

The Game of Five

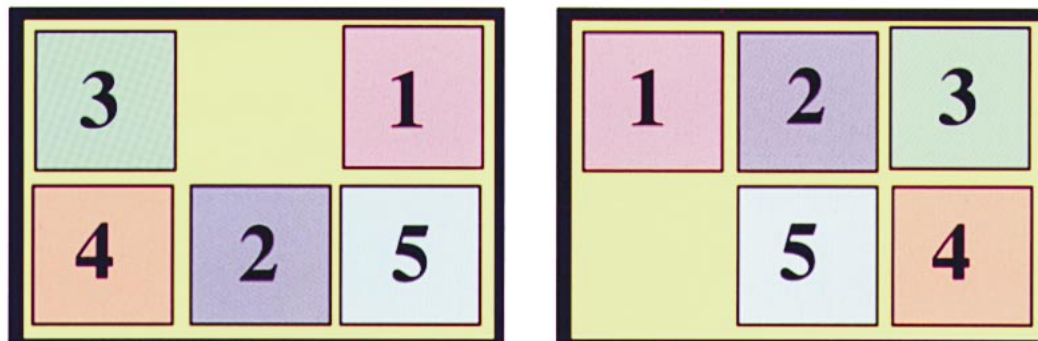
Problem-of-the-Week

The Problem

There are five numbered tiles in an open box. The box has room for six tiles, so there is an empty space which allows you to slide the tiles around.

The first arrangement is described by the notation 31524; the second arrangement, by 12345. Notice that the numbers in the first row are listed from left to right. Then, the second row is listed from right to left.

How many possible arrangements of the tiles are there? Show, if possible, how to change the first arrangement into the second.



Strategies and Hints

1. The arrangements 31524, 15243, 52431, 24315, and 43152 are equivalent. Explain why.
2. An *inversion* occurs when a higher digit comes before a lower one. For example, 31524 has four inversions, two for the digit 3 and two for the digit 5. An *even* arrangement has an even number of inversions; an *odd* arrangement has an odd number of inversions.
3. Why might categorizing an arrangement as odd or even help you determine whether the puzzle can be solved?
4. Give an example of an arrangement that can *not* be changed into 12345.