Lesson 10-7

Example 1 Recognize Geometric Sequences
Determine whether each sequence is geometric.

a. –2, 6, -18, 54, -162, …

Determine the pattern.

-2 6 -18 54 -162

×-3 ×-3 ×-3 ×-3

In this sequence, each term is found by multiplying –3 times the previous term. This sequence is geometric.

b. 6, 3, 0, -3, -6, …

Determine the pattern.

6 3 0 -3 -6

-3 -3 -3 -3

In this sequence, each term is found by subtracting 3 from the previous term. This sequence is arithmetic and *not* geometric.

Example 2 Continue Geometric Sequences
Find the next three terms in each geometric sequence.

a. 2, 5, 12.5, …

$\frac{5}{2} = 2.5$ Divide the second term by the first.

The common factor is 2.5. Use this information to find the next three terms.

2, 5, 12.5, 31.25 78.125 195.3125

×2.5 ×2.5 ×2.5

The next three terms are 31.25, 78.125, and 195.3125.

b. –3, 12, -48, …

$\frac{12}{-3} = -4$ Divide the second term by the first.

The common factor is –4. Use this information to find the next three terms.

-3, 12, -48, 192 -768 3072

×-4 ×-4 ×-4

The next three terms are 192, -768, and 3072.

Example 3 Use Geometric Sequences to Solve a Problem
INFLATION Suppose the cost of clothing has been increasing at an annual average rate of 4%. If Ashley bought $100 worth of clothing in 1999, what would she pay for the same amount of clothing in 2000, 2001, and 2002?

The cost for clothing is a geometric sequence in which the first term is 100 and the common ratio is 1.04.

Year	**Cost for Clothing**
1999	$100
2000	100(1.04) or $104
2001	104(1.04) or $108.16
2002	108.16(1.04) or $112.49

The cost of the same amount of clothing in 2000, 2001, and 2002 will be $104, $108.16, and about $112.49 respectively.

Example 4 *n*th Term of a Geometric Sequence

Find the eighth term of a geometric sequence in which $a_1 = -6$ and $r = 4$.

$a_n = a_1 \cdot r^{n-1}$	Formula for the *n*th term of a geometric sequence
$a_8 = -6 \cdot (4)^{8-1}$	$n = 8$, $a_1 = -6$, and $r = 4$
$a_8 = -6 \cdot 4^7$	$8 - 1 = 7$
$a_8 = -6 \cdot 16384$	$4^7 = 16384$
$a_8 = -98304$	$-6 \cdot 16384 = -98304$

The eighth term of the geometric sequence is –98304.

Example 5 Find Geometric Means

Find the geometric mean in the sequence $\frac{1}{2}$, ___, $\frac{1}{8}$.

In the sequence, $a_1 = \frac{1}{2}$ and $a_3 = \frac{1}{8}$. To find a_2, you must first find *r*.

$a_n = a_1 \cdot r^{n-1}$	Formula for the *n*th term of a geometric sequence
$a_3 = a_1 \cdot r^{3-1}$	$n = 3$
$\frac{1}{8} = \frac{1}{2} \cdot r^2$	$a_3 = \frac{1}{8}$ and $a_1 = \frac{1}{2}$
$\frac{\frac{1}{8}}{\frac{1}{2}} = \frac{\frac{1}{2} \cdot r^2}{\frac{1}{2}}$	Divide each side by $\frac{1}{2}$.
$\frac{1}{4} = r^2$	$\frac{1}{8} \div \frac{1}{2} = \frac{1}{8} \cdot \frac{2}{1}$ or $\frac{1}{4}$
$\pm\frac{1}{2} = r$	Take the square root of each side.

If $r = \frac{1}{2}$, the geometric mean is $\frac{1}{2}(\frac{1}{2})$ or $\frac{1}{4}$. If $r = -\frac{1}{2}$, the geometric mean is $\frac{1}{2}(-\frac{1}{2})$ or $-\frac{1}{4}$.

Therefore, the geometric mean is $\frac{1}{4}$ or $-\frac{1}{4}$.