Lesson 10-2

Example 1 Two Roots

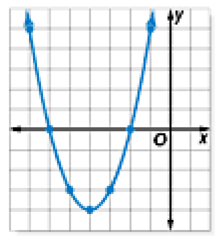
Solve $x^2 + 12 = -8x$ by graphing.

First rewrite the equation so one side is equal to zero.

$x^2 + 12 = -8x$	Original equation
$x^2 + 12 + 8x = -8x + 8x$	Add $8x$ to each side.
$x^2 + 8x + 12 = 0$	Simplify.

Graph the related function $f(x) = x^2 + 8x + 12$. The equation of the axis of symmetry is $x = -\frac{8}{2(1)}$ or $x = -4$. When $x = -4$, $f(x)$ equals $(-4)^2 + 8(-4) + 12$ or –4. So, the coordinates of the vertex are (-4, -4). Make a table of values to find other points to sketch the graph.

x	*f*(*x*)
-7	5
-6	0
-5	-3
-4	-4
-3	-3
-2	0
-1	5



To solve $x^2 + 8x + 12 = 0$, you need to know where the value of $f(x)$ is 0. This occurs at the x-intercepts. The x-intercepts of the parabola appear to be –6 and –2.

CHECK: Solve by factoring.

$x^2 + 8x + 12 = 0$	Original equation
$(x + 6)(x + 2) = 0$	Factor.
$x + 6 = 0$ or $x + 2 = 0$	Zero Product Property
$x = -6$ $\quad$ $x = -2$	Solve for x.

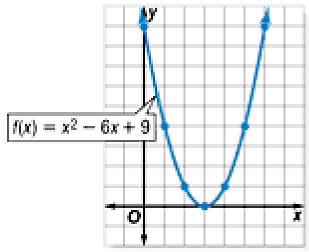
The solutions of the equation are -6 and -2.

Example 2 A Double Root

Solve $x^2 - 6x + 9 = 0$ by graphing.

Graph the related function $f(x) = x^2 - 6x + 9$.

x	$f(x)$
0	9
1	4
2	1
3	0
4	1
5	4
6	9



Notice that the vertex of the parabola is the x-intercept. Thus, one solution is 3. What is the other solution?

Try solving the equation by factoring.

$x^2 - 6x + 9 = 0$	Original equation
$(x - 3)(x - 3) = 0$	Factor.
$x - 3 = 0$ or $x - 3 = 0$	Zero Product Property
$x = 3$ $\quad$ $x = 3$	Solve for x.

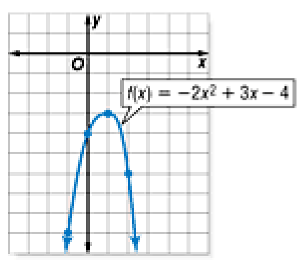
There are two identical factors for the quadratic function, so there is only one root, called a double root. The solution is 3.

Example 3 No Real Roots

Solve $-2x^2 + 3x - 4 = 0$ by graphing.

Graph the related function $f(x) = -2x^2 + 3x - 4$.

x	$f(x)$
-1	-9
0	-4
1	-3
2	-6



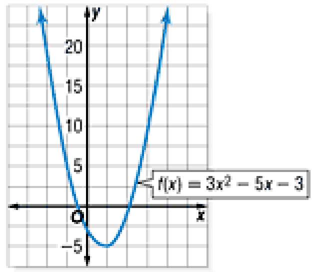
The graph has no x-intercept. Thus, there are no real number solutions for this equation. The symbol Ø, indicating an empty set, is often used to represent no real solutions.

Example 4 Rational Roots

Solve $3x^2 - 5x - 3 = 0$ by graphing. If integral roots cannot be found, estimate the roots by stating the consecutive integers between which the roots lie.

Graph the relation function $f(x) = 3x^2 - 5x - 3$.

x	$f(x)$
-2	19
-1	5
0	-3
1	-5
2	-1
3	9
4	25



Notice that the value of the function changes from positive to negative between the x values of –1 and 0 and between 2 and 3.

The x-intercepts of the graph are between –1 and 0 and between 2 and 3. So, one root is between –1 and 0, and the other root is between 2 and 3.

Example 5 Estimate Solutions to Solve a Problem

Alex will catch a ball that is thrown to him from a tree house. If the ball is thrown from 15 feet above the ground with an initial velocity of 24 feet per second, the function $y = -16t^2 + 24t + 10$ represents the height of the ball y in feet above Alex's hand after t seconds. (Alex will catch the ball when it is 5 feet above ground.) When will Alex catch the ball?

You need to find the solution of the equation $0 = -16t^2 + 24t + 10$. Use a graphing calculator to graph the related function $y = -16t^2 + 24t + 10$. The x-intercept is about 1.84. Therefore, the ball will reach Alex's hand in 1.84 seconds.