

Lesson 10–3

Example 1 Use the Product Property

Use $\log_3 2 \approx 0.6310$ to approximate the value of $\log_3 54$.

$$\begin{aligned}\log_3 54 &= \log_3 (3^3 \cdot 2) && \text{Replace 54 with } 27 \cdot 2 \text{ or } 3^3 \cdot 2. \\ &= \log_3 3^3 + \log_3 2 && \text{Product Property} \\ &= 3 + \log_3 2 && \text{Inverse Property of Exponents and Logarithms} \\ &= 3 + 0.6310 \text{ or } 3.6310 && \text{Replace } \log_3 2 \text{ with } 0.6310.\end{aligned}$$

Thus, $\log_3 54$ is approximately 3.6310.

Example 2 Use the Quotient Property

Use $\log_5 2 \approx 0.4307$ and $\log_5 7 \approx 1.2091$ to approximate $\log_5 3.5$.

$$\begin{aligned}\log_5 3.5 &= \log_5 \frac{7}{2} && \text{Replace 3.5 with the quotient } \frac{7}{2}. \\ &= \log_5 7 - \log_5 2 && \text{Quotient Property} \\ &\approx 1.2091 - 0.4307 \text{ or } 0.7784 && \text{Replace } \log_5 7 \text{ with } 1.2091 \text{ and } \log_5 2 \text{ with } 0.4307.\end{aligned}$$

Thus, $\log_5 3.5$ is approximately 0.7784.

CHECK Using the definition of logarithm and a calculator, $5^{0.7784} \approx 3.5$. ✓

Example 3 Use Properties of Logarithms

SOUND The loudness L of a sound in decibels is given by $L = 10 \log_{10} R$, where R is the sound's relative intensity. A rocket engine has a relative intensity of 10^{18} or 180 decibels. A jet taking off has a relative intensity of 10^{15} or 150 decibels. How does the intensity of 100 jets taking off at once compare to the intensity of the rocket engine?

$$\begin{aligned}\text{Let } L_1 \text{ be the loudness of one jet taking off.} &\rightarrow L_1 = 10 \log_{10} 10^{15} \\ \text{Let } L_2 \text{ be the loudness of 100 jets taking off.} &\rightarrow L_2 = 10 \log_{10} (100 \cdot 10^{15})\end{aligned}$$

Then the increase in loudness is $L_2 - L_1$.

$$\begin{aligned}L_2 - L_1 &= 10 \log_{10} (100 \cdot 10^{15}) - 10 \log_{10} 10^{15} && \text{Substitute for } L_1 \text{ and } L_2. \\ &= 10 (\log_{10} 10^2 + \log_{10} 10^{15}) - 10 \log_{10} 10^{15} && \text{Product Property} \\ &= 10 \log_{10} 10^2 + 10 \log_{10} 10^{15} - 10 \log_{10} 10^{15} && \text{Distributive Property} \\ &= 10 \log_{10} 10^2 && \text{Subtract.} \\ &= 10 (2) && \text{Inverse Property of Exponents and Logarithms} \\ &= 20 && \text{Multiply}\end{aligned}$$

The sound of 100 jets taking off is perceived by the human ear to be 20 decibels louder than one jet taking off or 170 decibels. The rocket engine at 180 decibels is 10 decibels louder than the 100 jets taking off.

CHECK

Substitute 2 into the original equation.

$$\begin{aligned} \log_7(2n+3) - \log_7(n-1) &= 1 && \text{Original equation} \\ \log_7[2(2)+3] - \log_7(2-1) &= 1 && \text{Substitute 2 for } n. \\ \log_7(7) - \log_7(1) &= 1 && \text{Simplify.} \\ 1 - 0 &= 1 && \log_7(7) = 1, \log_7(1) = 0 \\ 1 &= 1 && \text{Simplify.} \quad \checkmark \end{aligned}$$