12-5

Probability of Compound Events

Then
You calculated simple probability. (Lesson 0-11)

Now
- Find probabilities of independent and dependent events.
- Find probabilities of mutually exclusive events.

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Why?

Evita is flying from Cleveland to Honolulu. The airline reports that the flight from Cleveland to Honolulu has a 40% on-time record. The airline also reported that they lose luggage 5% of the time. What is the probability that both the flight will be on time and Evita's luggage will arrive?

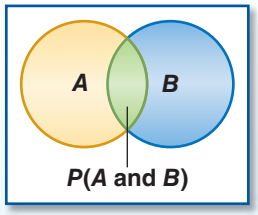
Independent and Dependent Events Recall that one event, like flying from Cleveland to Honolulu, is called a *simple event*. A **compound event** is made up of two or more simple events. So, the probability that the flight will be on time and the luggage arrives is an example of a compound event. The plane being on time may not affect whether luggage is lost. These two events are called **independent events** because the outcome of one event does not affect the outcome of the other.

Key Concept — **Probability of Independent Events** — For Your FOLDABLE

Words If two events, A and B, are independent, then the probability of both events occurring is the product of the probability of A and the probability of B.

Symbols $P(A \text{ and } B) = P(A) \cdot P(B)$

Model



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Real-World EXAMPLE 1 **Independent Events**

MARBLES **A bag contains 6 black marbles, 9 blue marbles, 4 yellow marbles, and 2 green marbles. A marble is selected, replaced, and a second marble is selected. Find the probability of selecting a black marble, then a yellow marble.**

First marble: $P(\text{black}) = \frac{6}{21}$ ← number of black marbles / total number of marbles

Second marble: $P(\text{yellow}) = \frac{4}{21}$ ← number of yellow marbles / total number of marbles

$P(\text{black, yellow}) = P(\text{black}) \cdot P(\text{yellow})$ Probability of independent events

$= \frac{6}{21} \cdot \frac{4}{21}$ or $\frac{24}{441}$ Substitution

The probability is $\frac{24}{441}$ or about 5.4%.

Check Your Progress

Find each probability.

1A. $P(\text{blue, green})$

1B. $P(\text{not black, blue})$

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When the outcome of one event affects the outcome of another event, they are **dependent events**. In Example 1, if the marble was not placed back in the bag, then drawing the two marbles would have been dependent events. The probability of drawing the second marble depends on what marble was drawn first.

Key Concept **Probability of Dependent Events** For Your FOLDABLE

Words If two events, A and B, are dependent, then the probability of both events occurring is the product of the probability of A and the probability of B after A occurs.

Symbols $P(A \text{ and } B) = P(A) \cdot P(B \text{ following } A)$

Recall that the complement of a set is the set of all objects that do *not* belong to the given set. In a standard deck of cards, the complement of drawing a heart is drawing a diamond, club, or spade. So, the probability of drawing a heart is $\frac{13}{52}$, and the probability of not drawing a heart is $\frac{52 - 13}{52}$ or $\frac{39}{52}$.

Notice that the sum of these probabilities is 1. This is true for any two complementary events.

Real-World Link

A standard deck of cards consists of 52 cards. There are 4 suits: hearts, diamonds, clubs, and spades. There are 13 cards of each suit. An ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, jack, queen, and king. The hearts and diamonds are red while the clubs and spades are black.

Real-World EXAMPLE 2 **Dependent Events**

CARDS **Cynthia randomly draws three cards from a standard deck one at a time without replacement. Find the probability that the cards are drawn in the given order.**

a. *P*(diamond, spade, diamond)

First card: $P(\text{diamond}) = \frac{13}{52}$ or $\frac{1}{4}$ ← number of diamonds / total number of cards

Second card: $P(\text{spade}) = \frac{13}{51}$ ← number of spades / number of cards remaining

Third card: $P(\text{diamond}) = \frac{12}{50}$ or $\frac{6}{25}$ ← number of diamonds remaining / number of cards remaining

$P(\text{diamond, spade, diamond}) = P(\text{diamond}) \cdot P(\text{spade}) \cdot P(\text{diamond})$

$= \frac{1}{4} \cdot \frac{13}{51} \cdot \frac{6}{25}$ or $\frac{13}{850}$ Substitution

The probability is $\frac{13}{850}$ or about 1.5%.

b. *P*(four, four, not a jack)

After Cynthia draws the first two fours from the deck of 52 cards, there are 50 cards left. Since neither of these cards are jacks, there are still four jacks left in the deck. So, there are $52 - 2 - 4$ or 46 cards that are not jacks.

$P(\text{four, four, not a jack}) = P(\text{four}) \cdot P(\text{four}) \cdot P(\text{not a jack})$

$= \frac{4}{52} \cdot \frac{3}{51} \cdot \frac{46}{50}$

$= \frac{552}{132{,}600}$ or $\frac{23}{5525}$

The probability is $\frac{23}{5525}$ or about 0.4%.

Problem-Solving Tip

Act It Out Acting out the situation can help you understand what the question is asking. Use a deck of cards to represent the situation described in the problem.

Check Your Progress **Find each probability.**

2A. P(two, five, not a five)

2B. P(heart, not a heart, heart)

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Mutually Exclusive Events Events that cannot occur at the same time are called **mutually exclusive events**. Suppose you wanted to find the probability of drawing a heart or a diamond. Since a card cannot be both a heart and a diamond, the events are mutually exclusive.

StudyTip

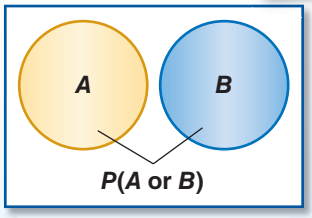
and* and *or While probabilities involving *and* deal with independent and dependent events, probabilities involving *or* deal with mutually exclusive and non-mutually exclusive events.

Key Concept **Mutually Exclusive Events** For Your FOLDABLE

Words If two events, A and B, are mutually exclusive, then the probability that either A or B occurs is the sum of their probabilities.

Symbols $P(A \text{ or } B) = P(A) + P(B)$

Model



Real-World EXAMPLE 3 **Mutually Exclusive Events**

A die is being rolled. Find each probability.

a. *P*(3 or 5)

Since a die cannot show both a 3 and a 5 at the same time, these events are mutually exclusive.

$P(\text{rolling a 3}) = \frac{1}{6}$ ← number of sides with a 3 / ← total number of sides

$P(\text{rolling a 5}) = \frac{1}{6}$ ← number of sides with a 5 / ← total number of sides

$P(3 \text{ or } 5) = P(\text{rolling a 3}) + P(\text{rolling a 5})$ Probability of mutually exclusive events

$= \frac{1}{6} + \frac{1}{6}$ Substitution

$= \frac{2}{6} \text{ or } \frac{1}{3}$ Add.

The probability of rolling a 3 or a 5 is $\frac{1}{3}$ or about 33%.

StudyTip

Alternative Method In Example 3a, you could have placed the number of possible outcomes over the total number of outcomes.

$\frac{1+1}{6} = \frac{2}{6} \text{ or } \frac{1}{3}$

b. *P*(at least 4)

Rolling at least a 4 means you can roll either a 4, 5, or a 6. So, you need to find the probability of rolling a 4, 5, or a 6.

$P(\text{rolling a 4}) = \frac{1}{6}$ ← number of sides with a 4 / ← total number of sides

$P(\text{rolling a 5}) = \frac{1}{6}$ ← number of sides with a 5 / ← total number of sides

$P(\text{rolling a 6}) = \frac{1}{6}$ ← number of sides with a 6 / ← total number of sides

$P(\text{at least 4}) = P(\text{rolling a 4}) + P(\text{rolling a 5}) + P(\text{rolling a 6})$ Mutually exclusive events

$= \frac{1}{6} + \frac{1}{6} + \frac{1}{6}$ Substitution

$= \frac{3}{6} \text{ or } \frac{1}{2}$ Add.

The probability of rolling at least a 4 is $\frac{1}{2}$ or about 50%.

Check Your Progress

3A. $P(\text{less than 3})$ **3B.** $P(\text{even})$

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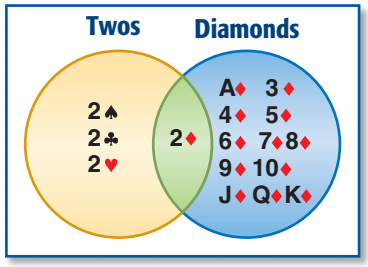
ReadingMath

A* or *B Unlike everyday language, the expression *A* or *B* allows the possibility of both *A* and *B* occurring.

Suppose you want to find the probability of randomly drawing a 2 or a diamond from a standard deck of cards. Since it is possible to draw a card that is both a 2 and a diamond, these events are not mutually exclusive.



$P(2)$	$P(\text{diamond})$	$P(2, \text{diamond})$
$\frac{4}{52}$	$\frac{13}{52}$	$\frac{1}{52}$

In the first two fractions above, the probability of drawing the two of diamonds is counted twice, once for a two and once for a diamond. To find the correct probability, subtract P(2 of diamonds) from the sum of the first two probabilities.

$$\begin{aligned} P(\text{2 or a diamond}) &= P(2) + P(\text{diamond}) - P(\text{2 of diamonds}) \\ &= \frac{4}{52} + \frac{13}{52} - \frac{1}{52} \\ &= \frac{16}{52} \text{ or } \frac{4}{13} \qquad \text{The probability is } \frac{4}{13} \text{ or about } 31\%. \end{aligned}$$

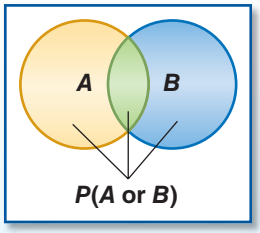
Watch Out!

Intersection of Events When determining the probability of events that are not mutually exclusive, you may count the intersection of the events twice since it occurs in both events. It only actually occurs once.

Key Concept **Events that are Not Mutually Exclusive** For Your FOLDABLE

Words If two events, *A* and *B*, are not mutually exclusive, then the probability that either *A* or *B* occurs is the sum of their probabilities decreased by the probability of both occurring.

Model



Symbols $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

Real-World EXAMPLE 4 **Events that are Not Mutually Exclusive**

STUDENT ATHLETES **Of 240 girls, 176 are on the Honor Roll, 48 play sports, and 36 are on the Honor Roll and play sports. What is the probability that a randomly selected student plays sports or is on the Honor Roll?**

Since some students play varsity sports and are on the Honor Roll, the events are not mutually exclusive.

$P(\text{sports}) = \frac{48}{240}$ $P(\text{Honor Roll}) = \frac{176}{240}$ $P(\text{sports and Honor Roll}) = \frac{36}{240}$

$$\begin{aligned} P(\text{sports or Honor Roll}) &= P(\text{sports}) + P(\text{HR}) - P(\text{sports and HR}) \\ &= \frac{48}{240} + \frac{176}{240} - \frac{36}{240} && \text{Substitution} \\ &= \frac{188}{240} \text{ or } \frac{47}{60} && \text{Simplify.} \end{aligned}$$

The probability is $\frac{47}{60}$ or about 78%.

Check Your Progress

4. **PETS** Out of 5200 households surveyed, 2107 had a dog, 807 had a cat, and 303 had both a dog and a cat. What is the probability that a randomly selected household has a dog or a cat?

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Check Your Understanding

Examples 1 and 2
pp. 771–772

Determine whether the events are *independent* or *dependent*. Then find the probability.

1. **BABYSITTING** A toy bin contains 12 toys, 8 stuffed animals, and 3 board games. Marsha randomly chooses 2 toys for the child she is babysitting to play with. What is the probability that she chose 2 stuffed animals as the first two choices?
2. **FRUIT** A fruit basket contains 6 apples, 5 bananas, 4 oranges, and 5 peaches. Drew randomly chooses one piece of fruit, eats it, and chooses another piece of fruit. What is the probability that he chose a banana and then an apple?
3. **MONEY** Nakos has 4 quarters, 3 dimes, and 2 nickels in his pocket. Nakos randomly picks two coins out of his pocket. What is the probability Nakos did not choose a dime either time, if he replaced the first coin back in his pocket before choosing a second coin?
4. **BOOKS** Joanna needs a book to prop up a table leg. She randomly selects a book, puts it back on the shelf, and selects another book. What is the probability that Joanna selected two math books?

Examples 3 and 4
pp. 773–774

A card is drawn from a standard deck of playing cards. Determine whether the events are *mutually exclusive* or *not* mutually exclusive. Then find the probability.

5. P(two or queen)
6. P(diamond or heart)
7. P(seven or club)
8. P(spade or ace)

Practice and Problem Solving

● = **Step-by-Step Solutions** begin on page R12.
Extra Practice begins on page 815.

Examples 1 and 2
pp. 771–772

Determine whether the events are *independent* or *dependent*. Then find the probability.

9. **COINS** If a coin is tossed 4 times, what is the probability of getting tails all 4 times?
10. **DICE** A die is rolled twice. What is the probability of rolling two different numbers?
11. **CANDY** A box of chocolates contains 10 milk chocolates, 8 dark chocolates, and 6 white chocolates. Sung randomly chooses a chocolate, eats it, and then randomly chooses another chocolate. What is the probability that Sung chose a milk chocolate and then a white chocolate?
12. **DICE** A die is rolled twice. What is the probability of rolling two of the same numbers?
13. **PETS** Chuck and Rashid went to a pet store to buy dog food. They chose from 10 brands of dry food, 6 brands of canned food, and 3 brands of pet snacks. What is the probability that Chuck and Rashid both chose dry food, if Chuck randomly chose first and liked the first brand he picked up?
14. **CARS** A rental agency has 12 white sedans, 8 gray sedans, 6 red sedans, and 3 green sedans for rent. Mr. Escobar rents a sedan, returns it because the radio is broken, and gets another sedan. What is the probability that Mr. Escobar was given a green sedan and then a gray sedan?

Real-World Link

About 65% of pet owners acquire their pets free or at low cost.

Source: National Council or Pet Population Study and Policy

Examples 3 and 4
pp. 773–774

Determine whether the events are *mutually exclusive* or *not* mutually exclusive. Then find the probability.

Real-World Link

A bowling museum and hall of fame is located in St. Louis, Missouri. The museum spans 50,000 square feet and is 3 stories tall.

Source: International Bowling Museum and Hall of Fame

15. BOWLING Cindy's bowling records indicate that for any frame, the probability that she will bowl a strike is 30%, a spare 45%, and neither 25%. What is the probability that Cindy will bowl either a spare or a strike for any given frame that she bowls?

16. SPORTS CARDS Dario owns 145 baseball cards, 102 football cards, and 48 basketball cards. What is the probability that Dario randomly selects a baseball or a football card from his collection?

17. SCHOLARSHIPS 3000 essays were received for a $5000 college scholarship. 2865 essays were the required length, 2577 of the applicants had the minimum required grade-point average, and 2486 had the required length and the minimum required grade-point average. What is the probability that an essay selected at random will have the required length or the applicant will have the minimum required grade-point average?

18. KITTENS Ruby's cat had a litter of 8 kittens. The litter included 2 orange females, 3 mixed-color females, 1 orange male, and 2 mixed-color males. Ruby wants to keep one kitten. What is the probability that she randomly chooses a kitten that is female or orange?

CHIPS A restaurant serves red tortilla chips, blue tortilla chips, and yellow tortilla chips. The bowl of chips Gabriel receives has 10 red chips, 8 blue chips, and 12 yellow chips. After Gabriel chooses a chip, he eats it. Find each probability.

19. P(red, blue)

20. P(blue, yellow)

21. P(yellow, not blue)

22. P(red, not yellow)

23. SOCKS Damon has 14 white socks, 6 black socks, and 4 blue socks in his drawer. If he chooses two socks at random, what is the probability that the first two socks he chooses are white?

Cards are being randomly drawn from a standard deck of cards. Once a card is drawn, it is not replaced. Find each probability.

24. P(heart or spade)

25. P(spade or club)

26. P(queen, then heart)

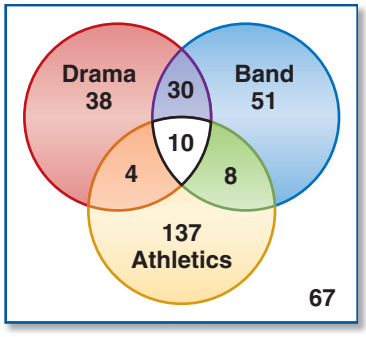
27. P(jack, then spade)

28. P(five, then red)

29. P(ace or black)

30. CANDY A bag of jelly beans contains 10 red, 6 green, 7 yellow, and 5 orange jelly beans. What is the probability of randomly choosing a red jelly bean, replacing, randomly choosing another red jelly bean, replacing, and then randomly choosing an orange jelly bean?

31. SPORTS The extracurricular activities in which the senior class at Valley View High School participate are shown in the Venn diagram.



a. How many students are in the senior class?

b. How many students participate in athletics?

c. If a student is randomly chosen, what is the probability that the student participates in athletics or drama?

d. If a student is randomly chosen, what is the probability that the student participates in only drama and band?

32. TILES Kirsten and José are playing a game. Kirsten places tiles numbered 1 to 50 in a bag. José selects a tile at random. If he selects a prime number or a number greater than 40, then he wins the game. What is the probability that José will on his first turn?

ReadingMath

Conditional Probability $P(B|A)$ is read the *probability* of *B, given A.*

33. MULTIPLE REPRESENTATIONS In this problem, you will explore conditional probability. **Conditional probability** is the probability that event B occurs given that event A has already occurred. It is calculated by dividing the probability of the occurrence of both events by the probability of the occurrence of the first event. The notation for conditional probability is $P(B \mid A)$.

a. GRAPHICAL Draw a Venn diagram to illustrate $P(A \text{ and } B)$.

b. VERBAL Write the formula for $P(B \mid A)$ given the Venn diagram.

c. ANALYTICAL A jar contains 12 marbles, of which 8 marbles are red and 4 marbles are green. If marbles are chosen without replacement, find $P(\text{red})$ and $P(\text{red, green})$.

d. ANALYTICAL Using the probabilities from part **c** and the Venn diagram in part **a**, determine the probability of choosing a green marble on the second selection, given that the first marble selected was red.

e. ANALYTICAL Write a formula for finding a conditional probability.

f. ANALYTICAL Use the definition from part **e** to answer the following: At a basketball game, 80% of the fans cheered for the home team. In the same crowd, 20% of the fans were waving banners and cheering for the home team. What is the probability that a fan waved a banner given that the fan cheered for the home team?

H.O.T. Problems

Use Higher-Order Thinking Skills

34. FIND THE ERROR George and Aliyah are determining the probability of randomly choosing a blue or red marble from a bag of 8 blue marbles, 6 red marbles, 8 yellow marbles, and 4 white marbles. Is either of them correct? Explain.

George

$P(\text{blue or red}) = P(\text{blue}) \cdot P(\text{red})$

$= \frac{8}{26} \cdot \frac{6}{26}$

$= \frac{48}{676}$

about 7%

Aliyah

$P(\text{blue or red}) = P(\text{blue}) + P(\text{red})$

$= \frac{8}{26} + \frac{6}{26}$

$= \frac{14}{26}$

about 54%

35. CHALLENGE In some cases, if one bulb in a string of holiday lights fails to work, the whole string will not light. If each bulb in a set has a 99.5% chance of working, what is the maximum number of lights that can be strung together with at least a 90% chance that the whole string will light?

36. REASONING Suppose there are three events A, B, and C that are not mutually exclusive. List all of the probabilities you would need to consider in order to calculate $P(A \text{ or } B \text{ or } C)$. Then write the formula you would use to calculate the probability.

37. OPEN ENDED Describe a situation in your life that involves dependent and independent events. Explain why the events are dependent or independent.

38. WRITING IN MATH Explain why the subtraction occurs when finding the probability of two events that are not mutually exclusive.

Standardized Test Practice

39. In how many ways can a committee of 4 be selected from a group of 12 people?

A 48
B 483
C 495
D 11,880

40. A total of 925 tickets were sold for $5925. If adult tickets cost $7.50 and children's tickets cost $3.00, how many adult tickets were sold?

F 700 **H** 325
G 600 **J** 225

41. SHORT RESPONSE A circular swimming pool with a diameter of 28 feet has a deck of uniform width built around it. If the area of the deck is 60π square feet, find its width.

42. The probability of heads landing up when you flip a coin is $\frac{1}{2}$. What is the probability of getting tails if you flip it again?

A $\frac{1}{4}$ **C** $\frac{1}{2}$
B $\frac{1}{3}$ **D** $\frac{3}{4}$

Spiral Review

43. SHOPPING The Millers have twelve grandchildren, 5 boys and 7 girls. For their anniversary, the grandchildren decided to pool their money and have three of them shop for the entire group. (Lesson 12-4)

a. Does this situation represent a *combination* or *permutation*?

b. How many ways are there to choose the three?

c. What is the probability that all three will be girls?

44. ECOLOGY A group of 1000 randomly selected teens were asked if they believed there was global warming. The results are shown in the table. Find the mean absolute deviation. (Lesson 12-3)

Teen Ecology Survey Results

Response	Number
Yes, strongly agree	312
Yes, mildly agree	340
No, I don't think so	109
No, absolutely not	116
Not sure	123

Solve each equation. State any extraneous solutions. (Lesson 11-8)

45. $\frac{4}{a} = \frac{3}{a-2}$

46. $\frac{3}{x} = \frac{1}{x-2}$

47. $\frac{x}{x+1} = \frac{x-6}{x-1}$

48. $\frac{2n}{3} + \frac{1}{2} = \frac{2n-3}{6}$

49. COOKING Hannah was making candy using a two-quart pan. As she stirred the mixture, she noticed that the pan was about $\frac{2}{3}$ full. If each piece of candy has a volume of about $\frac{3}{4}$ ounce, approximately how many pieces of candy will Hannah make? (*Hint:* There are 32 ounces in a quart.) (Lesson 11-3)

50. GEOMETRY A rectangle has a width of $3\sqrt{5}$ centimeters and a length of $4\sqrt{10}$ centimeters. Find the area of the rectangle. Write as a simplified radical expression. (Lesson 10-2)

Skills Review

Solve each equation. Check your solution. (Lesson 10-4)

51. $\sqrt{-3a} = 6$

52. $\sqrt{a} = 100$

53. $\sqrt{-k} = 4$

54. $5\sqrt{2} = \sqrt{x}$

55. $3\sqrt{7} = \sqrt{-y}$

56. $3\sqrt{4a} - 2 = 10$