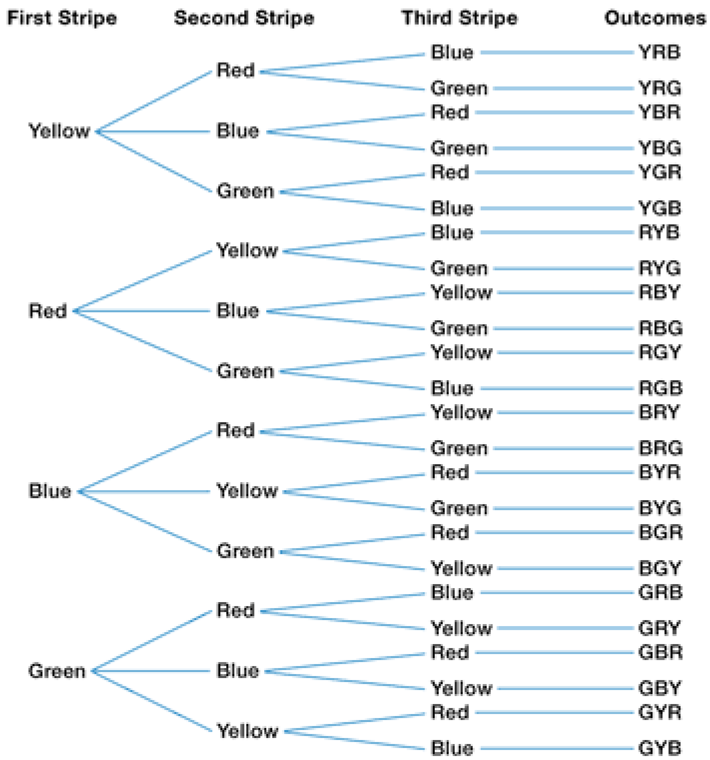
Lesson 14-2

Example 1 Tree Diagram Permutation

Mrs. Malone assigned a project to her art class. Each student is to design a flag. The background of the flag consists of 3 wide stripes. If there are 4 colors available for the 3 stripes and each stripe must be a different color. How many possible flags are there?



There are 24 different ways the students can color the background of their flags.

Example 2 Permutation

Find $_7P_3$.

$_nP_r = \frac{n!}{(n-r)!}$ Definition of $_nP_r$.

$_7P_3 = \frac{7!}{(7-3)!}$ $n = 7$, $r = 3$

$_7P_3 = \frac{7!}{4!}$ Subtract.

$_7P_3 = \frac{7\cdot6\cdot5\cdot4\cdot3\cdot2\cdot1}{4\cdot3\cdot2\cdot1}$ Definition of factorial.

$_7P_3 = 7 \cdot 6 \cdot 5$ or 210 Simplify.

There are 210 permutations of 7 objects taken 3 at a time.

Example 3 Permutation and Probability

There are 10 contestants in the state quiz bowl competition. Seven of the contestants are female. The top two contestants will go on to compete at the national quiz bowl competition.

a. How many different ways can first and second place be chosen?

Since the order of the contestants is important, this situation is a permutation of 10 contestants taken 2 at a time.

$_nP_r = \frac{n!}{(n-r)!}$ Definition of permutation

$_{10}P_2 = \frac{10!}{(10-2)!}$ $n = 10$, $r = 2$

$_{10}P_2 = \frac{10\cdot9\cdot8\cdot7\cdot6\cdot5\cdot4\cdot3\cdot2\cdot1}{8\cdot7\cdot6\cdot5\cdot4\cdot3\cdot2\cdot1}$ or 90 Definition of factorial

There are 90 possible ways that first and second place can be chosen.

b. What is the probability that the first and second place are both female?

Use the Fundamental Counting Principle to determine the number of ways for the first place to be female. There are 7 females. So, the choices for the first and second place if they are females is, 7 • 6 or 42. So, the number of favorable outcomes is 42. There are 42 ways for this event to occur out of the 90 possible permutations.

P(first and second female) = $\frac{42}{90}$ ← number of favorable outcomes ← number of possible outcomes

The probability that the top two contestants are female is $\frac{42}{90}$ or about 47%.

Example 4 Combination

Multiple-Choice Test Item

Shane can only take 3 books on his vacation. He has 8 books to choose from. How many different groups of books could be selected?

A. 56 **B.** 336 **C.** 6 **D.** 40,320

Read the Test Item

The order in which the books are chosen does not matter, so this situation represents a combination of 8 books taken 3 at a time.

Solve the Test Item

$${}_nC_r = \frac{n!}{(n-r)!r!}$$ Definition of combination

$${}_8C_3 = \frac{8!}{(8-3)!3!}$$ $n = 8$, $r = 3$

$$= \frac{8\cdot 7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}{5\cdot 4\cdot 3\cdot 2\cdot 1\cdot 3\cdot 2\cdot 1}$$ Definition of factorial

$$= \frac{8\cdot 7\cdot 6}{3\cdot 2\cdot 1} \text{ or } 56$$ Simplify.

There are 56 different groups of books that could be selected. Choice **A** is correct.

Example 5 Use Combinations

A literature teacher needs to choose 6 books out of 12 for her literature class to read. The 12 books consist of 4 British literature, 5 contemporary literature, and 3 Western literature books.

a. How many different ways can the teacher choose 6 books?

$${}_nC_r = \frac{n!}{(n-r)!r!}$$ Definition of combination

$${}_{12}C_6 = \frac{12!}{(12-6)!6!}$$ $n = 12$, $r = 6$

$$= \frac{12!}{6!6!}$$ $12 – 6 = 6$

$$= \frac{12\cdot 11\cdot 10\cdot 9\cdot 8\cdot 7}{6!}$$ Divide by the GCF, 6!.

$$= \frac{665280}{720} \text{ or } 924$$ Simplify.

There are 924 ways to choose 6 books out of 12.

b. If the books are chosen randomly, what is the probability that 2 British literature, 3 contemporary literature, and 1 Western literature book will be selected?

There are three questions to consider.

- How many ways can 2 British literature books be chosen from 4?
- How many ways can 3 contemporary literature books be chosen from 5?
- How many ways can 1 Western literature book be chosen from 3?

Using the Fundamental Counting Principle, the answer can be determined with the product of the three combinations.

$$\underbrace{{}_4C_2}_{\text{ways to choose 2 British literature books out of 4}} \cdot \underbrace{{}_5C_3}_{\text{ways to choose 3 contemporary literature books out of 5}} \cdot \underbrace{{}_3C_1}_{\text{ways to choose 1 Western literature book out of 3}}$$

$$({}_4C_2)({}_5C_3)({}_3C_1) = \frac{4!}{(4-2)!2!} \cdot \frac{5!}{(5-3)!3!} \cdot \frac{3!}{(3-1)!1!}$$ Definition of combination

$$= \frac{4!}{2!2!} \cdot \frac{5!}{2!3!} \cdot \frac{3!}{2!1!}$$ Simplify.

$$= \frac{4 \cdot 3}{2 \cdot 1} \cdot \frac{5 \cdot 4}{2 \cdot 1} \cdot \frac{3}{1}$$ Divide each by the GCF.

$$= \frac{720}{4} \text{ or } 180$$ Simplify.

There are 180 ways to choose this particular combination out of 924 possible combinations.

$$P(\text{2 British, 3 contemporary, 1 Western}) = \frac{180}{924} \quad \begin{matrix} \leftarrow \text{ number of favorable outcomes} \\ \leftarrow \text{ number of possible outcomes} \end{matrix}$$

$$= \frac{15}{77}$$ Simplify.

The probability that the literature teacher will randomly select 2 British literature books, 3 contemporary literature books, and 1 Western literature book is $\frac{15}{77}$ or about 19%.