Lesson 7–6

Example 1 Identify Possible Zeros

List all of the possible rational zeros of each function.

a. $p(x) = x^4 + 4x^3 - x^2 + 3x - 72$

Since the coefficient of x^4 is 1, the possible rational zeros must be a factor of the constant term 72. So, the possible rational zeros are the integers ±1, ±2, ±3, ±4, ±6, ±8, ±9, ±12, ±18, ±24, ±36, and ±72.

b. $h(x) = 3x^5 - 2x^3 + x + 2$

If $\frac{p}{q}$ is a rational root, then p is a factor of 2 and q is a factor of 3. The possible values of p are ±1 and ±2. The possible values of q are ±1 and ±3. So all of the possible rational zeros are as follows.

$\frac{p}{q} = \pm 1, \pm 2, \pm\frac{1}{3}$, and $\pm\frac{2}{3}$.

Example 2 Use the Rational Zero Theorem

Find all of the rational zeros for $h(x) = x^3 - 2x^2 - 29x + 30$.

The leading coefficient is 1, so the possible integer zeros are factors of 30, ±1, ±2, ±3, ±5, ±6, ±10, ±15, and ±30. From Descartes' Rule of Signs, you can tell that there are 2 or 0 positive real zeros. Make a table and test possible real zeros.

x	1	–2	–29	30
1	1	–1	–30	0

Since $(x - 1)$ is found to be a factor, try to factor the depressed polynomial so that other zeros do not have to be tested.

$$x^3 - 2x^2 - 29x + 30 = (x - 1)(x^2 - x - 30)$$
$$= (x - 1)(x + 5)(x - 6)$$

Now, use the Zero Product Property to find all of the zeros.

$(x - 1)(x + 5)(x - 6) = 0$

$x - 1 = 0$ or $x + 5 = 0$ or $x - 6 = 0$

$x = 1$ $x = -5$ $x = 6$

The rational zeros of this function are –5, 1, and 6.

Example 3 Find All Zeros

Find all the zeros of $g(x) = x^4 - x^3 - 11x^2 - x - 12$.

From the corollary to the Fundamental Theorem of Algebra, you know there are exactly 4 complex roots. According to Descartes' Rule of Signs, there is exactly 1 positive real root and 3 or 1 negative real roots. The possible rational zeros are ±1, ±2, ±3, ±4, ±6, and ±12. Make a table and test some possible rational zeros.

x	1	–1	–11	–1	–12
1	1	0	–11	–12	–24
2	1	1	–9	–19	–50
3	1	2	–5	–16	–60
4	1	3	1	3	0
–1	1	–2	–9	8	–20
–2	1	–3	–5	9	–30
–3	1	–4	1	–4	0

Since $f(4) = 0$, you know that $x = 4$ is a zero. The depressed polynomial is $x^3 + 3x^2 + x + 3$. You also know that $f(-3)$ is a zero, so $(x + 3)$ will be a factor of $x^3 + 3x^2 + x + 3$.

Factor $x^4 - x^3 - 11x^2 - x - 12$ completely since two of the factors are known.

$x^4 - x^3 - 11x^2 - x - 12 = 0$ — Original equation

$(x - 4)(x^3 + 3x^2 + x + 3) = 0$ — One factor is $(x - 4)$.

$(x - 4)(x + 3)(x^2 + 1) = 0$ — Another factor is $(x + 3)$.

$x - 4 = 0$ or $x + 3 = 0$ or $x^2 + 1 = 0$ — Zero Product Property

$x = 4$ $\quad$ $x = -3$ $\quad$ $x^2 = -1$ — Solve each equation.

$x = \pm\sqrt{-1}$

$x = \pm \boldsymbol{i}$

There is one positive real zero at $x = 4$, one negative real zero at $x = -3$, and two imaginary zeros at $x = \boldsymbol{i}$ and $x = -\boldsymbol{i}$. The zeros of this function are –3, 4, $\boldsymbol{i}$, and $-\boldsymbol{i}$.