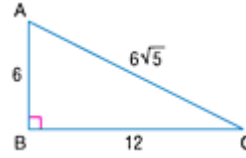


Lesson 11-7

Example 1 Sine, Cosine, and Tangent

Find the sine, cosine, and tangent of each acute angle of $\triangle ABC$. Round to the nearest ten thousandth.



Write each ratio and substitute the measures. Use a calculator to find each value.

$$\sin C = \frac{\textit{opposite leg}}{\textit{hypotenuse}}$$

$$= \frac{6}{6\sqrt{5}} \text{ or } 0.4472$$

$$\sin A = \frac{\textit{opposite leg}}{\textit{hypotenuse}}$$

$$= \frac{12}{6\sqrt{5}} \text{ or } 0.8944$$

$$\cos C = \frac{\textit{adjacent leg}}{\textit{hypotenuse}}$$

$$= \frac{12}{6\sqrt{5}} \text{ or } 0.8944$$

$$\cos A = \frac{\textit{adjacent leg}}{\textit{hypotenuse}}$$

$$= \frac{6}{6\sqrt{5}} \text{ or } 0.4472$$

$$\tan C = \frac{\textit{opposite leg}}{\textit{adjacent leg}}$$

$$= \frac{6}{12} \text{ or } 0.5$$

$$\tan A = \frac{\textit{opposite leg}}{\textit{adjacent leg}}$$

$$= \frac{12}{6} \text{ or } 2$$

Example 2 Find the Sine of an Angle

Find $\sin 75^\circ$ to the nearest ten thousandth.

Use a calculator. The calculator must be in degree mode.

Keystrokes: $75 \subseteq 0.9659258263$

Round to the nearest ten thousandth, $\sin 75^\circ \approx 0.9659$.

Example 3 Find the Measure of an Angle

Find the measure of $\angle F$ to the nearest degree.

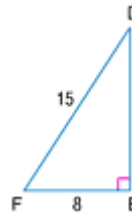
Since the lengths of the adjacent side and the hypotenuse are known, use the cosine ratio.

$$\cos F = \frac{\textit{adjacent leg}}{\textit{hypotenuse}}$$

$$= \frac{8}{15}$$

Definition of cosine

$FE = 8$ and $DF = 15$



Now use a calculator to find the measure of the angle whose cosine ratio is $\frac{8}{15}$.

Keystrokes: ψ [COS⁻¹] $8 \div 15 \subseteq 57.76904736$
To the nearest degree, the measure of $\angle F$ is 58° .

Example 4 Solve a Triangle**Find all of the missing measures in $\triangle LMN$.**You need to find the measures of $\angle N$, \overline{LM} , and \overline{LN} .

Step 1 Find the measure of $\angle N$. The sum of the measures of the angles in a triangle is 180.
 $180^\circ - 90^\circ - 35^\circ = 55^\circ$
The measure of $\angle N$ is 55° .

Step 2 Find the value of r , which is the measure of the side opposite $\angle N$.
Use the tangent ratio.

$$\tan 35^\circ = \frac{21}{r} \quad \text{Definition of tangent}$$

$$0.7002 \approx \frac{21}{r} \quad \text{Evaluate } \tan 35^\circ.$$

$$0.7002r \approx 21 \quad \text{Multiply each side by } r.$$

$$r \approx 29.99 \quad \text{Divide each side } 0.7002.$$

\overline{LM} is about 29.99 feet long.

Step 3 Find the value of s , which is the measure of the hypotenuse of $\triangle LMN$.
Use the sine ratio.

$$\sin 35^\circ = \frac{21}{s} \quad \text{Definition of sine.}$$

$$0.5736 \approx \frac{21}{s} \quad \text{Evaluate } \sin 35^\circ.$$

$$0.5736s \approx 21 \quad \text{Multiply each side by } s.$$

$$s \approx 36.61 \quad \text{Divide each side by } 0.5736.$$

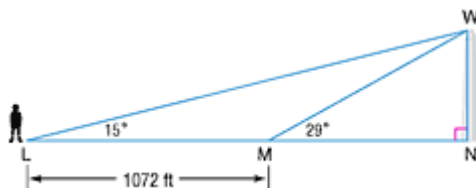
\overline{LN} is about 36.61 feet long.

So the missing measures are 55° , 29.99 ft., and 36.61 ft.

Example 5 Angle of Elevation

Tia wants to find the height of the Washington Monument. At point L, Tia measured the angle of elevation to point W to be 15° . At another point M, which was 1072 feet closer to the bottom of the Washington Monument, Tia measured the angle of elevation to point W to be 29° . Determine the height of the Washington Monument.

Explore Draw a diagram to model the situation. Two right triangles, $\triangle LWN$ and $\triangle MWN$, are formed. You know the angle of elevation for each triangle. To determine the height of the Washington Monument, find the length \overline{WN} , which is shared by both triangles.



Plan Let y represent the distance from the top of the Monument W to its base N . Let x represent MN in the first triangle and let $x + 1072$ represent LN .

Solve Write two equations involving the tangent ratio.

$$\begin{array}{l} \tan 29^\circ = \frac{y}{x} \qquad \text{and} \qquad \tan 15^\circ = \frac{y}{1072 + x} \\ x \tan 29^\circ = y \qquad \qquad \qquad (1072 + x) \tan 15^\circ = y \end{array}$$

Since both expressions are equal to y , use substitution to solve for x .

$$\begin{array}{l} x \tan 29^\circ = (1072 + x) \tan 15^\circ \\ x \tan 29^\circ = 1072 \tan 15^\circ + x \tan 15^\circ \\ x \tan 29^\circ - x \tan 15^\circ = 1072 \tan 15^\circ \\ x (\tan 29^\circ - \tan 15^\circ) = 1072 \tan 15^\circ \quad \text{Isolate } x. \\ x = \frac{1072 \tan 15^\circ}{\tan 29^\circ - \tan 15^\circ} \quad \text{Divide.} \\ x \approx 1003 \text{ feet} \quad \text{Use a calculator.} \end{array}$$

Use this value for x and the equation $x \tan 29^\circ = y$ to solve for y .

$$\begin{array}{l} x \tan 29^\circ = y \quad \text{Original equation} \\ 1003 \tan 29^\circ \approx y \quad \text{Replace } x \text{ with } 1003. \\ 556 \approx y \quad \text{Use a calculator.} \end{array}$$

The height of the Washington Monument is about 556 feet.

Examine Examine the solution by finding the angles of elevation.

$$\tan a = \frac{y}{x} \qquad \text{and} \qquad \tan b = \frac{y}{1072 + x}$$

$$\tan a = \frac{556}{1003}$$

$$\tan b = \frac{556}{1072 + 1003}$$

$$a = 29^\circ$$

$$b = 15^\circ$$

The solution checks.