Radical Equations

Why?
The waterline length of a sailboat is the length of the line made by the water’s edge when the boat is full. A sailboat’s hull speed is the fastest speed that it can travel. You can estimate hull speed \( h \) by using the formula \( h = 1.34\sqrt{l} \), where \( l \) is the length of the sailboat’s waterline.

Radical Equations Equations that contain variables in the radicand, like \( h = 1.34\sqrt{l} \), are called radical equations. To solve, isolate the desired variable on one side of the equation first. Then square each side of the equation to eliminate the radical.

**Example 1** Variable as a Radicand

**SAILING** Idris and Sebastian are sailing in a friend’s sailboat. They measure the hull speed at 9 nautical miles per hour. Find the length of the sailboat’s waterline. Round to the nearest foot.

**Understand** You know how fast the boat will travel and that it relates to the length.

**Plan** The boat travels at 9 nautical miles per hour. The formula for hull speed is \( h = 1.34\sqrt{l} \).

**Solve**

\[
\begin{align*}
h &= 1.34\sqrt{l} & \text{Formula for hull speed} \\
9 &= 1.34\sqrt{l} & \text{Substitute 9 for } h. \\
\frac{9}{1.34} &= \frac{1.34\sqrt{l}}{1.34} & \text{Divide each side by 1.34.} \\
6.71641791 &= \sqrt{l} & \text{Simplify.} \\
(6.71641791)^2 &= (\sqrt{l})^2 & \text{Square each side of the equation.} \\
45.11026954 &\approx l & \text{Simplify.}
\end{align*}
\]

The sailboat’s waterline length is about 45 feet.

**Check** Check the results by substituting your estimate back into the original formula. Remember that your result should be about 9.

\[
\begin{align*}
h &= 1.34\sqrt{l} & \text{Formula for hull speed} \\
9 &\div 1.34\sqrt{45} & h = 9 \text{ and } l = 45 \\
9 &\div 1.34(6.708203932) & \sqrt{45} = 6.708203932 \\
9 &\approx 8.98899327 & \text{Multiply.}
\end{align*}
\]
Lesson 10-4 Radical Equations

1. **DRIVING** The equation \( v = \sqrt{2.5r} \) represents the maximum velocity that a car can travel safely on an unbanked curve when \( v \) is the maximum velocity in miles per hour and \( r \) is the radius of the turn in feet. If a road is designed for a maximum speed of 65 miles per hour, what is the radius of the turn?

When a radicand is an expression, isolate the radical first. Then square both sides of the equation.

**EXAMPLE 2**

**Expression as a Radicand**

Solve \( \sqrt{a + 5} + 7 = 12 \).

\[
\begin{align*}
\sqrt{a + 5} + 7 &= 12 & \text{Original equation} \\
\sqrt{a + 5} &= 5 & \text{Subtract 7 from each side.} \\
(a + 5)^2 &= 5^2 & \text{Square each side.} \\
a + 5 &= 25 & \text{Simplify.} \\
a &= 20 & \text{Subtract 5 from each side.}
\end{align*}
\]

**Check Your Progress** Solve each equation.

2A. \( \sqrt{c - 3} - 2 = 4 \)  
2B. \( 4 + \sqrt{h + 1} = 14 \)

**Extraneous Solutions** Squaring each side of an equation sometimes produces a solution that is not a solution of the original equation. These are called **extraneous solutions**. Therefore, you must check all solutions in the original equation.

**EXAMPLE 3**

**Variable on Each Side**

Solve \( \sqrt{k + 1} = k - 1 \). Check your solution.

\[
\begin{align*}
\sqrt{k + 1} &= k - 1 & \text{Original equation} \\
(k + 1) &= (k - 1)^2 & \text{Square each side.} \\
k + 1 &= k^2 - 2k + 1 & \text{Simplify.} \\
0 &= k^2 - 3k & \text{Subtract } k \text{ and 1 from each side.} \\
0 &= k(k - 3) & \text{Factor.} \\
k &= 0 \text{ or } k - 3 = 0 & \text{Zero Product Property} \\
k &= 0 & \text{Solve.}
\end{align*}
\]

**CHECK** \( \sqrt{k + 1} = k - 1 \)

\[
\begin{align*}
0 + 1 &= 0 - 1 & \text{Original equation} \\
1 &= -1 & \text{False}
\end{align*}
\]

Since 0 does not satisfy the original equation, 3 is the only solution.
Example 1  

1. GEOMETRY The surface area of a basketball is $x$ square inches. What is the radius of the basketball if the formula for the surface area of a sphere is $SA = 4\pi r^2$?

Examples 2 and 3  

Solve each equation. Check your solution.

2. $\sqrt{10h} + 1 = 21$  
3. $\sqrt{7r + 2} + 3 = 7$  
4. $5 + \sqrt{g - 3} = 6$
5. $\sqrt{3x - 5} = x - 5$  
6. $\sqrt{2n + 3} = n$  
7. $\sqrt{a - 2} + 4 = a$

Practice and Problem Solving  

Example 1  

8. EXERCISE Suppose the function $S = \pi \sqrt{\frac{9.8\ell}{7}}$, where $S$ represents speed in meters per second and $\ell$ is the leg length of a person in meters, can approximate the maximum speed that a person can run.

a. What is the maximum running speed of a person with a leg length of 1.1 meters to the nearest tenth of a meter?

b. What is the leg length of a person with a running speed of 2.7 meters per second to the nearest tenth of a meter?

c. As a person’s leg length increases, does their speed increase or decrease? Explain.

Examples 2 and 3  

Solve each equation. Check your solution.

9. $\sqrt{a} + 11 = 21$  
10. $\sqrt{t} - 4 = 7$  
11. $\sqrt{n - 3} = 6$
12. $\sqrt{c + 10} = 4$  
13. $\sqrt{h - 5} = 2\sqrt{3}$  
14. $\sqrt{k + 7} = 3\sqrt{2}$
15. $y = \sqrt{12} - y$  
16. $\sqrt{u + 6} = u$  
17. $\sqrt{r + 3} = r - 3$
18. $\sqrt{1 - 2t} = 1 + t$  
19. $5\sqrt{a - 3} + 4 = 14$  
20. $2\sqrt{x - 11} - 8 = 4$

21. RIDES The amount of time $t$, in seconds, that it takes a simple pendulum to complete a full swing is called the period of the pendulum. It is given by $t = 2\pi \sqrt{\frac{\ell}{32}}$, where $\ell$ is the length of the pendulum, in feet.

a. The Giant Swing completes a period in about 8 seconds. About how long is the pendulum’s arm? Round to the nearest foot.

b. Does increasing the length of the pendulum increase or decrease the period? Explain.

Solve each equation. Check your solution.

22. $\sqrt{6a - 6} = a + 1$  
23. $\sqrt{x^2 + 9x + 15} = x + 5$  
24. $6\sqrt{\frac{5k}{4}} - 3 = 0$
25. $\sqrt{\frac{5y}{6} - 10} = 4$  
26. $\sqrt{2a^2 - 121} = a$  
27. $\sqrt{5x^2 - 9} = 2x$

28. GEOMETRY The formula for the slant height $c$ of a cone is $c = \sqrt{h^2 + r^2}$, where $h$ is the height of the cone and $r$ is the radius of its base. Find the height of the cone if the slant height is 4 and the radius is 2. Round to the nearest tenth.
29. **MULTIPLE REPRESENTATIONS** In this problem, you will solve a radical equation by graphing. Consider the equation \( \sqrt{2x - 7} = x - 7 \).

   a. **GRAPHICAL** Clear the Y= list. Enter the left side of the equation as \( Y1 = \sqrt{2x - 7} \). Enter the right side of the equation as \( Y2 = x - 7 \). Press **GRAPH**.

   b. **GRAPHICAL** Sketch what is shown on the screen.

   c. **ANALYTICAL** Use the **intersect** feature on the **CALC** menu to find the point of intersection.

   d. **ANALYTICAL** Solve the radical equation algebraically. How does your solution compare to the solution from the graph?

30. **PACKAGING** A cylindrical container of chocolate drink mix has a volume of 162 cubic inches. The radius \( r \) of the container can be found by using the formula \( r = \sqrt{\frac{V}{\pi h}} \), where \( V \) is the volume of the container and \( h \) is the height.

   a. If the radius is 2.5 inches, find the height of the container. Round your answer to the nearest hundredth.

   b. If the height of the container is 10 inches, find the radius of the container. Round to the nearest hundredth.

31. **FIND THE ERROR** Jada and Fina solved \( \sqrt{6 - b} = \sqrt{b + 10} \). Is either of them correct? Explain.

   **Jada**
   \[
   \sqrt{6 - b} = \sqrt{b + 10} \\
   (\sqrt{6 - b})^2 = (\sqrt{b + 10})^2 \\
   6 - b = b + 10 \\
   -2b = 4 \\
   b = -2 \\
   \]
   Check \( \sqrt{6 - (-2)} \neq \sqrt{(-2) + 10} \)

   **Fina**
   \[
   \sqrt{6 - b} = \sqrt{b + 10} \\
   (\sqrt{6 - b})^2 = (\sqrt{b + 10})^2 \\
   6 - b = b + 10 \\
   2b = 4 \\
   b = 2 \\
   \]
   Check \( \sqrt{6 - (2)} \neq \sqrt{(2) + 10} \)
   \( \sqrt{4} \neq \sqrt{12} \)
   No solution

32. **REASONING** Which equation has the same solution set as \( \sqrt{4} = \sqrt{x + 2} \)? Explain
   A. \( \sqrt{4} = \sqrt{x} + \sqrt{2} \)    B. \( 4 = x + 2 \)    C. \( 2 - \sqrt{2} = \sqrt{x} \)

33. **REASONING** Explain how solving the equation \( 5 = \sqrt{x} + 1 \) is different from solving the equation \( 5 = \sqrt{x} + 1 \).

34. **OPEN ENDED** Write a radical equation with a variable on each side. Then solve the equation.

35. **REASONING** Is the following equation sometimes, always or never true? Explain.
   \( \sqrt{(x - 2)^2} = x - 2 \)

36. **CHALLENGE** Solve \( \sqrt{x + 9} = \sqrt{3} + \sqrt{x} \).

37. **WRITING IN MATH** Write some general rules about how to solve radical equations. Demonstrate your rules by solving a radical equation.
38. **SHORT RESPONSE** Zack needs to drill a hole at each of the points A, B, C, D, and E on circle P.

If Zack drills holes so that \( \angle APE = 110^\circ \) and the other four angles are equal in measure, what is \( \angle CPD \)?

39. Which expression is undefined when \( w = 3 \)?
   
   A. \( \frac{w - 3}{w + 1} \)  
   B. \( \frac{w^2 - 3w}{3w} \)  
   C. \( \frac{w + 1}{w^2 - 3w} \)  
   D. \( \frac{3w}{3w^2} \)

40. What is the slope of a line that is parallel to the line?

41. What are the solutions of \( \sqrt{x + 3} - 1 = x - 4 \)?
   
   A. 1, 6  
   B. -1, -6  
   C. 1  
   D. 6

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**Spiral Review**

42. **ELECTRICITY** The voltage \( V \) required for a circuit is given by \( V = \sqrt{PR} \), where \( P \) is the power in watts and \( R \) is the resistance in ohms. How many more volts are needed to light a 100-watt light bulb than a 75-watt light bulb if the resistance of both is 110 ohms? (Lesson 10-3)

Simplify each expression. (Lesson 10-2)

43. \( \sqrt{6} \cdot \sqrt{8} \)
44. \( \sqrt{3} \cdot \sqrt{6} \)
45. \( 7\sqrt{3} \cdot 2\sqrt{6} \)
46. \( \sqrt{\frac{27}{a^2}} \)
47. \( \sqrt{\frac{5x^3}{4d^5}} \)
48. \( \frac{\sqrt{9x^3y}}{\sqrt{16x^2y^2}} \)

49. **PHYSICAL SCIENCE** A projectile is shot straight up from ground level. Its height \( h \), in feet, after \( t \) seconds is given by \( h = 96t - 16t^2 \). Find the value(s) of \( t \) when \( h \) is 96 feet. (Lesson 9-5)

Factor each trinomial, if possible. If the trinomial cannot be factored using integers, write prime. (Lesson 8-4)

50. \( 2x^2 + 7x + 5 \)
51. \( 6p^2 + 5p - 6 \)
52. \( 5d^2 + 6d - 8 \)
53. \( 8k^2 - 19k + 9 \)
54. \( 9g^2 - 12g + 4 \)
55. \( 2a^2 - 9a - 18 \)

Determine whether each expression is a monomial. Write yes or no. Explain. (Lesson 7-1)

56. 12  
57. \( 4x^3 \)  
58. \( a - 2b \)  
59. \( 4n + 5p \)  
60. \( \frac{x}{y^2} \)  
61. \( \frac{1}{5}abc^{14} \)

**Skills Review**

Simplify. (Lesson 1-1)

62. \( 9^2 \)
63. \( 10^6 \)
64. \( 4^5 \)
65. \( (8v)^2 \)
66. \( \left(\frac{w^3}{9}\right)^2 \)
67. \( (10y^2)^3 \)

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