Parallel and Perpendicular Lines

Why?
Notice the squares, rectangles and lines in the piece of art shown at the right. Some of the lines intersect forming right angles. Other lines do not intersect at all.

Parallel Lines Lines in the same plane that do not intersect are called parallel lines. Parallel lines have the same slope.

All vertical lines are parallel.

If two nonvertical lines in a plane have the same slope, then they are parallel.

You can write an equation of a line parallel to a given line if you know a point on the line and an equation of the given line. First find the slope of the given line. Then, substitute the point provided and the slope from the given line into the point-slope form.

EXAMPLE 1 Parallel Line Through a Given Point

Write an equation in slope-intercept form for the line that passes through \((-3, 5)\) and is parallel to the graph of \(y = 2x - 4\).

Step 1 The slope of the line with equation \(y = 2x - 4\) is 2. The line parallel to \(y = 2x - 4\) has the same slope, 2.

Step 2 Find the equation in slope-intercept form.

\[
\begin{align*}
    y - y_1 &= m(x - x_1) \\
    y - 5 &= 2[x - (-3)] \\
    y - 5 &= 2(x + 3) \\
    y - 5 &= 2x + 6 \\
    y - 5 + 5 &= 2x + 6 + 5 \\
    y &= 2x + 11
\end{align*}
\]

Write the equation in slope-intercept form.

\(y = 2x + 11\)

Check Your Progress

1. Write an equation in point-slope form for the line that passes through \((4, -1)\) and is parallel to the graph of \(y = \frac{1}{4}x + 7\).
**Perpendicular Lines** Lines that intersect at right angles are called **perpendicular lines**. The slopes of perpendicular lines are opposite reciprocals. That is, if the slope of a line is 4, the slope of the line perpendicular to it is \(-\frac{1}{4}\).

You can use the properties of the slopes of perpendicular lines to determine whether two lines are perpendicular.

**EXAMPLE 2** Slopes of Perpendicular Lines

**DESIGN** The outline of a company's new logo is shown on a coordinate plane.

**a.** Is \(\angle DFE\) a right angle in the logo?

If \(\overline{BE}\) and \(\overline{AD}\) are perpendicular, then \(\angle DFE\) is a right angle. Find the slopes of \(\overline{BE}\) and \(\overline{AD}\).

- **Slope of \(\overline{BE}\):** \(m = \frac{1 - 3}{7 - 2} = \frac{-2}{5}\)
- **Slope of \(\overline{AD}\):** \(m = \frac{6 - 1}{4 - 2} = \frac{5}{2}\)

The line segments are perpendicular because \(-\frac{2}{5} \times \frac{5}{2} = -1\). Therefore, \(\angle DFE\) is a right angle.

**b.** Is each pair of opposite sides parallel?

If a pair of opposite sides are parallel, then they have the same slope.

- **Slope of \(\overline{AC}\):** \(m = \frac{6 - 1}{2 - 2} = \text{undefined}\)

Since \(\overline{AC}\) and \(\overline{GE}\) are both parallel to the \(y\)-axis, they are vertical and are therefore parallel.

- **Slope of \(\overline{CG}\):** \(m = \frac{6 - 6}{7 - 2} = 0\)

Since \(\overline{CG}\) and \(\overline{AE}\) are both parallel to the \(x\)-axis, they are horizontal and are therefore parallel.

**Check Your Progress**

2. **CONSTRUCTION** On the plans for a treehouse, a beam represented by \(\overline{QR}\) has endpoints \(Q(-6, 2)\) and \(R(-1, 8)\). A connecting beam represented by \(\overline{ST}\) has endpoints \(S(-3, 6)\) and \(T(-8, 5)\). Are the beams perpendicular? Explain.
Lesson 4-4 Parallel and Perpendicular Lines

**EXAMPLE 3** Parallel or Perpendicular Lines

Determine whether the graphs of \( y = 5 \), \( x = 3 \), and \( y = -2x + 1 \) are parallel or perpendicular. Explain.

Graph each line on a coordinate plane.

From the graph, you can see that \( y = 5 \) is parallel to the \( x \)-axis and \( x = 3 \) is parallel to the \( y \)-axis. Therefore, they are perpendicular. None of the lines are parallel.

**Check Your Progress**

3. Determine whether the graphs of \( 6x - 2y = -2 \), \( y = 3x - 4 \), and \( y = 4 \) are parallel or perpendicular. Explain.

You can write the equation of a line perpendicular to a given line if you know a point on the line and the equation of the given line.

**EXAMPLE 4** Parallel Line Through a Given Point

Write an equation in slope-intercept form for the line that passes through \((-4, 6)\) and is perpendicular to the graph of \(2x + 3y = 12\).

**Step 1** Find the slope of the given line by solving the equation for \(y\).

\[
2x + 3y = 12 \quad \text{Original equation} \\
2x - 2x + 3y = -2x + 12 \quad \text{Subtract } 2x \text{ from each side.} \\
3y = -2x + 12 \quad \text{Simplify.} \\
\frac{3y}{3} = \frac{-2x + 12}{3} \quad \text{Divide each side by } 3. \\
y = -\frac{2}{3}x + 4 \quad \text{Simplify.}
\]

The slope is \(-\frac{2}{3}\).

**Step 2** The slope of the perpendicular line is the opposite reciprocal of \(-\frac{2}{3}\) or \(\frac{3}{2}\). Find the equation of the perpendicular line.

\[
y - y_1 = m(x - x_1) \quad \text{Point-slope form} \\
y - 6 = \frac{3}{2}(x - (-4)) \quad (x_1, y_1) = (-4, 6) \text{ and } m = \frac{3}{2} \\
y - 6 = \frac{3}{2}(x + 4) \quad \text{Simplify.} \\
y - 6 = \frac{3}{2}x + 6 \quad \text{Distributive Property} \\
y - 6 + 6 = \frac{3}{2}x + 6 + 6 \quad \text{Add } 6 \text{ to each side.} \\
y = \frac{3}{2}x + 12 \quad \text{Simplify.}
\]

**Check Your Progress**

4. Write an equation in slope-intercept form for the line that passes through \((4, 7)\) and is perpendicular to the graph of \(y = \frac{2}{3}x - 1\).
Check Your Understanding

Example 1  
Write an equation in slope-intercept form for the line that passes through the given point and is parallel to the graph of each equation.

1. \((-1, 2), y = \frac{1}{2}x - 3\)  
2. \((0, 4), y = -4x + 5\)

Example 2  
3. **GARDENS** A garden is in the shape of a quadrilateral with vertices \(A(-2, 1), B(3, -3), C(5, 7),\) and \(D(-3, 4).\) Two paths represented by \(AC\) and \(BD\) cut across the garden. Are the paths perpendicular? Explain.

Example 3  
5. \(y = -2x, 2y = x, 4y = 2x + 4\)  
6. \(y = \frac{1}{2}x, 3y = x, y = -\frac{1}{2}x\)

Example 4  
7. \((-2, 3), y = -\frac{1}{2}x - 4\)  
8. \((-1, 4), y = 3x + 5\)

9. \((2, 3), 2x + 3y = 4\)  
10. \((3, 6), 3x - 4y = -2\)
Lesson 4-4 Parallel and Perpendicular Lines

**Practice and Problem Solving**

**Example 1**  
Write an equation in slope-intercept form for the line that passes through the given point and is parallel to the graph of each equation.

11. \((3, -2), y = x + 4\)  
12. \((4, -3), y = 3x - 5\)  
13. \((0, 2), y = -5x + 8\)  
14. \((-4, 2), y = -\frac{1}{2}x + 6\)  
15. \((-2, 3), y = -\frac{3}{4}x + 4\)  
16. \((9, 12), y = 13x - 4\)

**Example 2**  
17. GEOMETRY A trapezoid is a quadrilateral that has exactly one pair of parallel opposite sides. Determine whether \(ABCD\) is a trapezoid. Explain your reasoning.

18. GEOMETRY The quadrilateral \(CDEF\) is a kite. Determine whether the diagonals of the kite are perpendicular. Explain your reasoning.

19. Determine whether \(y = -6x + 4\) and \(y = \frac{1}{6}x\) are perpendicular. Explain.

20. MAPS On a map, Elmwood Drive passes through \(R(4, -11)\) and \(S(0, -9)\), and Taylor Road passes through \(J(6, -2)\) and \(K(4, -5)\). If they are straight lines, are the two streets perpendicular? Explain.

**Example 3**  
Determine whether the graphs of the following equations are parallel or perpendicular. Explain.

21. \(2x - 8y = -24, 4x + y = -2, x - 4y = 4\)

22. \(3x - 9y = 9, 3y = x + 12, 2x - 6y = 12\)

**Example 4**  
Write an equation in slope-intercept form for the line that passes through the given point and is perpendicular to the graph of the equation.

23. \((-3, -2), y = -2x + 4\)  
24. \((-5, 2), y = \frac{1}{2}x - 3\)  
25. \((-4, 5), y = \frac{1}{3}x + 6\)

26. \((2, 6), y = -\frac{1}{4}x + 3\)  
27. \((3, 8), y = 5x - 3\)  
28. \((4, -2), y = 3x + 5\)

Write an equation in slope-intercept form for a line perpendicular to the graph of the equation that passes through the \(x\)-intercept of that line.

29. \(y = -\frac{1}{2}x - 4\)  
30. \(y = \frac{2}{3}x - 6\)  
31. \(y = 5x + 3\)

32. Write an equation in slope-intercept form for the line that is perpendicular to the graph of \(3x + 2y = 8\) and passes through the \(y\)-intercept of that line.

Determine whether the graphs of each pair of equations are parallel, perpendicular, or neither.

33. \(y = 4x + 3\)  
34. \(y = -2x\)  
35. \(3x + 5y = 10\)

36. \(4x + y = 3\)  
37. \(2x + 5y = 15\)  
38. \(5x - 3y = -6\)

36. \(-3x + 4y = 8\)  
37. \(3x + 5y = 15\)  
38. \(4x + 14y = -42\)
39. Write an equation of the line that is parallel to the graph of \( y = 7x - 3 \) and passes through the origin.

40. EXCAVATION  Scientists excavating a dinosaur mapped the site on a coordinate plane. If one bone lies from \((-5, 8)\) to \((10, -1)\) and a second bone lies from \((-10, -3)\) to \((-5, -6)\), do the bones lie parallel to one another? Explain.

41. ARCHAEOLOGY In the ruins of an ancient civilization, an archaeologist found pottery at \((2, 6)\) and hair accessories at \((4, -1)\). A pole is found with one end at \((7, 10)\) and the other end at \((14, 12)\). Is the pole perpendicular to the line formed by the pottery and the hair accessories? Explain.

42. GRAPHICS To create a design on a computer, Andeana must enter the coordinates for points on the design. One line segment she drew has endpoints of \((-2, 1)\) and \((4, 3)\). The other coordinates that Andeana entered are \((2, -7)\) and \((8, -3)\). Could these points be the vertices of a rectangle? Explain.

43. MULTIPLE REPRESENTATIONS In this problem, you will explore parallel and perpendicular lines.
   a. GRAPHICAL Graph the points \(A(-3, 3), B(3, 5),\) and \(C(-4, 0)\) on a coordinate plane.
   b. ANALYTICAL Determine the coordinates of a fourth point \(D\) that would form a parallelogram. Explain your reasoning.
   c. ANALYTICAL What is the minimum number of points that would be moved in order to make the parallelogram into a rectangle? Describe which points should be moved and explain why.

H.O.T. Problems Use Higher-Order Thinking Skills

44. CHALLENGE If the line through \((-2, 4)\) and \((5, d)\) is parallel to the graph of \(y = 3x + 4\), what is the value of \(d\)?

45. REASONING Is a horizontal line perpendicular to a vertical line sometimes, always, or never? Explain your reasoning.

46. OPEN ENDED Graph a line that is parallel and a line that is perpendicular to \(y = 2x - 1\).

47. FIND THE ERROR Carmen and Chase are finding an equation of the line that is perpendicular to the graph of \(y = \frac{1}{3}x + 2\) and passes through the point \((-3, 5)\). Is either of them correct? Explain your reasoning.

\[
\begin{align*}
\text{Carmen} & : \\
& y - 5 = -3[x - (-3)] \\
& y - 5 = -3(x + 3) \\
& y = -3x - 9 + 5 \\
& y = -3x - 4
\end{align*}
\]

\[
\begin{align*}
\text{Chase} & : \\
& y - 5 = 3[x - (-3)] \\
& y - 5 = 3(x + 3) \\
& y = 3x + 9 + 5 \\
& y = 3x + 14
\end{align*}
\]

48. WRITING IN MATH Illustrate how you can determine whether two lines are parallel or perpendicular. Write an equation whose graph is parallel to the line shown in the graph at the right, and an equation whose graph is perpendicular to the line shown in the graph. Explain your reasoning.
49. Which of the following is an algebraic translation of the following phrase?

5 less than the quotient of a number and 8

A \[ \frac{n}{8} - 5 \]  
B \[ \frac{n}{8} \]  
C \[ 5 - \frac{n}{8} \]  
D \[ \frac{n}{8} - 5 \]

50. A line through which two points would be parallel to a line with a slope of \( \frac{3}{4} \)?

F \((0, 5)\) and \((-4, 2)\)  
H \((0, 0)\) and \((0, -2)\)  
G \((0, 2)\) and \((-4, 1)\)  
J \((0, -2)\) and \((-4, -2)\)

51. Which equation best fits the data in the table?

<table>
<thead>
<tr>
<th>(x)</th>
<th>(y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td>4</td>
<td>11</td>
</tr>
</tbody>
</table>

A \(y = x + 4\)  
B \(y = 2x + 3\)  
C \(y = 7\)  
D \(y = 4x - 5\)

52. SHORT RESPONSE  Tyler is filling his 6000-gallon pool at a constant rate. After 4 hours, the pool contained 800 gal. How many total hours will it take to completely fill the pool?

Spiral Review

Write each equation in standard form. (Lesson 4-3)

53. \(y - 13 = 4(x - 2)\)  
54. \(y - 5 = -2(x + 2)\)  
55. \(y + 3 = -5(x + 1)\)  
56. \(y + 7 = \frac{1}{2}(x + 2)\)  
57. \(y - 1 = \frac{5}{6}(x - 4)\)  
58. \(y - 2 = -\frac{2}{5}(x - 8)\)

59. CANOE RENTAL  Latanya and her friends rented a canoe for 3 hours and paid a total of $45. (Lesson 4-2)

a. Write a linear equation to find the total cost \(C\) of renting the canoe for \(h\) hours.

b. How much would it cost to rent the canoe for 8 hours?

Write an equation of the line that passes through each point with the given slope. (Lesson 4-2)

60. \((5, -2), m = 3\)  
61. \((-5, 4), m = -5\)  
62. \((3, 0), m = -2\)  
63. \((3, 5), m = 2\)  
64. \((-3, -1), m = -3\)  
65. \((-2, 4), m = -5\)

Simplify each expression. If not possible, write simplified. (Lesson 1-4)

66. \(13m + m\)  
67. \(14a^2 + 13b^2 + 27\)  
68. \(3(x + 2x)\)

69. FINANCIAL LITERACY  At a Farmers’ Market, merchants can rent a small table for $5.00 and a large table for $8.50. One time, 25 small and 10 large tables were rented. Another time, 35 small and 12 large were rented. (Lesson 1-2)

a. Write an expression to show the total amount of money collected.

b. Evaluate the expression.

Skills Review

Express each relation as a graph. Then determine the domain and range. (Lesson 1-6)

70. \([(3, 8), (3, 7), (2, -9), (1, -9), (-5, -3)]\)  
71. \([(3, 4), (4, 3), (2, 2), (5, -4), (-4, 5)]\)  
72. \([(0, 2), (-5, 1), (0, 6), (-1, 9), (-4, -5)]\)  
73. \([(7, 6), (3, 4), (4, 5), (-2, 6), (-3, 2)]\)