Lesson 12-9

Example 1 Use Cross Products

Solve $\frac{m+1}{m} = \frac{m+1}{m-6}$**.**

$\frac{m+1}{m} = \frac{m+1}{m-6}$	Original equation
$(m + 1)(m - 6) = m(m + 1)$	Cross multiply.
$m^2 - 5m - 6 = m^2 + m$	Distributive Property
$-6 = 6m$	Add $5m$ and $-m^2$ to each side.
$-1 = m$	Divide each side by 6.

Example 2 Use the LCD

Solve $\frac{1}{2} + \frac{a-2}{a} = \frac{a+2}{2a}$**.**

$\frac{1}{2} + \frac{a-2}{a} = \frac{a+2}{2a}$	Original equation
$(2a)(\frac{1}{2} + \frac{a-2}{a}) = (2a)(\frac{a+2}{2a})$	The LCD is $2a$.
$(\frac{2a}{1} \cdot \frac{1}{2}) + (\frac{2a}{1} \cdot \frac{a-2}{a}) = (\frac{2a}{1})(\frac{a+2}{2a})$	Distributive Property
$a + 2(a - 2) = a + 2$	Simplify.
$a + 2a - 4 = a + 2$	Multiply.
$3a - 4 = a + 2$	Combine like terms.
$2a = 6$	Add 4 and $-a$ to each side.
$a = 3$	Divide each side by 2.

Example 3 Multiple Solutions

Solve $\frac{1}{r+1} - \frac{1}{2r} = \frac{3}{40}$.

$\frac{1}{r+1} - \frac{1}{2r} = \frac{3}{40}$	Original equation
$40r(r+1)\left(\frac{1}{r+1} - \frac{1}{2r}\right) = 40r(r+1)\left(\frac{3}{40}\right)$	The LCD is $40r(r + 1)$.
$\left(\frac{40r(r+1)}{1} \cdot \frac{1}{r+1}\right) - \left(\frac{40r(r+1)}{1} \cdot \frac{1}{2r}\right) = \left(\frac{40r(r+1)}{1}\right)\left(\frac{3}{40}\right)$	Distributive Property
$40r - 20(r+1) = 3r(r+1)$	Simplify.
$40r - 20r - 20 = 3r^2 + 3r$	Distributive Property
$20r - 20 = 3r^2 + 3r$	Combine like terms.
$0 = 3r^2 - 17r + 20$	Set equal to 0.
$0 = (3r - 5)(r - 4)$	Factor.
$3r - 5 = 0$ or $r - 4 = 0$	
$3r = 5$ or $r = 4$	
$r = \frac{5}{3}$	

CHECK:

$r = \frac{5}{3}$:

$$\frac{1}{r+1} - \frac{1}{2r} = \frac{3}{40}$$

$$\frac{1}{\frac{5}{3}+1} - \frac{1}{2\left(\frac{5}{3}\right)} \stackrel{?}{=} \frac{3}{40}$$

$$\frac{1}{\frac{8}{3}} - \frac{1}{\frac{10}{3}} \stackrel{?}{=} \frac{3}{40}$$

$$\frac{3}{8} - \frac{3}{10} \stackrel{?}{=} \frac{3}{40}$$

$$\frac{3}{40} = \frac{3}{40}$$

$r = 4$:

$$\frac{1}{r+1} - \frac{1}{2r} = \frac{3}{40}$$

$$\frac{1}{4+1} - \frac{1}{2(4)} \stackrel{?}{=} \frac{3}{40}$$

$$\frac{1}{5} - \frac{1}{8} \stackrel{?}{=} \frac{3}{40}$$

$$\frac{3}{40} = \frac{3}{40}$$

The solutions are $\frac{5}{3}$ and 4.

Example 4 Work Problem

Jane paints 1 house in 3 days. When Jane and Becca paint a house together, they can paint the same house in 2 days. How long would it take Becca to paint the house herself?

Explore Since it takes Jane 3 days to paint the house, she can paint $\frac{1}{3}$ of the house in one day. The amount of the house Becca can do in one day can be represented by $\frac{1}{t}$. To determine how long it takes Becca to paint the house, use the formula Jane's work + Becca's work = 1 completed house.

Plan The time that both of them worked was 2 days. Each rate multiplied by this time results in the amount of work done by each person.

Solve Jane's work + Becca's work = Total work

$$\frac{1}{3}(2) \quad + \quad \frac{1}{t}(2) \quad = 1$$

$$\frac{2}{3} + \frac{2}{t} = 1$$ Multiply.

$$3t\left(\frac{2}{3} + \frac{2}{t}\right) = 3t \cdot 1$$ The LCD is $3t$.

$$3t\left(\frac{2}{3}\right) + 3t\left(\frac{2}{t}\right) = 3t$$ Distributive Property

$$2t + 6 = 3t$$ Simplify.

$$6 = t$$ Add $-2t$ to each side.

Examine The time that it would take Becca to paint the house by herself is 6 days. This seems reasonable because the combined efforts of the two took longer than half of Jane's usual time.

Example 5 Rate Problem

Eric and Ryan live 135 miles apart. It takes Eric 2 hours and 15 minutes to drive to Ryan's house. It takes Ryan 2 hours and 30 minutes to drive to Eric's house. If they leave at the same time traveling to each other's house, when will they meet?

Determine the rates of each.

Eric: $\frac{135\,mi}{2.25\,hr}$ 2 hours and 15 minutes = 2.25 hours

Ryan: $\frac{135\,mi}{2.5\,hr}$ 2 hours and 30 minutes = 2.5 hours

Next, since both are leaving at the same time, the time both have traveled when they meet will be the same. And since they started at opposite ends of the route, the sum of their distances is equal to the total route, 135 miles.

	r	***t***	***d***
Eric	$\frac{135 \text{ mi}}{2.25 \text{ hr}}$	*t* hours	$\frac{135t}{2.25}$ mi
Ryan	$\frac{135 \text{ mi}}{2.5 \text{ hr}}$	*t* hours	$\frac{135t}{2.5}$ mi

$$\frac{135t}{2.25} + \frac{135t}{2.5} = 135$$ The sum of the distance is 135.

$$45\left(\frac{135t}{2.25} + \frac{135t}{2.5}\right) = 135 \cdot 45$$ The LCD is 45.

$$\frac{45}{1} \cdot \frac{135t}{2.25} + \frac{45}{1} \cdot \frac{135t}{2.5} = 6075$$ Distributive Property

$$2700t + 2430t = 6075$$ Simplify.

$$5130t = 6075$$ Add.

$$t \approx 1.18$$ Divide each side by 5130.

Eric and Ryan will pass at about 1.18 hours or about 1 hour 11 minutes after leaving home.

Example 6 No Solution

Solve $1 + \frac{3t}{t-2} = \frac{6}{t-2}$**.**

$1 + \frac{3t}{t-2} = \frac{6}{t-2}$	Original equation
$(t-2)(1 + \frac{3t}{t-2}) = (t-2)(\frac{6}{t-2})$	The LCD is $t - 2$.
$(t-2)(1) + (t-2)(\frac{3t}{t-2}) = (t-2)(\frac{6}{t-2})$	Distributive Property
$t - 2 + 3t = 6$	Simplify.
$4t - 2 = 6$	Add like terms
$4t = 8$	Add 2 to each side.
$t = 2$	Divide each side by 4.

Since 2 is an excluded value for t, the number 2 is an extraneous solution. Thus, the equation has no solution.

Example 7 Extraneous Solution

Solve $p - \frac{2p}{p+3} = \frac{6}{p+3}$**.**

$p - \frac{2p}{p+3} = \frac{6}{p+3}$	Original equation
$(p+3)(p - \frac{2p}{p+3}) = (p+3)(\frac{6}{p+3})$	The LCD is $p + 3$.
$(p+3)(p) - (p+3)(\frac{2p}{p+3}) = (p+3)(\frac{6}{p+3})$	Distributive Property
$p^2 + 3p - 2p = 6$	Multiply.
$p^2 + p - 6 = 0$	Set equal to zero.
$(p+3)(p-2) = 0$	Factor.

$p + 3 = 0$ or $p - 2 = 0$

$p = -3$ $\quad p = 2$

The number –3 is an extraneous solution, since –3 is an excluded value for p. Thus, 2 is the solution of the equation.