Lesson 12-7

Example 1 LCM of Monomials

Find the LCM of $21r^3st$ and $12r^2s^2$.

Find the prime factors of each coefficient and variable expression.

$21r^3st = 3 \cdot 7 \cdot r \cdot r \cdot r \cdot s \cdot t$

$12r^2s^2 = 2 \cdot 2 \cdot 3 \cdot r \cdot r \cdot s \cdot s$

Use each prime factor the greatest number of times it appears in any of the factorizations.

$21r^3st = 3 \cdot 7 \cdot r \cdot r \cdot r \cdot s \cdot t$

$12r^2s^2 = 2 \cdot 2 \cdot 3 \cdot r \cdot r \cdot s \cdot s$

LCM = $2 \cdot 2 \cdot 3 \cdot 7 \cdot r \cdot r \cdot r \cdot s \cdot s \cdot t$ or $84r^3s^2t$

Example 2 LCM of Polynomials

Find the LCM of $2x^2 - x - 3$ and $x^2 - 3x - 4$.

Express each polynomial in factored form.

$2x^2 - x - 3 = (2x - 3)(x + 1)$

$x^2 - 3x - 4 = (x + 1)(x - 4)$

Use each factor the greatest number of times it appears.

LCM = $(2x - 3)(x + 1)(x - 4)$

Example 3 Monomial Denominators

Find $\frac{4}{b^2} + \frac{3}{2b}$.

Factor each denominator.

$b^2 = b \cdot b$

$2b = 2 \cdot b$

LCD = $2b^2$

$$\frac{4}{b^2} + \frac{3}{2b} = \frac{2(4)}{2(b^2)} + \frac{b(3)}{b(2b)}$$ Multiply $\frac{4}{b^2}$ by $\frac{2}{2}$ and $\frac{3}{2b}$ by $\frac{b}{b}$.

$$= \frac{8}{2b^2} + \frac{3b}{2b^2}$$ Multiply.

$$= \frac{8 + 3b}{2b^2}$$ Add the numerators.

Example 4 Polynomial Denominators

Find $\frac{4}{x^2-2x-3}+\frac{x}{x^2-9}$**.**

$$\frac{4}{x^2-2x-3}+\frac{x}{x^2-9}$$

$$=\frac{4}{(x-3)(x+1)}+\frac{x}{(x-3)(x+3)}$$ Factor denominators.

$$=\frac{4}{(x-3)(x+1)}\cdot\frac{x+3}{x+3}+\frac{x}{(x-3)(x+3)}\cdot\frac{x+1}{x+1}$$ The LCD is $(x-3)(x+3)(x+1)$.

$$=\frac{4x+12}{(x-3)(x+3)(x+1)}+\frac{x^2+x}{(x-3)(x+3)(x+1)}$$ $4(x+3)=4x+12$, $x(x+1)=x^2+x$

$$=\frac{4x+12+x^2+x}{(x-3)(x+3)(x+1)}$$ Add numerators.

$$=\frac{x^2+5x+12}{(x-3)(x+3)(x+1)}$$ Simplify.

Example 5 Binomials in Denominators

Find $\frac{1}{2r+14}-\frac{2}{r-3}$**.**

$$\frac{1}{2r+14}-\frac{2}{r-3}=\frac{1}{2(r+7)}-\frac{2}{r-3}$$ Factor.

$$=\frac{1(r-3)}{2(r+7)(r-3)}-\frac{2(2)(r+7)}{(r-3)(2)(r+7)}$$ The LCD is $2(r+7)(r-3)$.

$$=\frac{1(r-3)-2(2)(r+7)}{2(r+7)(r-3)}$$ Subtract the numerators.

$$=\frac{r-3-4(r+7)}{2(r+7)(r-3)}$$ Multiply.

$$=\frac{r-3-4r-28}{2(r+7)(r-3)}$$ Distributive Property

$$=\frac{-3r-31}{2(r+7)(r-3)} \text{ or } -\frac{3r+31}{2(r+7)(r-3)}$$ Simplify.

Example 6 Polynomials in Denominators

Multiple-Choice Test Item

Find $\frac{n-1}{n+4} - \frac{3}{4n^2-64}$ **.**

A. $\frac{4n^2-20n+16}{4(n-4)(n+4)}$

B. $\frac{4n^2-20n+13}{4(n+4)^2}$

C. $\frac{4n^2-20n+13}{4(n-4)(n+4)}$

D. $\frac{4n^2-20n+16}{4(n-4)^2}$

Read the Test Item

The expression $\frac{n-1}{n+4} - \frac{3}{4n^2-64}$ represents the difference of two rational expressions with unlike denominators.

Solve the Test Item

Step 1 Factor the denominator of the second expression.

$$\frac{n-1}{n+4} - \frac{3}{4n^2-64} = \frac{n-1}{n+4} - \frac{3}{4(n-4)(n+4)}$$

The LCD is 4(n – 4) (n + 4).

Step 2 Change each rational expression into an equivalent expression with the LCD.

$$\frac{n-1}{n+4} - \frac{3}{4(n-4)(n+4)} = \frac{n-1}{n+4} \bullet \frac{4(n-4)}{4(n-4)} - \frac{3}{4(n-4)(n+4)}$$

$$= \frac{(n-1)(4)(n-4)}{4(n-4)(n+4)} - \frac{3}{4(n-4)(n+4)}$$

$$= \frac{4n^2-20n+16}{4(n-4)(n+4)} - \frac{3}{4(n-4)(n+4)}$$

Step 3 Subtract the numerators.

$$\frac{4n^2-20n+16}{4(n-4)(n+4)} - \frac{3}{4(n-4)(n+4)} = \frac{4n^2-20n+16-3}{4(n-4)(n+4)}$$

$$= \frac{4n^2-20n+13}{4(n-4)(n+4)}$$

The answer is C.