Lesson 10-3

Example 1 Irrational Roots

Solve $x^2 + 12x + 36 = 11$ by taking the square root of each side. Round to the nearest tenth if necessary.

$x^2 + 12x + 36 = 11$	Original equation
$(x + 6)^2 = 11$	$x^2 + 12x + 36$ is a perfect square trinomial
$\sqrt{(x+6)^2} = \sqrt{11}$	Take the square root of each side.
$\lvert x+6 \rvert = \sqrt{11}$	Simplify.
$x + 6 = \pm\sqrt{11}$	Definition of absolute value
$x + 6 - 6 = \pm\sqrt{11} - 6$	Subtract 6 from each side.
$x = -6 \pm \sqrt{11}$	Simplify.

Use a calculator to evaluate each value of x.

$x = -6 + \sqrt{11}$ $\quad\quad$ $x = -6 - \sqrt{11}$

$\approx$ -2.7 $\quad\quad$ $\approx$ -9.3

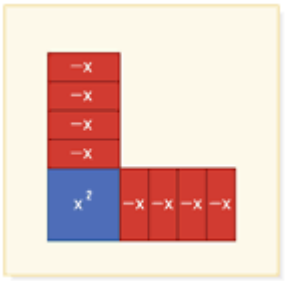
The solution set is {-9.3, -2.7}.

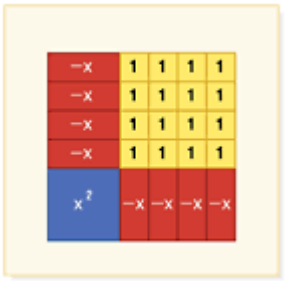
Example 2 Complete the Square

Find the value of c that makes $x^2 - 8x + c$ a perfect square.

Method 1 Use algebra tiles.

Arrange the tiles for $x^2 - 8x$ so that two sides of the figure are congruent.





To make the figure a square, add 16 positive 1-tiles.

$x^2 - 8x + 16$ is a perfect square.

Method 2 Complete the square.

Step 1 Find $\frac{1}{2}$ of -8. $\quad\quad$ $\frac{-8}{2} = -4$

Step 2 Square the result of Step 1. $\quad\quad$ $(-4)^2 = 16$

Step 3 Add the result of Step 2 to $x^2 - 8x$. $\quad\quad$ $x^2 - 8x + 16$

Thus, $c = 16$. Notice that $x^2 - 8x + 16 = (x - 4)^2$.

Example 3 Solve an Equation by Completing the Square

Solve $4x^2 + 16x + 7 = 0$ by completing the square.

Step 1 Isolate the x^2 and x terms.

$4x^2 + 16x + 7 = 0$ Original equation

$4x^2 + 16x + 7 - 7 = 0 - 7$ Subtract 7 from each side.

$4x^2 + 16x = -7$ Simplify.

$4(x^2 + 4x) = -7$ Factor out 4.

$x^2 + 4x = \frac{-7}{4}$ Divide each side by 4.

Step 2 Complete the square and solve.

$x^2 + 4x + 4 = \frac{-7}{4} + 4$ Since $\left(\frac{4}{2}\right)^2 = 4$, add 4 to each side.

$(x + 2)^2 = \frac{9}{4}$ Factor $x^2 + 4x + 4$.

$x + 2 = \pm\frac{3}{2}$ Take the square root of each side.

$x + 2 - 2 = \pm\frac{3}{2} - 2$ Subtract 2 from each side.

$x = -2 \pm \frac{3}{2}$ Simplify.

$x = -2 + \frac{3}{2}$ or $x = -2 - \frac{3}{2}$

$= -\frac{1}{2}$ $= -\frac{7}{2}$

CHECK Substitute each value for x in the original equation.

$4x^2 + 16x + 7 = 0$ $4x^2 + 16x + 7 = 0$

$4(-\frac{1}{2})^2 + 16(-\frac{1}{2}) + 7 =? 0$ $4(-\frac{7}{2})^2 + 16(-\frac{7}{2}) + 7 = 0$

$1 + -8 + 7 =? 0$ $49 + -56 + 7 =? 0$

$0 = 0$ $0 = 0$

The solution set is $\{-\frac{7}{2}, -\frac{1}{2}\}$.

Example 4 Solve a Quadratic Equation in Which $a \neq 1$.

Solve $5x^2 - 80x - 5 = 0$.

$5x^2 - 80x - 5 = 0$	Original equation
$\frac{5x^2 - 80x - 5}{5} = \frac{0}{5}$	Divide each side by 5.
$x^2 - 16x - 1 = 0$	Simplify.
$x^2 - 16x - 1 + 1 = 0 + 1$	Add 1 to each side.
$x^2 - 16x = 1$	Simplify.
$x^2 - 16x + 64 = 1 + 64$	Since $\left(\frac{-16}{2}\right)^2 = 64$, add 64 to each side.
$(x - 8)^2 = 65$	Factor $x^2 - 16x + 64$.
$x - 8 = \pm\sqrt{65}$	Take the square root of each side.
$x - 8 + 8 = \pm\sqrt{65} + 8$	Add 8 to each side.
$x = 8 \pm \sqrt{65}$	Simplify.

Use a calculator to evaluate each value of x.

$x = 8 + \sqrt{65}$ or $x = 8 - \sqrt{65}$

≈ 16.1 ≈ -0.1