

Lesson 9-1

Example 1 Classify Numbers as Prime or Composite

Factor each number. Then classify each number as *prime* or *composite*.

a. 31

The only whole numbers that can be multiplied to get 31 are 1 and 31. Therefore, the factors of 31 are 1 and 31. Since the only factors of 31 are 1 and itself, 31 is a prime number.

b. 54

To find the factors of 54, list all pairs of whole numbers whose product is 54.

$$1 \times 54 \quad 2 \times 27 \quad 3 \times 18 \quad 6 \times 9$$

Therefore, the factors of 54, in increasing order, are 1, 2, 3, 6, 9, 18, 27, and 54. Since 54 has more than two factors, it is a composite number.

Example 2 Prime Factorization of a Positive Integer

Find the prime factorization of 72.

Method 1

$$\begin{aligned} 72 &= 2 \cdot 36 && \text{The least prime factor of 72 is 2.} \\ &= 2 \cdot 2 \cdot 18 && \text{The least prime factor of 36 is 2.} \\ &= 2 \cdot 2 \cdot 2 \cdot 9 && \text{The least prime factor of 18 is 2.} \\ &= 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 && \text{The least prime factor of 9 is 3.} \end{aligned}$$

All of the factors in the last row are prime. Thus, the prime factorization of 72 is $2 \cdot 2 \cdot 2 \cdot 3 \cdot 3$.

Method 2

Use a factor tree.

$$\begin{array}{ccccccc} & & & & 72 & & \\ & & & & / \quad \backslash & & \\ & & & & 4 \quad 18 & & 72 = 4 \cdot 18 \\ & & & & / \quad \backslash & & 4 = 2 \cdot 2 \text{ and } 18 = 2 \cdot 9 \\ & & & & 2 \quad 2 \quad 2 \quad 9 & & \\ & & & & / \quad \backslash & & 9 = 3 \cdot 3 \\ & & & & 3 \quad 3 & & \end{array}$$

All of the factors in the last branch of the factor tree are prime. Thus, the prime factorization of 72 is $2 \cdot 2 \cdot 2 \cdot 3 \cdot 3$ or $2^3 \cdot 3^2$.

Example 3 Prime Factorization of a Negative Integer

Find the prime factorization of -96 .

$$\begin{aligned} -96 &= -1 \cdot 96 && \text{Express } -96 \text{ as } -1 \text{ times } 96. \\ &= -1 \cdot 2 \cdot 48 && 96 = 2 \cdot 48 \\ &= -1 \cdot 2 \cdot 2 \cdot 24 && 48 = 2 \cdot 24 \\ &= -1 \cdot 2 \cdot 2 \cdot 2 \cdot 12 && 24 = 2 \cdot 12 \\ &= -1 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 6 && 12 = 2 \cdot 6 \\ &= -1 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 && 6 = 2 \cdot 3 \end{aligned}$$

Thus, the prime factorization of -96 is $-1 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3$ or $-1 \cdot 2^5 \cdot 3$.

Example 4 Prime Factorization of a Monomial

Factor each monomial completely.

a. $-21xy^4$

$$\begin{aligned} -21xy^4 &= -1 \cdot 21 \cdot x \cdot y \cdot y \cdot y \cdot y && \text{Express } -21 \text{ as } -1 \text{ times } 21 \text{ and } y^4 = y \cdot y \cdot y \cdot y \\ &= -1 \cdot 3 \cdot 7 \cdot x \cdot y \cdot y \cdot y \cdot y && 21 = 3 \cdot 7 \end{aligned}$$

Thus, $-21xy^4$ in factored form is $-1 \cdot 3 \cdot 7 \cdot x \cdot y \cdot y \cdot y \cdot y$.

b. $26rs^2t$

$$26rs^2t = 2 \cdot 13 \cdot r \cdot s \cdot s \cdot t \quad 26 = 2 \cdot 13$$

Thus, $26rs^2t$ in factored form is $2 \cdot 13 \cdot r \cdot s \cdot s \cdot t$.

Example 5 GCF of a Set of Monomials

Find the GCF of each set of monomials.

a. 12 and 24

$$12 = 2 \cdot 2 \cdot 3$$

Factor each number.

$$24 = \textcircled{2} \cdot \textcircled{2} \cdot 2 \cdot \textcircled{3}$$

Circle the common prime factors, if any.

The GCF of 12 and 24 is $2 \cdot 2 \cdot 3$ or 12.

b. $25ab^2c$ and $45ab$

$$25ab^2c = 5 \cdot 5 \cdot a \cdot b \cdot b \cdot c$$

Factor each number.

$$45ab = 3 \cdot 3 \cdot \textcircled{5} \cdot \textcircled{a} \cdot \textcircled{b}$$

Circle the common prime factors.

The GCF of $25ab^2c$ and $45ab$ is $5 \cdot a \cdot b$ or $5ab$.

Example 6 Use Factors

A company sells pencils packaged in boxes of 24 or 64. Within the boxes, the pencils are packaged into smaller boxes. The smaller boxes contain the same number of pencils before they are placed in the larger boxes. What number of pencils should be placed in the smaller boxes to make the least number of smaller boxes?

Find the factors of 24 and 64.

The factors of 24 are 1, 2, 3, 4, 6, 8, 12, and 24.

The factors of 64 are 1, 2, 4, 8, 16, 32, and 64.

To find the number of pencils that will make the least number of boxes, we will find the largest factor that will go into both numbers.

The largest number that will go into both 24 and 64 is 8. Therefore, the company should put 8 pencils into each smaller box. The box of 24 would then have 3 smaller boxes and the box of 64 would have 8 smaller boxes.