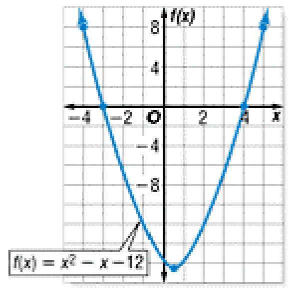
Lesson 6–2

Example 1 Two Real Solutions

Solve $x^2 - x - 12 = 0$ by graphing.

Graph the related quadratic function $f(x) = x^2 - x - 12$. The equation of the axis of symmetry is $x = -\frac{-1}{2(1)}$ or $\frac{1}{2}$. Make a table using x–values around $\frac{1}{2}$. Then, graph each point.

x	–4	–3	$\frac{1}{2}$	4	5
$f(x)$	8	0	$-12\frac{1}{4}$	0	8



From the table and the graph, you can see that the zeros of the function are –3 and 4. Therefore, the solutions of the equation are –3 and 4.

CHECK: Check your solutions by substituting each solution into the equation to see if it is satisfied.

$$x^2 - x - 12 = 0 \qquad\qquad x^2 - x - 12 = 0$$

$$(-3)^2 - (-3) - 12 \stackrel{?}{=} 0 \qquad\qquad (4)^2 - (4) - 12 \stackrel{?}{=} 0$$

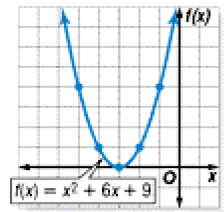
$$0 = 0 \ \checkmark \qquad\qquad 0 = 0 \ \checkmark$$

Example 2 **One Real Solution**
Solve $x^2 + 9 = -6x$ by graphing.

Write the equation in the $ax^2 + bx + c = 0$ form.

$x^2 + 9 = -6x \quad \rightarrow \quad x^2 + 6x + 9 = 0$ Add 6*x* to each side.

Graph the related quadratic function
$f(x) = x^2 + 6x + 9$.



x	–5	–4	–3	–2	–1
***f*(x)**	4	1	0	1	4

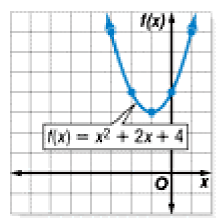
Notice that the graph has only one *x*–intercept, –3.
Thus, the equation's only solution is –3.

Example 3 **No Real Solution**
Solve $x^2 + 2x = -4$ by graphing.

Write the equation in $ax^2 + bx + c = 0$ form.

$x^2 + 2x = -4 \quad \rightarrow \quad x^2 + 2x + 4 = 0$ Add 4 to each side.

Graph the related quadratic function
$f(x) = x^2 + 2x + 4$.



x	–3	–2	–1	0	1
f(*x*)	7	4	3	4	7

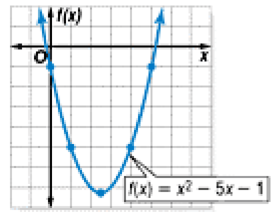
Notice that the graph has no *x*–intercepts. This means that the original equation has no real solution.

Example 4 Estimate Roots

Solve $x^2 - 5x - 1 = 0$ by graphing. If exact roots cannot be found, state the consecutive integers between which the roots are located.

The equation of the axis of symmetry of the related function is $x = -\frac{-5}{2(1)}$ or 2.5.

x	0	1	2.5	5	6
***f*(*x*)**	-1	–5	–7.25	-1	5



The x–intercepts of the graph are between –1 and 0 and between 5 and 6. So, one solution is between –1 and 0, and the other is between 5 and 6.

Example 5 **Write and Solve an Equation**

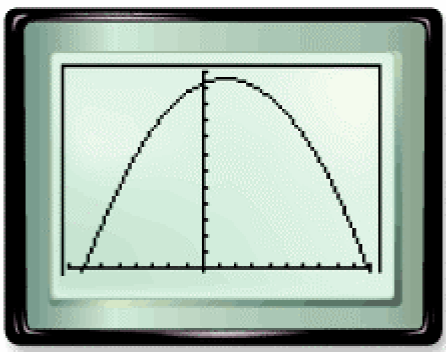
BUILDINGS The John Hancock Center is a commercial and residential building in Chicago, IL. Suppose a ball is tossed straight up from the top of the 1,127–foot tall structure with an initial speed of 44 feet per second. The height $h(t)$ of the ball t seconds after it is thrown is given by $h(t) = -16t^2 + at + b$, where a is the initial speed of the ball and b is the initial height from which it is thrown. How long was the ball in the air before it hit the street below?

You need to find t when $a = 44$, $b = 1127$, and $h(t) = 0$. (The height of the ball will be 0 when it hits the street.) Substitute these values into the given function.

$h(t) = -16t^2 + at + b$ Given function

$0 = -16t^2 + 44t + 1127$ Replace a with 44, b with 1127, and $h(t)$ with 0.

Graph the related function $y = -16t^2 + 44t + 1127$ using a graphing calculator. Adjust your window so that the x–intercepts of the graph are visible.



Use the ZERO feature, 2nd [CALC], to find the positive zero of the function, since time cannot be negative. Use the arrow keys to locate a left bound for the zero and press ENTER. Then, locate a right bound and press ENTER twice. The positive zero of the function is approximately 9.9. The ball will be in the air about 10 seconds before hitting the street.