Lesson 11–7

Example 1 **Use Pascal's Triangle**

Expand $(n + 2)^5$.

Write the sixth row of Pascal's triangle.

1 5 10 10 5 1

Use the patterns of a binomial expansion and the coefficients to write the expansion of $(n + 2)^5$.

$$\begin{aligned}(n + 2)^5 &= 1n^5(2)^0 + 5n^4(2)^1 + 10n^3(2)^2 + 10n^2(2)^3 + 5n(2)^4 + 1n^0(2)^5 \\ &= n^5 + 10n^4 + 40n^3 + 80n^2 + 80n + 32\end{aligned}$$

Example 2 **Use the Binomial Theorem**

Expand $(x + 4)^7$.

The expansion will have eight terms. Use the sequence 1, $\frac{7}{1}$, $\frac{7 \cdot 6}{1 \cdot 2}$, $\frac{7 \cdot 6 \cdot 5}{1 \cdot 2 \cdot 3}$ to find the coefficients for the first four terms. Then use symmetry to find the remaining coefficients.

$$\begin{aligned}&(x + 4)^7 \\ &= 1x^7(4)^0 + \frac{7}{1}x^6(4)^1 + \frac{7 \cdot 6}{1 \cdot 2}x^5(4)^2 + \frac{7 \cdot 6 \cdot 5}{1 \cdot 2 \cdot 3}x^4(4)^3 + \frac{7 \cdot 6 \cdot 5}{1 \cdot 2 \cdot 3}x^3(4)^4 + \frac{7 \cdot 6}{1 \cdot 2}x^2(4)^5 + \frac{7}{1}x^1(4)^6 + 1x^0(4)^7 \\ &= x^7 + 28x^6 + 336x^5 + 2240x^4 + 8960x^3 + 21{,}504x^2 + 28{,}672x + 16{,}384\end{aligned}$$

Example 3 **Factorials**

Evaluate $\frac{16!}{4!12!}$.

$$\begin{aligned}\frac{16!}{4!\ 12!} &= \frac{16 \cdot 15 \cdot 14 \cdot 13 \cdot \cancel{12} \cdot \cancel{11} \cdot \cancel{10} \cdot \cancel{9} \cdot \cancel{8} \cdot \cancel{7} \cdot \cancel{6} \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{4 \cdot 3 \cdot 2 \cdot 1 \cdot \cancel{12} \cdot \cancel{11} \cdot \cancel{10} \cdot \cancel{9} \cdot \cancel{8} \cdot \cancel{7} \cdot \cancel{6} \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}} \\ &= 1820\end{aligned}$$

Example 4 Use a Factorial Form of the Binomial Theorem

Expand $(5x - 2)^6$.

$$(5x - 2)^6 = \sum_{k=0}^{6} \frac{6!}{(6-k)!k!}(5x)^{6-k}(-2)^k$$ Binomial Theorem, factorial form

$$= \frac{6!}{6!\ 0!}(5x)^6(-2)^0 + \frac{6!}{5!\ 1!}(5x)^5(-2)^1 + \frac{6!}{4!\ 2!}(5x)^4(-2)^2 + \frac{6!}{3!\ 3!}(5x)^3(-2)^3 + \frac{6!}{2!\ 4!}(5x)^2(-2)^4 +$$

$$\frac{6!}{1!\ 5!}(5x)^1(-2)^5 + \frac{6!}{0!\ 6!}(5x)^0(-2)^6$$ Let k = 0, 1, 2, 3, 4, 5, and 6.

$$= \frac{6\cdot5\cdot4\cdot3\cdot2\cdot1}{6\cdot5\cdot4\cdot3\cdot2\cdot1\cdot1}(15{,}625x^6)(1) + \frac{6\cdot5\cdot4\cdot3\cdot2\cdot1}{5\cdot4\cdot3\cdot2\cdot1\cdot1}(3125x^5)(-2) + \frac{6\cdot5\cdot4\cdot3\cdot2\cdot1}{4\cdot3\cdot2\cdot1\cdot2\cdot1}(625x^4)(4) +$$

$$\frac{6\cdot5\cdot4\cdot3\cdot2\cdot1}{3\cdot2\cdot1\cdot3\cdot2\cdot1}(125x^3)(-8) + \frac{6\cdot5\cdot4\cdot3\cdot2\cdot1}{2\cdot1\cdot4\cdot3\cdot2\cdot1}(25x^2)(16) + \frac{6\cdot5\cdot4\cdot3\cdot2\cdot1}{1\cdot5\cdot4\cdot3\cdot2\cdot1}(5x^1)(-32) +$$

$$\frac{6\cdot5\cdot4\cdot3\cdot2\cdot1}{1\cdot6\cdot5\cdot4\cdot3\cdot2\cdot1}(1)(64)$$

$$= 15{,}625x^6 - 37{,}500x^5 + 37{,}500x^4 - 20{,}000x^3 + 6000x^2 - 960x + 64$$ Simplify.

Example 5 Find a Particular Term

Find the seventh term in the expansion of $(m - 5)^8$.

First, use the Binomial Theorem to write the expansion in sigma notation.

$$(m - 5)^8 = \sum_{k=0}^{8} \frac{8!}{(8-k)!\ k!}\ m^{8-k}(-5)^k$$

In the seventh term, $k = 6$.

$$\frac{8!}{(8-k)!\ k!}\ m^{8-k}(-5)^k = \frac{8!}{(8-6)!\ (6)!}m^{8-6}(-5)^6$$ $k = 6$

$$= \frac{8\cdot7}{2\cdot1}m^2(-5)^6$$ $\frac{8!}{2!\ 6!} = \frac{8\cdot7\cdot6!}{2!6!}$ or $\frac{8\cdot7}{2\cdot1}$

$$= 437{,}500m^2$$ Simplify.