**Lesson 6–6**

## Example 1 Graph a Quadratic Function in Vertex Form

**Analyze $y = (x - 3)^2 - 2$. Then draw its graph.**

This function can be rewritten as $y = [x - (3)]^2 - 2$. Then $h = 3$ and $k = -2$.

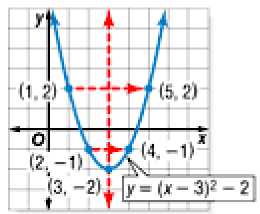
The vertex is at $(h, k)$ or $(3, -2)$, and the axis of symmetry is $x = 3$. The graph has the same shape as the graph of $y = x^2$, but it is translated 3 units right and 2 units down. Now use this information to draw the graph.

**Step 1** Plot the vertex, $(3, -2)$.

**Step 2** Draw the axis of symmetry, $x = 3$.

**Step 3** Find and plot two points on one side of the axis of symmetry, such as $(2, -1)$ and $(1, 2)$.

**Step 4** Use symmetry to complete the graph.




## Example 2 Write $y = x^2 + bx + c$ in Vertex Form

**Write $y = x^2 - 2x + 7$ in vertex form. Then analyze the function.**

| | |
|---|---|
| $y = x^2 - 2x + 7$ | Notice that $x^2 - 2x + 7$ is not a perfect square. |
| | Complete the square by adding $\left(\frac{-2}{2}\right)^2$ or 1. |
| $y = (x^2 - 2x + 1) + 7 - 1$ | Balance this addition by subtracting 1. |
| $y = (x - 1)^2 + 6$ | Write $(x^2 - 2x + 1)$ as a perfect square. |

This function can be rewritten as $y = [x - (1)]^2 + 6$. Written in this way, you can see that $h = 1$ and $k = 6$.

The vertex is at $(1, 6)$, and the axis of symmetry is $x = 1$. Since $a = 1$, the graph opens up and has the same shape as the graph of $y = x^2$, but it is translated 1 unit right and 6 units up.

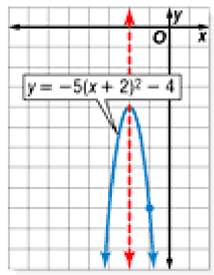
**CHECK:** You can check the vertex and axis of symmetry using the formula $x = -\frac{b}{2a}$. In the original equation, $a = 1$ and $b = -2$, so the axis of symmetry is $x = -\frac{(-2)}{2(1)}$ or 1. Thus, the $x$–coordinate of the vertex is 1, and the $y$–coordinate of the vertex is $y = (1)^2 - 2(1) + 7$ or 6.

**Example 3** **Write $y = ax^2 + bx + c$ in Vertex Form, $a \neq 1$**
**Write $y = –5x^2 – 20x – 24$ in vertex form. Then analyze and graph the function.**

| | |
|---|---|
| $y = –5x^2 – 20x – 24$ | Original equation |
| $y = –5(x^2 + 4x) – 24$ | Group $ax^2 + bx$ and factor, dividing by $a$. |
| $y = –5(x^2 + 4x + 4) – 24 – (–5)(4)$ | Complete the square by adding 4 inside the parentheses. Notice that this is an overall addition of –5(4). Balance this addition by subtracting –5(4). |
| $y = –5(x + 2)^2 – 4$ | Write $x^2 + 4x + 4$ as a perfect square. |

The vertex form of this function is $y = –5[x^2 – (–2)]^2 – 4$. So, $h = –2$ and $k = –4$.

The vertex is at (–2, –4), and the axis of symmetry is $x = –2$. Since $a = –5$, the graph opens downward and is narrower than the graph of $y = x^2$. It is also translated 2 units left and 4 units down.




Now graph the function. Two points on the graph to the right of $x = –2$ are (–1, –9) and (0, –24). Use symmetry to complete the graph.

## Example 4 **Write an Equation Given Points**

**Write an equation for the parabola whose vertex is at (–3, –4) and passes through (–1, 2).**

The vertex of the parabola is at (–3, –4), so $h = -3$ and $k = -4$. Since (–1, 2) is a point on the graph of the parabola, let $x = -1$ and $y = 2$. Substitute these values into the vertex form of the equation and solve for $a$.

| | |
|---|---|
| $y = a(x - h)^2 + k$ | Vertex form |
| $2 = a[-1 - (-3)]^2 + (-4)$ | Substitute 2 for $y$, –1 for $x$, –3 for $h$, and –4 for $k$. |
| $2 = a(4) - 4$ | Simplify. |
| $6 = 4a$ | Add 4 to each side. |
| $1.5 = a$ | Divide each side by 4. |

The equation of the parabola in vertex form is $y = 1.5(x + 3)^2 - 4$.

**CHECK:** A graph of $y = 1.5(x + 3)^2 - 4$ verifies that the parabola passes through the point at (–1, 2).