

Lesson 11–1

Example 1 Find the Next Terms

Find the next four terms of the arithmetic sequence $-\frac{17}{4}, -\frac{7}{2}, -\frac{11}{4}, \dots$.

Find the common difference d by subtracting two consecutive terms.

$$\left(-\frac{7}{2}\right) - \left(-\frac{17}{4}\right) = \frac{3}{4} \text{ and } -\frac{11}{4} - \left(-\frac{7}{2}\right) = \frac{3}{4} \quad \text{So, } d = \frac{3}{4}.$$

Now add $\frac{3}{4}$ to the third term of the sequence, and then continue adding $\frac{3}{4}$ until the next four terms are found.

$$\begin{array}{ccccccc} -\frac{11}{4} & & -2 & & -\frac{5}{4} & & -\frac{1}{2} & & \frac{1}{4} \\ & \nearrow & & \nearrow & & \nearrow & & \nearrow & \\ & +\frac{3}{4} & & +\frac{3}{4} & & +\frac{3}{4} & & +\frac{3}{4} & \end{array}$$

The next four terms of the sequence are $-2, -\frac{5}{4}, -\frac{1}{2},$ and $\frac{1}{4}$.

Example 2 Find a Particular Term

GEOLOGY Along the San Andreas Fault in California, the plates of the earth are moving over time. A section of the Pacific plate is moving northwest at the rate of about 1.3 centimeters per year. About how far will the plate move in 100 years?

Explore Since the problem asks for the distance in 100 years, it is easier to describe the movement in 10-year periods. Every 10 years, the plate moves $10(1.3) = 13$ centimeters. If you list the distances moved by the plate using 10-year periods, the distances form an arithmetic sequence with common difference 13. For example, the distances for the 10-year periods will be 13, 26, 39, ...

Plan You can use the formula for the n th term of an arithmetic sequence with $a_1 = 13$ and $d = 13$ to find a_{10} , the distance moved by the plate after 100 years.

Solve

$a_n = a_1 + (n - 1)d$	Formula for the n th term
$a_{10} = 13 + (10 - 1)13$	$n = 10, a_1 = 13, d = 13$
$a_{10} = 130$	Simplify.

The fault will move 130 centimeters in 100 years.

Examine You can find the terms of the sequence by adding 13. a_4 through a_{10} are 52, 65, 78, 91, 104, 117, and 130. Therefore, 130 centimeters is correct.

Example 3 Write an Equation for the n th Term

Write an equation for the n th term of the arithmetic sequence 1, -4, -9, -14,

In this sequence, $a_1 = 1$ and $d = -5$. Use the n th term formula to write an equation.

$$\begin{array}{ll} a_n = a_1 + (n - 1)d & \text{Formula for the } n\text{th term} \\ a_n = 1 + (n - 1)(-5) & a_1 = 1, d = -5 \\ a_n = 1 + (-5n) + 5 & \text{Distributive Property} \\ a_n = -5n + 6 & \text{Simplify.} \end{array}$$

An equation is $a_n = -5n + 6$.

Example 4 Find Arithmetic Means

Find the three arithmetic means between 17 and -3.

You can use the n th term formula to find the common difference. In the sequence 17, $\underline{\quad ? \quad}$, $\underline{\quad ? \quad}$, $\underline{\quad ? \quad}$, -3, ..., a_1 is 17 and a_5 is -3.

$$\begin{array}{ll} a_n = a_1 + (n - 1)d & \text{Formula for the } n\text{th term} \\ a_5 = 17 + (5 - 1)d & n = 5, a_1 = 17 \\ -3 = 17 + 4d & a_5 = -3 \\ -20 = 4d & \text{Subtract 17 from each side.} \\ -5 = d & \text{Divide each side by 4.} \end{array}$$

Now use the value of d to find the three arithmetic means.

$$\begin{array}{ccccccc} 17 & & 12 & & 7 & & 2 \\ \curvearrowright & & \curvearrowright & & \curvearrowright & & \\ & +(-5) & & +(-5) & & +(-5) & \end{array}$$

The arithmetic means are 12, 7, and 2.

CHECK $2 + (-5) = -3 \checkmark$