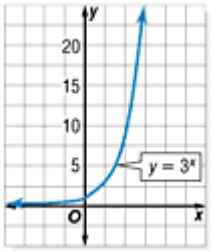
Lesson 10-5

Example 1 Graph an Exponential Function with $a > 1$

a. Graph $y = 3^x$. State the y-intercept.

x	3^x	y
-2	3^{-2}	$\frac{1}{9}$
-1	3^{-1}	$\frac{1}{3}$
0	3^0	1
1	3^1	3
2	3^2	9
3	3^3	27



Graph the ordered pairs and connect the points with a smooth curve. The y-intercept is 1. *Notice that the y-values change little for small values of x, but they increase quickly as the values of x becomes greater.*

b. Use the graph to determine the approximate value of $3^{-1.5}$.

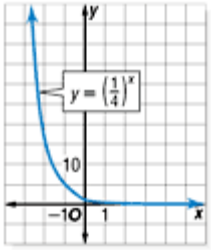
The graph represents all real values of x and their corresponding values of y for $y = 3^x$. So, the value of y is about 0.2 when $x = -1.5$. Use a calculator to confirm this value.

$3^{-1.5} \approx 0.19245009$.

Example 2 Graph Exponential Functions with $0 < a < 1$

a. Graph $y = \left(\frac{1}{4}\right)^x$. State the y-intercept.

x	$\left(\frac{1}{4}\right)^x$	y
-3	$\left(\frac{1}{4}\right)^{-3}$	64
-2	$\left(\frac{1}{4}\right)^{-2}$	16
-1	$\left(\frac{1}{4}\right)^{-1}$	4
0	$\left(\frac{1}{4}\right)^{0}$	1
1	$\left(\frac{1}{4}\right)^{1}$	$\frac{1}{4}$
2	$\left(\frac{1}{4}\right)^{2}$	$\frac{1}{16}$



Graph the ordered pairs and connect the points with a smooth curve. The y-intercept is 1.

b. Use the graph to determine the approximate value of $\left(\frac{1}{4}\right)^{1.5}$.

The value of y is about 0.1 when $x = 1.5$. Use a calculator to confirm this value.

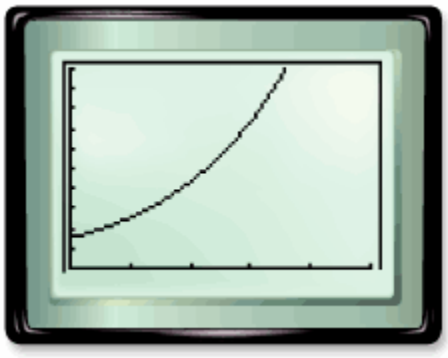
$\left(\frac{1}{4}\right)^{1.5} = \frac{1}{8}$.

Example 3 Use Exponential Functions to Solve Problems

Connor has 4 weeks before his math final exam. He plans to study for 3 hours the first week and increase the time he will study $S(x)$ in hours according to the function $S(x) = 3(1.7)^x$, where x represents the number of weeks of studying.

a. Graph the function. What values of $S(x)$ and x are meaningful in the context of the problem?

Use a graphing calculator to graph the function. Only values where $S(x) > 0$ and $x > 0$ are meaningful in the context of the problem



b. How many hours did he study the second week?

$S(x) = 3(1.7)^x$	Original equation
$S(2) = 3(1.7)^2$	$x = 2$
$S(2) = 8.67$	Use a calculator.

He studied 8.67 hours during the second week.

c. Connor has scheduled 20 hours to study during the fourth week. According to the function, has he scheduled enough time?

$S(x) = 3(1.7)^x$	Original equation
$S(4) = 3(1.7)^4$	$x = 4$
$S(4) = 25.0563$	Use a calculator.

According to the function, he should schedule 25.0563 hours. He has not scheduled enough time.

Example 4 Identify Exponential Behavior

Determine whether each set of data displays exponential behavior.

a.

x	0	1	2	3	4	5
y	50	46	42	38	34	30

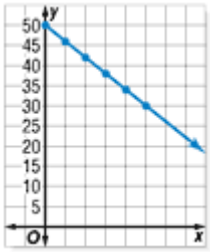
Method 1 Look for a Pattern

The domain values are at regular intervals of 1. The range values have a common difference of –4.

50	46	42	38	34	30
	-4	-4	-4	-4	-4

The data do not display exponential behavior. They display linear behavior.

Method 2 Graph the Data



This is a graph of a line, not an exponential function.

b.

x	0	10	20	30	40	50
y	1	5	25	125	625	3125

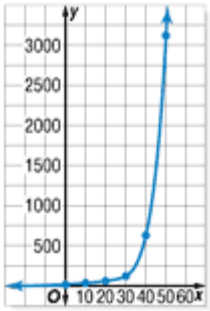
Method 1 Look for a Pattern

The domain values are at regular intervals of 10. The range values have a common factor of 5.

1	5	25	125	625	3125
	×5	×5	×5	×5	×5

Since the domain values are at regular intervals and the range values have a common factor, the data are probably exponential. The equation for the data may involve 5^x.

Method 2 Graph the Data



The graph shows a rapidly increasing value of *y* as *x* increases. This is a characteristic of exponential behavior.