

## Lesson 10-4

### Example 1 Integral Roots

Use two methods to solve  $x^2 + 12x = -20$ .

First, rewrite the equation so one side is equal to zero.

$$\begin{aligned}x^2 + 12x &= -20 && \text{Original equation} \\x^2 + 12x + 20 &= -20 + 20 && \text{Add 20 to each side.} \\x^2 + 12x + 20 &= 0 && \text{Simplify.}\end{aligned}$$

### Method 1 Quadratic Formula

For this equation,  $a = 1$ ,  $b = 12$ , and  $c = 20$ .

$$\begin{aligned}x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} && \text{Quadratic Formula} \\&= \frac{-12 \pm \sqrt{12^2 - 4(1)(20)}}{2(1)} && a = 1, b = 12, \text{ and } c = 20 \\&= \frac{-12 \pm \sqrt{144 - 80}}{2} && \text{Multiply.} \\&= \frac{-12 \pm \sqrt{64}}{2} && \text{subtract.} \\&= \frac{-12 \pm 8}{2} && \text{Take the square root of 64.} \\x &= \frac{-12 - 8}{2} \quad \text{or} \quad x = \frac{-12 + 8}{2} \\&= -10 \qquad \qquad \qquad = -2\end{aligned}$$

### Method 2 Factoring

$$\begin{aligned}x^2 + 12x + 20 &= 0 && \text{Original equation} \\(x + 10)(x + 2) &= 0 && \text{Factor } x^2 + 12x + 20. \\x + 10 = 0 \quad \text{or} \quad x + 2 &= 0 && \text{Zero Product Property} \\&= -10 \qquad \qquad \qquad = -2 && \text{Solve.}\end{aligned}$$

The solution set is  $\{-10, -2\}$ .

### Example 2 Irrational Roots

Solve  $-3x^2 + 5x + 9 = 0$  by using the Quadratic Formula. Round to the nearest tenth if necessary.

For this equation,  $a = -3$ ,  $b = 5$ , and  $c = 9$ .

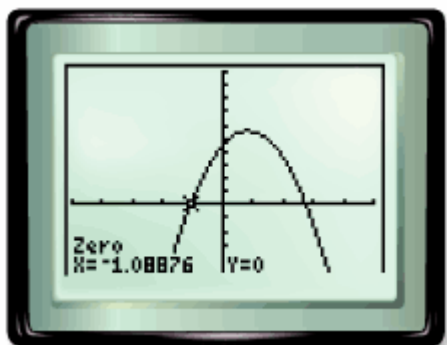
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{Quadratic Formula}$$
$$= \frac{-5 \pm \sqrt{5^2 - 4(-3)(9)}}{2(-3)} \quad a = -3, b = 5, \text{ and } c = 9$$

$$= \frac{-5 \pm \sqrt{25 + 108}}{-6} \quad \text{Multiply.}$$

$$= \frac{-5 \pm \sqrt{133}}{-6} \quad \text{Add.}$$

$$x = \frac{-5 - \sqrt{133}}{-6} \quad \text{or} \quad x = \frac{-5 + \sqrt{133}}{-6}$$
$$\approx 2.8 \quad \approx -1.1$$

Check the solutions by using the CALC menu on a graphing calculator to determine the zeros of the related quadratic function.



The approximate solution set is  $\{-1.1, 2.8\}$ .

**Example 3 Use the Quadratic Formula to Solve a Problem**  
 A box is to be formed by cutting  $x$  inch by  $x$  inch squares from each corner of a piece of cardboard and then folding the sides. If the dimensions of the piece of cardboard are 9 inches by 11 inches and the area of the base of the box will be approximately 49 square inches, what will be the value of  $x$ ?



The area of the base can be found using the formula  $A = lw$ , where the length in this case is  $11 - 2x$  and the width is  $9 - 2x$ .

$$A = lw \text{ Formula for area of the base}$$

$$49 = (11 - 2x)(9 - 2x) \quad A = 49, l = 11 - 2x, w = 9 - 2x$$

$$49 = 99 - 40x + 4x^2 \quad \text{Multiply.}$$

$$0 = 4x^2 - 40x + 50 \quad \text{Set equal to 0.}$$

This equation cannot be factored, and completing the square would involve a lot of computation. To find accurate solutions, use the Quadratic Formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{Quadratic Formula}$$

$$= \frac{-(-40) \pm \sqrt{(-40)^2 - 4(4)(50)}}{2(4)} \quad a = 4, b = -40, c = 50$$

$$= \frac{40 \pm \sqrt{800}}{8}$$

$$x \approx 8.5 \text{ or } x \approx 1.5$$

Since 8.5 would result in a length of  $11 - 2(8.5)$  or  $-6$  inches and a width of  $-8$  inches, use the value of 1.5 inches. For the box to have a base area of approximately 49 square inches, you will cut squares out of the corner of the cardboard that measure 1.5 inches by 1.5 inches.

**Example 4 Use the Discriminant**

**State the value of the discriminant for each equation. Then determine the number of real roots of the equation.**

a.  $4x^2 + 1 = -4x$

**Step 1** Rewrite the equation in standard form.

$$4x^2 + 1 = -4x \quad \text{Original equation}$$

$$4x^2 + 1 + 4x = -4x + 4x \quad \text{Add } 4x \text{ to each side.}$$

$$4x^2 + 4x + 1 = 0 \quad \text{Simplify.}$$

**Step 2** Find the discriminant.

$$b^2 - 4ac = 4^2 - 4(4)(1) \quad a = 4, b = 4, \text{ and } c = 1$$

$$= 0 \quad \text{Simplify.}$$

Since the discriminant is zero, the equation has 1 real root.

**b.  $2x^2 - 7x + 3 = 0$**

$$b^2 - 4ac = (-7)^2 - 4(2)(3) \\ = 25$$

$a = 2, b = -7, \text{ and } c = 3$   
Simplify.

Since the discriminant is positive, the equation has two real roots.