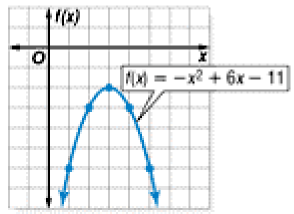
Lesson 6–1

Example 1 Graph a Quadratic Function

Graph $f(x) = -x^2 + 6x - 11$ by making a table of values.

First, choose integer values for x. Then, evaluate the function for each x value. Graph the resulting coordinate pairs and connect the points with a smooth curve.

x	$-x^2 + 6x - 11$	$f(x)$	$(x, f(x))$
1	$-(1)^2 + 6(1) - 11$	–6	(1, –6)
2	$-(2)^2 + 6(2) - 11$	–3	(2, –3)
3	$-(3)^2 + 6(3) - 11$	–2	(3, –2)
4	$-(4)^2 + 6(4) - 11$	–3	(4, –3)
5	$-(5)^2 + 6(5) - 11$	–6	(5, –6)



Example 2 Axis of Symmetry, y–Intercept, Vertex

Consider the quadratic function $f(x) = 6 + 5x - x^2$.

a. Find the y–intercept, the equation of the axis of symmetry, and the x–coordinate of the vertex.

Begin by rearranging the terms of the function so that the quadratic term is first, the linear term is second, and the constant term is last. Then identify a, b, and c.

$$f(x) = ax^2 + bx + c$$

$$f(x) = 6 + 5x - x^2 \quad \rightarrow \quad f(x) = -x^2 + 5x + 6$$

So, $a = -1$, $b = 5$, and $c = 6$.

The y–intercept is 6. You can find the equation of the axis of symmetry using a and b.

$x = -\frac{b}{2a}$ Equation of the axis of symmetry

$x = -\frac{5}{2(-1)}$ $a = -1$, $b = 5$

$x = \frac{5}{2}$ or 2.5 Simplify.

The equation of the axis of symmetry is $x = 2.5$. Therefore, the x–coordinate of the vertex is 2.5.

b. Make a table of values that includes the vertex.

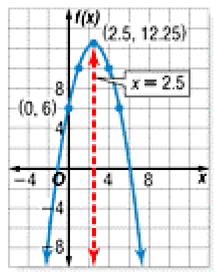
Choose some values for x that are less than 2.5 and some that are greater than 2.5. This ensures that points on each side of the axis of symmetry are graphed.

x	$-x^2 + 5x + 6$	$f(x)$	$(x, f(x))$
0	$-(0)^2 + 5(0) + 6$	6	(0, 6)
1	$-(1)^2 + 5(1) + 6$	10	(1, 10)
2.5	$-(2.5)^2 + 5(2.5) + 6$	12.25	(2.5, 12.25)
4	$-(4)^2 + 5(4) + 6$	10	(4, 10)
5	$-(5)^2 + 5(5) + 6$	6	(5, 6)

← Vertex

c. Use this information to graph the function.

Graph the vertex and the y–intercept. Then graph the points from your table connecting them and the y–intercept with a smooth curve. As a check, draw the axis of symmetry, $x = 2.5$, as a dashed line. The graph of the function should be symmetrical about this line.

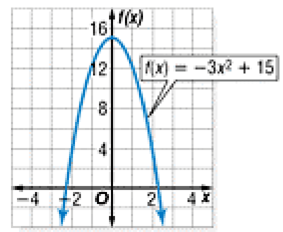


Example 3 **Maximum or Minimum Value**

Consider the function $f(x) = -3x^2 + 15$.

a. Determine whether the function has a maximum or a minimum value.

For this function, $a = -3$, $b = 0$, and $c = 15$. Since $a < 0$, the graph opens down and the function has a maximum value.



b. State the maximum or minimum value of the function.

The maximum value of the function is the y–coordinate of the vertex.

The x–coordinate of the vertex is $-\frac{0}{2(-3)}$ or 0.

Find the y–coordinate of the vertex by evaluating the function for $x = 0$.

$f(x) = -3x^2 + 15$	Original function
$f(0) = -3(0)^2 + 15$ or 15	$x = 0$

Therefore, the maximum value of the function is 15.

Example 4 Find a Maximum Value

ENTERTAINMENT An amusement park currently sells an average of 650 all–day ride passes per day. The price of each pass is $12.00. The owner would like to increase the price of each pass and has hired a research company to help her determine the effect of a price increase. The research company determined that for each $1.50 increase in the price, 50 fewer people will buy passes for the park.

a. How much should the passes cost in order to maximize the income for the amusement park?

Words The income is the number of tickets multiplied by the price per ticket.

Variables Let x = the number of $1.50 price increases. Then $12 + 1.50x$ = the price per ticket and $650 - 50x$ = the number of tickets sold. Let $I(x)$ = income as a function of x.

Equation: The income | is | the number of tickets | multiplied by | the price per ticket

$$I(x) = (650 - 50x) \cdot (12 + 1.50x)$$

$= 650(12) + 650(1.50x) - 50x(12) - 50x(1.50x)$

$= 7800 + 975x - 600x - 75x^2$ Multiply.

$= 7800 + 375x - 75x^2$ Simplify.

$= -75x^2 + 375x + 7800$ Rewrite in $ax^2 + bx + c$ form.

$I(x)$ is a quadratic function with $a = -75$, $b = 375$, and $c = 7800$. Since $a < 0$, the function has a maximum value at the vertex of the graph. The x–coordinate of the vertex is $-\dfrac{375}{2(-75)}$ or 2.5.

This means the amusement park should make 2.5 price increases of $1.50 to maximize their income. Thus, the ticket price should be 12 + 1.50(2.5) or $15.75.

b. What is the maximum income the amusement park owner can expect to make?

To determine maximum income, find the maximum value of the function by evaluating $I(x)$ for $x = 2.5$.

$I(x) = -75x^2 + 375x + 7800$ Income function

$I(2.5) = -75(2.5)^2 + 375(2.5) + 7800$ $x = 2.5$

$= 8268.75$ Use a calculator.

Thus, the maximum income the amusement park can expect is $8268.75. Graph this function on a graphing calculator and use the CALC menu to confirm this solution.