Lesson 9-6

Example 1 Factor Perfect Square Trinomials
Determine whether each trinomial is a perfect square trinomial. If so, factor it.

a. $4x^2 + 12x + 9$

1. Is the first term a perfect square? Yes, $4x^2 = (2x)^2$.
2. Is the last term a perfect square? Yes, $9 = (3)^2$.
3. Is the middle term equal to $2(2x)(3)$? Yes, $12x = 2(2x)(3)$.

$4x^2 + 12x + 9$ is a perfect square trinomial.

$4x^2 + 12x + 9 = (2x)^2 + 2(2x)(3) + (3)^2$ — Write as $a^2 + 2ab + b^2$.
$= (2x + 3)^2$ — Factor using pattern.

b. $4a^2 - 8a + 16$

1. Is the first term a perfect square? Yes, $4a^2 = (2a)^2$.
2. Is the last term a perfect square? Yes, $16 = (4)^2$.
3. Is the middle term equal to $2(2a)(4)$? No, $8a \neq 2(2a)(4)$.

$4a^2 - 8a + 16$ is not a perfect square trinomial.

Example 2 Factor Completely
Factor each polynomial.

a. $3x^2 - 12x + 12$

This polynomial has a GCF of 3. First, factor out the GCF and you are left with $3(x^2 - 4x + 4)$. The resulting trinomial has the first term as a perfect square $x^2 = (x)^2$, the last term is also a perfect square $4 = 2^2$, and the middle term is equal to $2(x)(2)$ or $4x$. Therefore, the polynomial is a perfect square trinomial.

$3x^2 - 12x + 12$
$= 3(x^2 - 4x + 4)$ — 3 is the GCF.
$= 3[(x)^2 - 2(x)(2) + (2)^2]$ — Write as $a^2 - 2ab + b^2$.
$= 3(x - 2)^2$ — $a = x$ and $b = 2$.

b. $2x^3 - x^2 - 15x$

This polynomial has three terms that have a GCF of x. The resulting trinomial is then in the form $ax^2 + bx + c$. Are there two numbers m and n whose product is $2 \cdot -15$ or -30 and whose sum is -1? Yes, the product of 5 and -6 is -30 and their sum is -1.

$2x^3 - x^2 - 15x$
$= x(2x^2 - x - 15)$ — x is the GCF.
$= x(2x^2 + mx + nx - 15)$ — Write the pattern.
$= x(2x^2 + 5x + -6x - 15)$ — $m = 5$ and $n = -6$
$= x[(2x^2 + 5x) + (-6x - 15)]$ — Group terms with common factors.
$= x[x(2x + 5) + -3(2x + 5)]$ — Factor out the GCF from each grouping.
$= x(2x + 5)(x - 3)$ — $2x + 5$ is the common factor.

Example 3 Solve Equations with Repeated Factors

Solve $16x^2 + 8x + 1 = 0$.

$16x^2 + 8x + 1 = 0$	Original equation
$(4x)^2 + 2(4x)(1) + (1)^2 = 0$	Recognize $16x^2 - 8x + 1$ as a perfect square trinomial.
$(4x + 1)^2 = 0$	Factor the perfect square trinomial.
$4x + 1 = 0$	Set repeated factor equal to zero.
$4x = -1$	Solve for x.
$x = -\frac{1}{4}$	

Thus, the solution set is $\{-\frac{1}{4}\}$. Check this solution in the original equation.

Example 4 Use the Square Root Property to Solve Equations

Solve each equation. Check your solutions.

a. $(x-2)^2 = \frac{4}{9}$

$(x-2)^2 = \frac{4}{9}$	Original equation
$x - 2 = \pm\sqrt{\frac{4}{9}}$	Square Root Property
$x - 2 = \pm\frac{2}{3}$	$\frac{4}{9} = \frac{2}{3} \cdot \frac{2}{3}$
$x = 2 \pm \frac{2}{3}$	Add 2 to each side.
$x = 2 + \frac{2}{3}$ or $x = 2 - \frac{2}{3}$	Separate into two equations.
$= \frac{8}{3}$ $\quad = \frac{4}{3}$	Simplify.

The solution set is $\{\frac{4}{3}, \frac{8}{3}\}$. Check each solution in the original equation.

b. $\mathbf{x^2 + \frac{1}{2}x + \frac{1}{16} = 36}$

$x^2 + \frac{1}{2}x + \frac{1}{16} = 36$	Original equation
$(x)^2 + 2(x)(\frac{1}{4}) + (\frac{1}{4})^2 = 36$	Recognize perfect square trinomial.
$(x + \frac{1}{4})^2 = 36$	Factor perfect square trinomial.
$x + \frac{1}{4} = \pm\sqrt{36}$	Square Root Property
$x + \frac{1}{4} = \pm 6$	$36 = 6 \cdot 6$
$x = -\frac{1}{4} \pm 6$	Subtract $\frac{1}{2}$ from each side.
$x = -\frac{1}{4} + 6$ or $x = -\frac{1}{4} - 6$	Separate into two equations.
$= \frac{23}{4}$ $\quad = -\frac{-25}{4}$	Simplify.

The solution set is $\{\frac{-25}{4}, \frac{23}{4}\}$. Check each solution in the original equation.

c. $\mathbf{(x + 1)^2 = 10}$

$(x + 1)^2 = 10$	Original equation
$x + 1 = \pm\sqrt{10}$	Square Root Property
$x = -1 \pm \sqrt{10}$	Subtract 1 from each side.

Since 10 is not a perfect square, the solution set is $\{-1 \pm \sqrt{10}\}$. Using a calculator, the approximate solutions are $-1 + \sqrt{10}$ or about 2.16 and $-1 - \sqrt{10}$ or about –4.16.

Check: You can check your answer using a graphing calculator. Graph $y = (x + 1)^2$ and $y = 10$. Using the INTERSECT feature of your graphing calculator, find where $(x + 1)^2 = 10$. The check of 2.16 as one of the approximate solutions is shown at the right.