3$ **32.** $(k^2 - 2k - 8), (k + 2)^2$

Examples 5 and 7
pp. 708 and 710

Find each sum or difference.

33. $\frac{5}{4x} + \frac{1}{10x}$ **34.** $\frac{6}{r} + \frac{2}{r^2}$ **35.** $\frac{3}{2a} + \frac{1}{5b}$

36. $\frac{6g}{g + 5} - \frac{g - 2}{2g}$ **37.** $\frac{7}{4k + 8} - \frac{k}{k + 2}$ **38.** $\frac{5}{2d + 2} - \frac{d}{d + 5}$

39. $\frac{-2}{7r} + \frac{4}{t}$ **40.** $\frac{n}{n - 2} + \frac{n}{n + 1}$ **41.** $\frac{d}{d + 5} + \frac{7}{d - 1}$

42. $\frac{4}{a} - \frac{1}{3a}$ **43.** $\frac{6}{5t^2} - \frac{2}{3t}$ **44.** $\frac{7}{4r} - \frac{3}{t}$

45. $\frac{w - 3}{w^2 - w - 20} + \frac{w}{w + 4}$ **46.** $\frac{n}{2n + 10} + \frac{1}{n^2 - 25}$

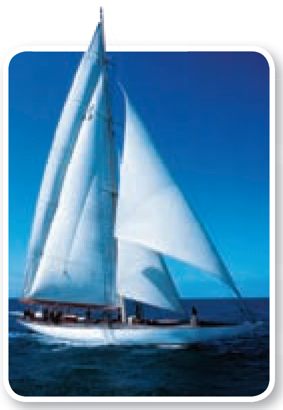
47. $\frac{2x}{x^2 + 8x + 15} - \frac{x + 3}{x + 5}$ **48.** $\frac{r - 3}{r^2 + 6r + 9} - \frac{r - 9}{r^2 - 9}$

Example 6
p. 709

49. **TRAVEL** Grace walks to her friend's house 2 miles away and then jogs back home. Her jogging speed is 2.5 times her walking speed w.

a. Write and simplify an expression to represent the amount of time Grace spends going to and coming from her friend's house.

b. If Grace walks about 3.5 miles per hour, how many minutes did she spend going to and from her friend's house?



Real-World Link

In a recent year, there were nearly 12.8 million boats registered in the U.S.

Source: USCG Boating

50. **BOATS** A boat travels 3 miles downstream at a rate 2 miles per hour faster than the current, or $x + 2$ miles per hour. It then travels 6 miles upstream at a rate 2 miles per hour slower than the current, or $x - 2$ miles per hour.

a. Write and simplify an expression to represent the total time it takes the boat to travel 3 miles downstream and 6 miles upstream.

b. If the rate of the current x is 4 miles per hour, how long did it take the boat to travel the 9 miles?

51. **SCHOOL** Mr. Kim had 18 more geometry tests to grade than algebra tests. He graded 12 tests on Saturday and 20 tests on Sunday. Write an expression for the fraction of tests he graded if a represents the number of algebra tests.

52. **PLAYS** A total of 1248 people attended the school play. The same number x attended each of the two Sunday performances. There were twice as many people at the Saturday performance than at both Sunday performances. Write an expression to represent the fraction of people who attended the Saturday performance.

Find each sum or difference.

53. $\frac{x + 5}{x^2 - 4} - \frac{3}{x^2 - 4}$ **54.** $\frac{18y}{9y + 2} - \frac{-4}{-2 - 9y}$

55. $\frac{k^2 - 26}{k - 5} - \frac{1}{5 - k}$ **56.** $\frac{8}{c - 1} + \frac{c}{1 - c}$

57. $\frac{2}{x - 1} + \frac{3}{x + 1} - \frac{4x - 2}{x^2 - 1}$ **58.** $\frac{x^2 - x - 12}{x^2 - 11x + 30} - \frac{x - 4}{18 - x}$

59. $\frac{a^2 - 5a}{3a - 18} - \frac{7a - 36}{3a - 18}$ **60.** $\frac{8n - 3}{n^2 + 8n + 12} - \frac{5n - 9}{n^2 + 8n + 12}$

61. $\frac{x^2 - 16}{x^3} + \frac{x^3 + 1}{x^4}$ **62.** $\frac{x}{7x - 3} + \frac{x + 2}{15x + 30}$

63. $\frac{5x}{3x^2 + 19x - 14} - \frac{1}{9x^2 - 12x + 4}$ **64.** $\frac{2x + 7}{x^2 - y^2} + \frac{-5}{x^2 - 2xy + y^2}$

65. TRIATHLONS In a sprint triathlon, athletes swim 400 meters, bike 20 kilometers, and run 5 kilometers. An athlete bikes 12 times as fast as she swims and runs 5 times as fast as she swims.

a. Simplify $\frac{400}{x} + \frac{20{,}000}{12x} + \frac{5000}{5x}$, an expression that represents the time it takes the athlete to complete the sprint triathlon.

b. If the athlete swims 40 meters per minute, find the total time it takes her to complete the triathlon.

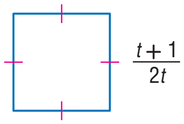
Real-World Link

A regular triathlon includes a 1.5-kilometer swim, a 40-kilometer bike ride, and a 10-kilometer run. The first Olympic triathlons were held in 2000 in Sydney, Australia.

Source: USA Triathlon

GEOMETRY **Write an expression for the perimeter of each figure.**

66.



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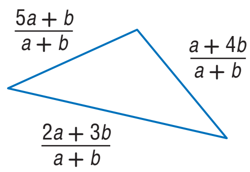
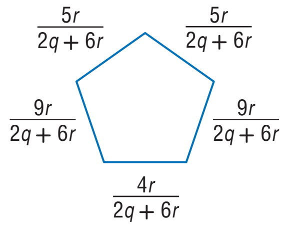


68.



69 BIKES Marina rides her bike at an average rate of 10 miles per hour. On one day, she rides 9 miles and then rides around a large loop x miles long. On the second day, she rides 5 miles and then rides around the loop three times.

a. Write an expression to represent the total time she spent riding her bike on those two days. (*Hint:* Use $t = \frac{d}{r}$, where t is time, d is distance, and r is rate.) Then simplify the expression.

b. If the loop is 2 miles long, how long did Marina ride her bike on those two days?

70. TRAVEL The Showalter family drives 80 miles to a college football game. On the return trip home, their average speed is about 3 miles per hour slower.

a. Let x represent the average speed of the car on the way to the game. Write and simplify an expression to represent the total time it took driving to the game and then back home.

b. If their average speed on the way to the game was 68 miles per hour, how long did it take the Showalter family to drive to the game and back? Round to the nearest tenth.

H.O.T. Problems

Use **H**igher-**O**rder **T**hinking Skills

71. CHALLENGE Find $\left(\frac{4}{7y-2} + \frac{7y}{2-7y}\right)\left(\frac{y+5}{6} - \frac{y+3}{6}\right)$.

72. WRITING IN MATH Describe in words the steps you use to find the LCM in an addition or subtraction of rational expressions with unlike denominators.

73. CHALLENGE Is the following statement *sometimes*, *always*, or *never* true? Explain your reasoning.

$$\frac{a}{x} + \frac{b}{y} = \frac{ay+bx}{xy}$$

74. OPEN ENDED Describe a real-life situation that could be expressed by adding two rational expressions that are fractions. Explain what the denominator and numerator represent in both expressions.

75. WRITING IN MATH Describe how to add rational expressions with denominators that are additive inverses.

Standardized Test Practice

76. SHORT RESPONSE An object is launched upwards at 19.6 meters per second from a 58.8-meter-tall platform. The equation for the object's height h meters at time t seconds after launch is $h(t) = -4.9t^2 + 19.6t + 58.8$. How long after the launch does the object strike the ground?

77. Simplify $\frac{2}{5} + \frac{3}{25} + \frac{1}{10}$.

A $\frac{2}{5}$ C $\frac{31}{50}$

B $\frac{3}{5}$ D $\frac{5}{31}$

78. STATISTICS Courtney has grades of 84, 65, and 76 on three math tests. What grade must she obtain on the next test to have an average of exactly 80 for the four tests?

F 84 H 98

G 80 J 95

79. Simplify $\frac{2}{x} + \frac{3}{x^2} + \frac{1}{2x}$.

A $\frac{3x + 2}{x^2}$ C $\frac{5x + 6}{2x^2}$

B $\frac{6}{2x^2}$ D $\frac{6 + x}{x^2}$

Spiral Review

Find each quotient. (Lesson 11-5)

80. $(6x^2 + 10x) \div 2x$ **81.** $(15y^3 + 14y) \div 3y$ **82.** $(10a^3 - 20a^2 + 5a) \div 5a$

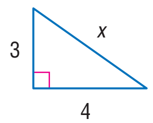
Convert each rate. Round to the nearest tenth if necessary. (Lesson 11-4)

83. 23 feet per second to miles per hour

84. 118 milliliters per second to quarts per hour (*Hint*: 1 liter ≈ 1.06 quarts)

Find the length of the missing side. If necessary, round to the nearest hundredth. (Lesson 10-5)

85.



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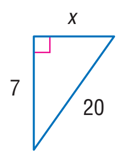

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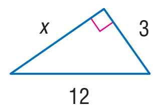

88. AMUSEMENT The height h in feet of a car above the exit ramp of a free-fall ride can be modeled by $h(t) = -16t^2 + s$. t is the time in seconds after the car drops, and s is the starting height of the car in feet. If the designer wants the ride to last 3 seconds, what should the starting height of the car be in feet? (Lesson 8-6)

Express each number in scientific notation. (Lesson 7-3)

89. 12,300 **90.** 0.0000375 **91.** 1,255,000

92. FINANCIAL LITERACY Ruben has \$13 to order pizza. The pizza costs \$7.50 plus \$1.25 per topping. He plans to tip 15% of the total cost of the pizza. Write and solve an inequality to find out how many toppings he can order. (Lesson 5-3)

Skills Review

Find each quotient. (Lesson 11-4)

93. $\frac{12}{3x^2} \div \frac{6}{x}$ **94.** $\frac{g^4}{2} \div \frac{g^3}{8d^2}$ **95.** $\frac{4y - 8}{y + 1} \div (y - 2)$