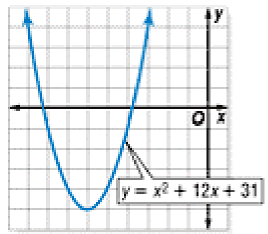
Lesson 6–7

Example 1 Graph a Quadratic Inequality

Graph $y \le x^2 + 12x + 31$.

Step 1 Graph the related quadratic equation, $y = x^2 + 12x + 31$.

Since the inequality symbol is ≤, the parabola should be solid.



Step 2 Test a point outside the parabola, such as (0, 0).

$y \le x^2 + 12x + 31$

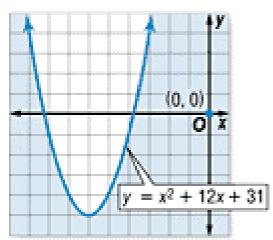
$0 \overset{?}{\le} (0)^2 + 12(0) + 31$

$0 \overset{?}{\le} 0 + 0 + 31$

$0 \le 31$ ✓

So, (0, 0) is a solution of the inequality.

Step 3 Shade the region outside the parabola.



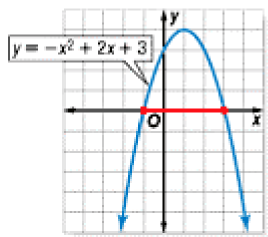
Example 2 **Solve $ax^2 + bx + c \geq 0$**

Solve $-x^2 + 2x + 3 \geq 0$ by graphing.

The solution consists of the x value for which the graph of the related quadratic function lies *on and above* the x-axis. Begin by finding the roots of the related equation.

$-x^2 + 2x + 3 = 0$	Related equation
$-1(x^2 - 2x - 3) = 0$	Factor out -1.
$-1(x + 1)(x - 3) = 0$	Factor the trinomial.
$x + 1 = 0 \quad x - 3 = 0$	Zero Product Property
$x = -1 \quad x = 3$	Solve each equation.

Sketch the graph of a parabola that has x–intercepts at -1 and 3. The graph should open downward since $a < 0$.



The graph lies on the x- axis at $x = -1$ and $x = 3$. It lies above the x-axis between $x = -1$ and $x = 3$. Therefore, the solution set is $\{x \mid -1 \leq x \leq 3\}$.

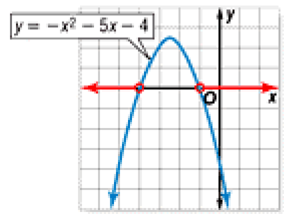
Example 3 Solve $ax^2 + bx + c < 0$

Solve $-x^2 - 5x - 4 < 0$ by graphing.

The solution consists of the x values for which the graph of the related quadratic function lies *below* the x-axis. Begin by finding the roots of the related equation.

$-x^2 - 5x - 4 = 0$	Related equation
$-1(x^2 + 5x + 4) = 0$	Factor out –1.
$-1(x + 4)(x + 1) = 0$	Factor the trinomial.
$x + 4 = 0 \quad x + 1 = 0$	Zero Product Property
$x = -4 \quad x = -1$	Solve each equation.

Sketch the graph of a parabola that has x–intercepts of -4 and -1. The graph should open downward since $a < 0$.



The graph lies below the x-axis to the left of the $x = -4$ and to the right of the $x = -1$. Therefore, the solution set is $\{x | x < -4 \text{ or } x > -1\}$.

CHECK: Test one value of x less than -4, one between -4 and -1, and one greater than -1 in the original inequality.

Test $x = -5$.

$-x^2 - 5x - 4 < 0$

$-(-5)^2 - 5(-5) - 4 \overset{?}{<} 0$

$-4 < 0$ ✓

Test $x = -2$.

$-x^2 - 5x - 4 < 0$

$-(-2)^2 - 5(-2) - 4 \overset{?}{<} 0$

$2 < 0$ X

Test $x = 0$.

$-x^2 - 5x - 4 < 0$

$-(0)^2 - 5(0) - 4 \overset{?}{<} 0$

$-4 < 0$ ✓

Example 4 Write an Inequality

NUMBER THEORY The product of two consecutive even integers is greater than 24. What are the possible values for the two integers?

Explore Let x = one of the integers. Then the next consecutive even integer = $x + 2$.

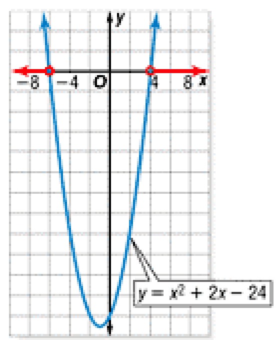
Plan Since the product of the two numbers is greater than 24, you know that $x(x + 2) > 24$.

$x(x + 2) > 24$	Original inequality
$x^2 + 2x > 24$	Distributive Property
$x^2 + 2x - 24 > 0$	Subtract 24 from each side.

Solve Solve the related equation.

$x^2 + 2x - 24 = 0$	Related equation
$(x + 6)(x - 4) = 0$	Factor the trinomial.
$x + 6 = 0 \qquad x - 4 = 0$	Zero Product Property
$x = -6 \qquad x = 4$	Solve each equation.

Sketch the graph of a parabola that has x–intercepts at –6 and 4. The graph should open up since $a > 0$.



The graph lies above the x-axis to the left of the x = -6 and to the right of the x = 4. Therefore, the solution set of $x^2 + 2x - 24 > 0$ is $\{x| x < -6 \text{ or } x > 4\}$. Since the problem specifies solutions that are even integers, the solution set is any pair of even consecutive integers such that the least of the pair is less than or equal to –8 or is greater than or equal to 6.

Examine If the least of the pair is –8, then the other even integer is –8 + 2 = –6. The product is (–8)(–6) or 48 which is greater than 24. If the least of the pair is 6, then the other even integer is 6 + 2 = 8. The product is (6)(8) or 48 which is greater than 24.

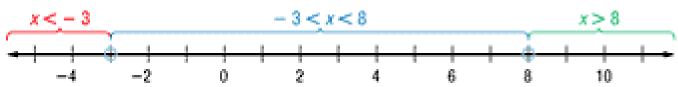
Example 5 **Solve a Quadratic Inequality**

Solve $-x^2 + 5x \leq -24$ algebraically.

First solve the related quadratic equation $-x^2 + 5x = -24$.

$-x^2 + 5x = -24$	Related quadratic equation
$-x^2 + 5x + 24 = 0$	Add 24 to each side.
$x^2 - 5x - 24 = 0$	Multiply each side by –1 to make factoring easier.
$(x - 8)(x + 3) = 0$	Factor.
$x - 8 = 0$ or $x + 3 = 0$	Zero Product Property
$x = 8$ $\quad$ $x = -3$	Solve each equation.

Plot –3 and 8 on a number line. Use dots since these values are solutions of the original inequality. Notice that the number line is now separated into three intervals.



Test a value in each interval to see if it satisfies the original inequality.

$x < -3$	$-3 < x < 8$	$x > 8$
Test $x = -4$.	Test $x = 0$.	Test $x = 9$.
$-x^2 + 5x \leq -24$	$-x^2 + 5x \leq -24$	$-x^2 + 5x \leq -24$
$-(-4)^2 + 5(-4) \overset{?}{\leq} -24$	$-(0)^2 + 5(0) \overset{?}{\leq} -24$	$-(9)^2 + 5(9) \overset{?}{\leq} -24$
$-36 \leq -24$ ✓	$0 \leq -24$ **X**	$-36 \leq -24$ ✓

The two intervals $x < -3$ and $x > 8$ satisfy the inequality. The two points –3 and 8 also satisfy the inequality. Therefore, the solution set is $\{x \mid x \leq -3 \text{ or } x \geq 8\}$. This is shown on the number line below.