

## Lesson 11–3

### Example 1 Find the Next Term

#### Multiple–Choice Test Item

Find the missing term in the geometric sequence:  $-64, 16, -4, 1, \underline{\hspace{1cm}}$ .

- A.  $-0.25$       B.  $0.25$       C.  $4$       D.  $-4$

#### Read the Test Item

Since  $\frac{16}{-64} = -0.25$ ,  $\frac{-4}{16} = -0.25$ , and  $\frac{1}{-4} = -0.25$ , the sequence has a common ratio of  $-0.25$ .

#### Solve the Test Item

To find the missing term, multiply the last given term by  $-0.25$ .

$$1(-0.25) = -0.25$$

The answer is A.

### Example 2 Find a Particular Term

Find the sixth term of a geometric sequence for which  $a_1 = -10$  and  $r = -1.5$ .

$$\begin{aligned} a_n &= a_1 \cdot r^{n-1} && \text{Formula for } n\text{th term} \\ a_6 &= (-10) \cdot (-1.5)^{6-1} && n = 6, a_1 = -10, r = -1.5 \\ a_6 &= (-10) \cdot (-7.59375) && (-1.5)^5 = -7.59375 \\ a_6 &= 75.9375 \end{aligned}$$

The sixth term is 75.9375.

### Example 3 Write an Equation for the $n$ th Term

Write an equation for the  $n$ th term of the geometric sequence  $-45, 15, -5, -\frac{5}{3}, \dots$ .

In this sequence,  $a_1 = -45$  and  $r = \frac{15}{-45}$  or  $-\frac{1}{3}$ . Use the  $n$ th term formula to write an equation.

$$\begin{aligned} a_n &= a_1 \cdot r^{n-1} && \text{Formula for } n\text{th term} \\ a_n &= -45 \cdot \left(-\frac{1}{3}\right)^{n-1} && a_1 = -45, r = -\frac{1}{3} \end{aligned}$$

An equation is  $a_n = -45 \cdot \left(-\frac{1}{3}\right)^{n-1}$ .

**Example 4 Find a Term Given the Fourth Term and the Ratio**

Find the fifteenth term of a geometric sequence for which  $a_4 = 25.5$  and  $r = -2$ .

First, find the value of  $a_1$ .

$$\begin{aligned} a_n &= a_1 \cdot r^{n-1} && \text{Formula for } n\text{th term} \\ a_4 &= a_1 \cdot (-2)^{4-1} && n = 15, r = -2 \\ 25.5 &= -8a_1 && a_4 = 25.5 \\ -3.1875 &= a_1 && \text{Divide each side by } -8. \end{aligned}$$

Now find  $a_{15}$ .

$$\begin{aligned} a_n &= a_1 \cdot r^{n-1} && \text{Formula for } n\text{th term} \\ a_{15} &= -3.1875 \cdot (-2)^{15-1} && n = 15, a_1 = -3.1875, r = -2, \\ a_{15} &= -52,224 \end{aligned}$$

The fifteenth term is  $-52,224$ .

**Example 5 Find Geometric Means**

Find four geometric means between 15,625 and 5.

Use the  $n$ th term formula to find the value of  $r$ . In the sequence 15,625,  $\underline{\quad}$ ,  $\underline{\quad}$ ,  $\underline{\quad}$ ,  $\underline{\quad}$ , 5,  $a_1$  is 15,625 and  $a_6$  is 5.

$$\begin{aligned} a_n &= a_1 \cdot r^{n-1} && \text{Formula for } n\text{th term} \\ a_6 &= 15,625 \cdot r^{6-1} && n = 6, a_1 = 15,625 \\ 5 &= 15,625r^5 && a_6 = 5 \\ 0.00032 &= r^5 && \text{Divide each side by } 15,625. \\ 0.2 &= r && \text{Take the fifth root of each side.} \end{aligned}$$

Since 5 is an odd root, there is only one solution to  $0.00032 = r^5$ , which is positive 0.2. Use  $r = 0.2$  to find four geometric means.

$$\begin{aligned} r &= 0.2 \\ a_2 &= 15,625(0.2) \text{ or } 3125 \\ a_3 &= 3125(0.2) \text{ or } 625 \\ a_4 &= 625(0.2) \text{ or } 125 \\ a_5 &= 125(0.2) \text{ or } 25 \end{aligned}$$

The geometric means are 3125, 625, 125, and 25.