Lesson 9-5

Example 1 Factor the Difference of Squares
Factor each binomial.

a. $a^2 - 64$

$a^2 - 64 = a^2 - 8^2$	Write in the form $a^2 - b^2$.
$= (a + 8)(a - 8)$	Factor the difference of squares.

b. $16m^4 - n^2$

$16m^4 - n^2 = (4m^2)^2 - (n)^2$	$16m^4 = 4m^2 \cdot 4m^2$ and $n^2 = n \cdot n$
$= (4m^2 + n)(4m^2 - n)$	Factor the difference of squares.

Example 2 Factor Out a Common Factor
Factor $5x^2y - 500y$.

$5x^2y - 500y = 5y(x^2 - 100)$	The GCF of $5x^2y$ and $500y$ is $5y$.
$= 5y(x^2 - 10^2)$	$100 = 10^2$
$= 5y(x + 10)(x - 10)$	Factor the difference of squares.

Example 3 Apply a Factoring Technique More than Once
Factor $16x^5 - 625x$.

$16x^5 - 625x$	Original polynomial
$= x(16x^4 - 625)$	The GCF of $16x^5$ and $625x$ is x.
$= x[(4x^2)^2 - 25^2]$	$16x^4 = 4x^2 \cdot x^2$ and $625 = 25 \cdot 25$
$= x(4x^2 + 25)(4x^2 - 25)$	Factor the difference of squares.
$= x(4x^2 + 25)[(2x)^2 - 5^2]$	$4x^2 = 2x \cdot 2x$ and $25 = 5 \cdot 5$
$= x(4x^2 + 25)(2x + 5)(2x - 5)$	Factor the difference of squares.

Example 4 Apply Several Different Factoring Techniques
Factor $a^4 - 5a^3 - 4a^2 + 20a$.

$a^4 - 5a^3 - 4a^2 + 20a$	Original polynomial
$= a(a^3 - 5a^2 - 4a + 20)$	Factor out the GCF.
$= a[(a^3 - 5a^2) + (-4a + 20)]$	Group terms with common factors.
$= a[a^2(a - 5) + 4(-a + 5)]$	Factor each grouping.
$= a[a^2(a - 5) + 4(-1)(a - 5)]$	$(-a + 5) = -1(a - 5)$
$= a[a^2(a - 5) - 4(a - 5)]$	Simplify.
$= a[(a - 5)(a^2 - 4)]$	$a - 5$ is the common factor.
$= a[(a - 5)(a^2 - 2^2)]$	$a^2 = a \cdot a$ and $4 = 2 \cdot 2$
$= a(a - 5)(a + 2)(a - 2)$	Factor the difference of squares.

Example 5 Solve Equations by Factoring
Solve each equation. Check your solutions.

a. $\boldsymbol{a^3 + 2a^2 - a = 2}$

$a^3 + 2a^2 - a = 2$	Original equation
$a^3 + 2a^2 - a - 2 = 0$	Subtract 2 from each side.
$(a^3 + 2a^2) + (-a - 2) = 0$	Group terms with a common factor.
$a^2(a + 2) + -1(a + 2) = 0$	Factor each grouping.
$(a + 2)(a^2 - 1) = 0$	$a + 2$ is the common factor.
$(a + 2)(a + 1)(a - 1) = 0$	$a^2 = a \cdot a$ and $1 = 1 \cdot 1$

Applying the Zero Product Property, set each factor equal to 0 and solve the resulting three equations.

$a + 2 = 0$ or $a + 1 = 0$ or $a - 1 = 0$

$a = -2$ $a = -1$ $a = 1$

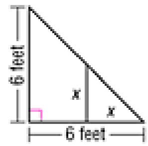
The solution set is {-2, -1, 1}. Check each solution in the original equation.

b. $\boldsymbol{4b^2 - 1 = 0}$

$4b^2 - 1 = 0$	Original equation
$(2b)^2 - 1^2 = 0$	$4b^2 = 2b \cdot 2b$ and $1 = 1 \cdot 1$
$(2b + 1)(2b - 1) = 0$	Factor the difference of squares.
$2b + 1 = 0$ or $2b - 1 = 0$	Zero Product Property
$2b = -1$ $2b = 1$	Solve each equation.
$b = -\frac{1}{2}$ $b = \frac{1}{2}$	

The solution set is $\{-\frac{1}{2}, \frac{1}{2}\}$. Check each solution in the original equation.

Example 6 Use Differences of Two Squares
OPEN ENDED TEST ITEM



A triangle is cut off the corner of a right triangle. The right triangle has legs that both measure 6 feet. The cut is a right triangle with both legs of measure *x* feet.

a. Write an equation in terms of *x* that represents the area *A* of the paper after the corner is removed.

b. What value of *x* will result in an area that is $\frac{3}{4}$ the area of the original triangular piece of paper? Show how you arrived at your answer.

Read the Test Item

A is the area of the triangle minus the area of the triangular corner to be removed.

Solve the Test Item

a. The area of the triangle is $\frac{1}{2} \cdot 6 \cdot 6$ or 18 square feet, and the area of the cut triangle is $\frac{1}{2} \cdot x \cdot x$ or $\frac{1}{2}x^2$ square feet. Thus $A = 18 - \frac{1}{2}x^2$.

b. Find *x* so that *A* is $\frac{3}{4}$ the area of the original triangular piece of paper.

$A = \frac{3}{4}$ area of original triangle

$18 - \frac{1}{2}x^2 = \frac{3}{4}(18)$	$A = 18 - \frac{1}{2}x^2$ and area of original triangle is 18.
$18 - \frac{1}{2}x^2 = \frac{27}{2}$	Simplify.
$18 - \frac{1}{2}x^2 - \frac{27}{2} = 0$	Subtract $\frac{27}{2}$ from each side.
$\frac{9}{2} - \frac{1}{2}x^2 = 0$	Simplify.
$9 - x^2 = 0$	Multiply each side by 2 to remove fractions.
$(3 + x)(3 - x) = 0$	Factor the difference of squares.
$3 + x = 0$ or $3 - x = 0$	Zero Product Property
$x = -3$ $x = 3$	Solve each equation.

Since length cannot be negative, the only reasonable solution is 3.