9-6

Exponential Functions

Then
You simplified numerical expressions involving exponents. (Lesson 1-2)

Now
- Graph exponential functions.
- Identify data that display exponential behavior.

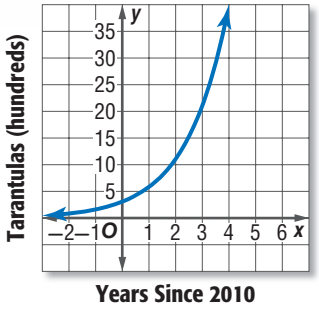
New Vocabulary
exponential function

Math Online
glencoe.com
- Extra Examples
- Personal Tutor
- Self-check Quiz
- Homework Help

Why?

Tarantulas can appear scary with their large hairy bodies and legs, but they are harmless to humans. The graph shows a tarantula spider population that increases over time. Notice that the graph is neither linear nor quadratic.

The graph represents the function $y = 3(2)^x$. This is an example of an *exponential* function.



Graph Exponential Functions An **exponential function** is a function of the form $y = ab^x$, where $a \neq 0$, $b > 0$, and $b \neq 1$. Notice that the base is a constant and the exponent is a variable. Exponential functions are nonlinear and nonquadratic functions.

Key Concept — **Exponential Function** — For Your FOLDABLE

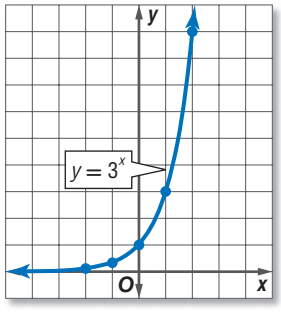
Words An exponential function is a function that can be described by an equation of the form $y = ab^x$, where $a \neq 0$, $b > 0$, and $b \neq 1$.

Examples $y = 2(3)^x$ $\quad y = 4^x$ $\quad y = \left(\frac{1}{2}\right)^x$

EXAMPLE 1 **Graph with $a > 0$ and $b > 1$**

a. Graph $y = 3^x$. Find the y-intercept, and state the domain and range.

x	3^x	y
-2	3^{-2}	$\frac{1}{9}$
-1	3^{-1}	$\frac{1}{3}$
0	3^0	1
1	3^1	3
2	3^2	9



Graph the ordered pairs, and connect the points with a smooth curve. The graph crosses the y-axis at 1, so the y-intercept is 1. The domain is all real numbers, and the range is all positive real numbers.

b. Use the graph to approximate the value of $3^{0.7}$.

The graph represents all real values of x and their corresponding values of y for $y = 3^x$. So, when $x = 0.7$, y is about 2. Use a calculator to confirm this value: $3^{0.7} \approx 2.157669$.

Check Your Progress

1A. Graph $y = 7^x$. Find the y-intercept, and state the domain and range.

1B. Use the graph to approximate the value of $y = 7^{0.5}$ to the nearest tenth. Use a calculator to confirm the value.

Personal Tutor glencoe.com

The graphs of functions of the form $y = ab^x$, where $a > 0$ and $b > 1$, all have the same shape as the graph in Example 1. The greater the base or b-value, the faster the graph rises as you move from left to right on the graph. The graphs of functions of the form $y = ab^x$, where $a > 0$ and $0 < b < 1$, also have the same general shape.

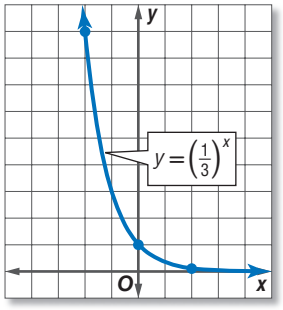
StudyTip

$a < 0$ If the value of a is less than 0, the graph will be reflected across the x-axis.

EXAMPLE 2 Graph with $a > 0$ and $0 < b < 1$

a. Graph $y = \left(\frac{1}{3}\right)^x$. Find the y-intercept, and state the domain and range.

x	$\left(\frac{1}{3}\right)^x$	y
-2	$\left(\frac{1}{3}\right)^{-2}$	9
0	$\left(\frac{1}{3}\right)^{0}$	1
2	$\left(\frac{1}{3}\right)^{2}$	$\frac{1}{9}$



The y-intercept is 1. The domain is all real numbers, and the range is all positive real numbers. Notice that as x increases, the y-values decrease less rapidly.

b. Use the graph to approximate the value of $\left(\frac{1}{3}\right)^{-1.5}$.

When $x = -1.5$, the value of y is about 5. Use a calculator to confirm this value:

KEYSTROKES: (1 ÷ 3) ^ −1.5 ENTER 5.196152.

Check Your Progress

2A. Graph $y = \left(\frac{1}{2}\right)^x - 1$. Find the y-intercept, and state the domain and range.

2B. Use the graph to approximate the value of $\left(\frac{1}{2}\right)^{-2.5} - 1$ to the nearest tenth. Use a calculator to confirm the value.

Personal Tutor glencoe.com

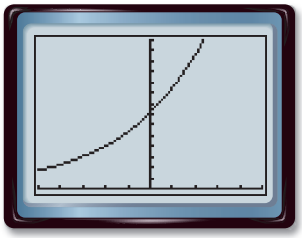
Exponential functions occur in many real world situations.

Real-World EXAMPLE 3 Use Exponential Functions to Solve Problems

SODA The consumption of soda has increased each year since 2000. The function $C = 179(1.029)^t$ models the amount of soda consumed in the world, where C is the amount consumed in billions of liters and t is the number of years since 2000.

a. Graph the function. What values of C and t are meaningful in the context of the problem?

Since t represents time, $t > 0$. At $t = 0$, the consumption is 179 billion liters. Therefore, in the context of this problem $C > 179$ is meaningful.



[−50, 50] scl: 10 by [0, 350] scl: 25

Real-World Link

The United States is the largest soda consumer in the world. In a recent year, the United States accounted for one third of the world's total soda consumption.

Source: Worldwatch Institute

b. How much soda was consumed in 2005?

$C = 179(1.029)^t$ Original equation
$= 179(1.029)^5$ $t = 5$
$= 206.5$ Use a calculator.

The world soda consumtion in 2005 was approximately 206.5 billion liters.

Check Your Progress

3. A certain bacteria doubles every 20 minutes. Beginning with 10 cells in a culture, the population can be represented by the function $B = 10(2)^t$, where B is the number of bacteria cells and t is the time in 20 minute increments. How many will there be after 2 hours?

Personal Tutor glencoe.com

Identify Exponential Behavior Recall from Lesson 3-3 that linear functions have a constant rate of change. Exponential functions do not have constant rates of change, but they do have constant ratios.

EXAMPLE 4 Identify Exponential Behavior

Determine whether the set of data shown below displays exponential behavior. Write *yes* or *no*. Explain why or why not.

x	0	5	10	15	20	25
y	64	32	16	8	4	2

Problem-Solving Tip

Make an Organized List Making an organized list of x-values and corresponding y-values is helpful in graphing the function. It can also help you identify patterns in the data.

Method 1 Look for a pattern.

The domain values are at regular intervals of 5. Look for a common factor among the range values.

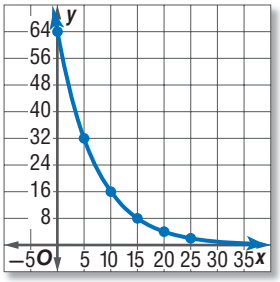
$$64 \quad 32 \quad 16 \quad 8 \quad 4 \quad 2$$
$$\times\frac{1}{2} \quad \times\frac{1}{2} \quad \times\frac{1}{2} \quad \times\frac{1}{2} \quad \times\frac{1}{2}$$

The range values differ by the common factor of $\frac{1}{2}$.

Since the domain values are at regular intervals and the range values differ by a positive common factor, the data are probably exponential. Its equation may involve $\left(\frac{1}{2}\right)^x$.

Method 2 Graph the data.

Plot the points and connect them with a smooth curve. The graph shows a rapidly decreasing value of y as x increases. This is a characteristic of exponential behavior in which the base is between 0 and 1.



Study Tip

Checking Answers The graph of an exponential function may resemble part of the graph of a quadratic function. Be sure to check for a pattern as well as to look at a graph.

Check Your Progress

4. Determine whether the set of data shown below displays exponential behavior. Write *yes* or *no*. Explain why or why not.

x	0	3	6	9	12	15
y	12	16	20	24	28	32

Personal Tutor glencoe.com

Check Your Understanding

Examples 1 and 2
pp. 567–568

Graph each function. Find the *y*-intercept, and state the domain and range. Then use the graph to determine the approximate value of the given expression. Use a calculator to confirm the value.

1. $y = 2^x$; $2^{1.5}$

2. $y = -5^x$; $-5^{0.5}$

3. $y = -\left(\frac{1}{5}\right)^x$; $-\left(\frac{1}{5}\right)^{-0.5}$

4. $y = 3\left(\frac{1}{4}\right)^x$; $3\left(\frac{1}{4}\right)^{0.5}$

Graph each function. Find the *y*-intercept, and state the domain and range.

5. $f(x) = 6^x + 3$

6. $f(x) = 2 - 2^x$

Example 3
pp. 568–569

7. **BIOLOGY** The function $f(t) = 100(1.05)^t$ models the growth of a fruit fly population, where $f(t)$ is the number of flies and t is time in days.

a. What values for the domain and range are reasonable in the context of this situation? Explain.

b. After two weeks, approximately how many flies are in this population?

Example 4
p. 569

Determine whether the set of data shown below displays exponential behavior. Write *yes* or *no*. Explain why or why not.

8.

x	1	2	3	4	5	6
y	−4	−2	0	2	4	6

9.

x	2	4	6	8	10	12
y	1	4	16	64	256	1024

Practice and Problem Solving

= **Step-by-Step Solutions** begin on page R12.
Extra Practice begins on page 815.

Examples 1 and 2
pp. 567–568

Graph each function. Find the *y*-intercept, and state the domain and range. Then use the graph to determine the approximate value of the given expression. Use a calculator to confirm the value.

10. $y = 2 \cdot 8^x$, $2(8)^{-0.5}$

11. $y = 2 \cdot \left(\frac{1}{6}\right)^x$; $2\left(\frac{1}{6}\right)^{1.5}$

12. $y = \left(\frac{1}{12}\right)^x$; $\left(\frac{1}{12}\right)^{0.5}$

13. $y = -3 \cdot 9^x$, $-3(9)^{-0.5}$

14. $y = -4 \cdot 10^x$, $-4(10)^{-0.5}$

15. $y = 3 \cdot 11^x$, $3(11)^{-0.2}$

Graph each function. Find the *y*-intercept, and state the domain and range.

16. $y = 4^x + 3$

17. $y = \frac{1}{2}(2^x - 8)$

18. $y = 5(3^x) + 1$

19. $y = -2(3^x) + 5$

Example 3
pp. 568–569

20. **BIOLOGY** A population of bacteria in a culture increases according to the model $p = 300(2.7)^{0.02t}$, where t is the number of hours and $t = 0$ corresponds to 9:00 A.M.

a. Use this model to estimate the number of bacteria at 11 A.M.

b. Graph the function and name the p-intercept. Describe what the p-intercept represents, and describe a reasonable domain and range for this situation.

Example 4
p. 569

Determine whether the set of data shown below displays exponential behavior. Write *yes* or *no*. Explain why or why not.

21.

x	−4	0	4	8	12
y	2	−4	8	−16	32

22.

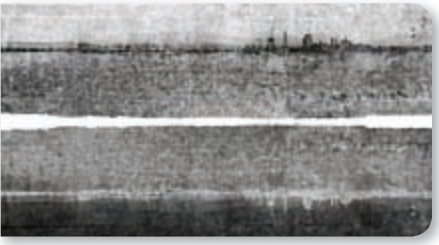
x	−6	−3	0	3
y	5	10	15	20

23.

x	−8	−6	−4	−2
y	0.25	0.5	1	2

24.

x	20	30	40	50	60
y	1	0.4	0.16	0.064	0.0256



Real-World Link

The world's largest photograph, named The Great Picture, was created by a group of photographers known as The Legacy Project. The photograph has an area of 3375 square feet.

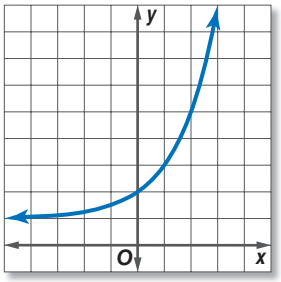
Source: Photoshop Support

25. PHOTOGRAPHY Jameka is enlarging a photograph to make a poster for school. She will enlarge the picture repeatedly at 150%. The function $P = 1.5^x$ models the new size of the picture being enlarged, where x is the number of enlargements. How big is the picture after it has been enlarged 4 times?

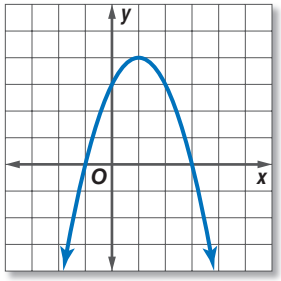
26. FINANCIAL LITERACY Daniel invested \$500 into a savings account. The equation $A = 500(1.005)^{12t}$ models the value of Daniel's investment A after t years. How much will Daniel's investment be worth in 8 years?

Identify each function as *linear*, *quadratic*, or *exponential*.

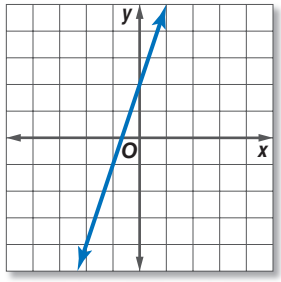
27.



28.



29.



30. $y = 4^x + 3$

31. $y = 2x(x - 1)$

32. $5x + y = 8$

33. GRADUATION The number of graduates at a high school has increased by a factor of 1.055 every year since 2001. In 2001, 110 students graduated. The function $N = 110(1.055)^t$ models. N, the number of students expected to graduate t year after 2001. How many students will graduate in 2012?

Describe the graph of each equation as a transformation of the graph of $y = 2^x$.

34. $y = 2^x + 6$

35. $y = 3(2)^x$

36. $y = -\frac{1}{4}(2)^x$

37. $y = -3 + 2^x$

38. $y = \left(\frac{1}{2}\right)^x$

39. $y = -5(2)^x$

40. DEER The deer population at a national park doubles every year. In 2000, there were 25 deer in the park. The function $N = 25(2)^t$ models the number of deer N in the national park t years after 2000. What will the deer population in the park be in 2015?

H.O.T. Problems

Use Higher-Order Thinking Skills

41. CHALLENGE Write an exponential function that passes through the points at (0, 3) and (1, 6).

42. REASONING Determine whether the graph of $y = ab^x$, where $a \neq 0$, $b > 0$, and $b \neq 1$, *sometimes*, *always*, or *never* has an x-intercept. Explain your reasoning.

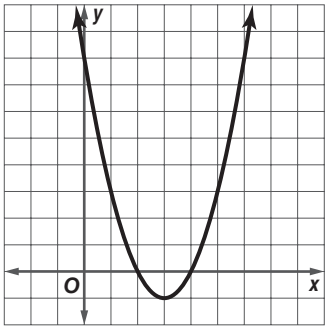
43. OPEN ENDED Choose an exponential function that represents a real-world situation, and graph the function. Analyze the graph.

44. REASONING Compare and contrast an exponential function of the form $y = ab^x + c$, where $a \neq 0$, $b > 0$, and $b \neq 1$ and a quadratic function of the form $y = ax^2 + c$.

45. WRITING IN MATH Explain how to determine whether a set of data displays exponential behavior.

Standardized Test Practice

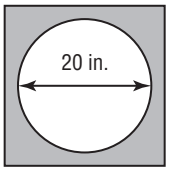
46. SHORT RESPONSE What are the zeros of the function graphed below?



47. Hinto invested $300 into a savings account. The equation $A = 300(1.005)^{12t}$ models the amount in Hinto's account A after t years. How much will be in Hinto's account after 7 years?

A $25,326 **C** $385.01
B $456.11 **D** $301.52

48. GEOMETRY Ayana placed a circular piece of paper on a square picture as shown below. If the picture extends 4 inches beyond the circle on each side, what is the perimeter of the square piece of paper?



F 64 in. **H** 94 in.
G 80 in. **J** 112 in.

49. Which of the following shows $4x^2 - 8x - 12$ factored completely?

A $4(x - 3)(x + 1)$
B $4(x + 3)(x - 1)$
C $(4x + 12)(x - 1)$
D $(x - 3)(4x + 4)$

Spiral Review

Solve each equation by using the Quadratic Formula. Round to the nearest tenth if necessary. (Lesson 9-5)

50. $6x^2 - 3x - 30 = 0$ **51.** $4x^2 + 18x = 10$ **52.** $2x^2 + 6x = 7$

Solve each equation by taking the square root of each side. Round to the nearest tenth if necessary. (Lesson 9-4)

53. $x^2 = 25$ **54.** $x^2 + 6x + 9 = 16$ **55.** $x^2 - 14x + 49 = 15$

Evaluate each product. Express the results in both scientific notation and standard form. (Lesson 7-3)

56. $(1.9 \times 10^2)(4.7 \times 10^6)$ **57.** $(4.5 \times 10^{-3})(5.6 \times 10^4)$ **58.** $(3.8 \times 10^{-4})(6.4 \times 10^{-8})$

59. DEMOLITION DERBY When a car hits an object, the damage is measured by the collision impact. For a certain car the collision impact I is given by $I = 2v^2$, where v represents the speed in kilometers per minute. What is the collision impact if the speed of the car is 4 kilometers per minute? (Lesson 7-1)

Use elimination to solve each system of equations. (Lesson 6-3)

60. $x + y = -3$
$x - y = 1$

61. $3a + b = 5$
$2a + b = 10$

62. $3x - 5y = 16$
$-3x + 2y = -10$

Skills Review

Find the next three terms of each arithmetic sequence. (Lesson 3-5)

63. 1, 3, 5, 7, …

64. −6, −4, −2, 0, …

65. 6.5, 9, 11.5, 14, …

66. 10, 3, −4, −11, …

67. $\frac{1}{2}, \frac{5}{4}, 2, \frac{11}{4}, \ldots$

68. $1, \frac{3}{4}, \frac{1}{2}, \frac{1}{4}, \ldots$