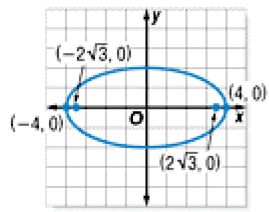
Lesson 8–4

Example 1 Write an Equation for a Graph

Write an equation for the ellipse shown below.

The length of the major axis is the distance between the points at (4, 0) and (–4, 0). This distance is 8 units.

$2a = 8$ Length of major axis = 8.
$a = 4$ Divide each side by 2.



The foci are located at $(2\sqrt{3}, 0)$ and $(-2\sqrt{3}, 0)$, so $c = 2\sqrt{3}$.

$c^2 = a^2 - b^2$ Equation relating a, b, and c
$12 = 16 - b^2$ $c = 2\sqrt{3}$ and $a = 4$
$b^2 = 4$ Solve for b^2.

Since the major axis is horizontal, substitute 16 for a^2 and 4 for b^2 in the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. An equation of the ellipse is $\frac{x^2}{16} + \frac{y^2}{4} = 1$.

Example 2 Write an Equation Given the Lengths of the Axes

Write an equation for the ellipse with endpoints of the major axis at (0, 5) and (0, –5) and the endpoints of the minor axis at (-2, 0) and (2, 0).

The length of the major axis is 5 – (–5) or 10 units.

$2a = 10$ Length of major axis = 10
$a = 5$ Divide each side by 2.

The length of the minor axis is 2 – (-2) or 4 units.

$2b = 4$ Length of minor axis = 4
$b = 2$ Divide each side by 2.

The major axis is vertical, so substitute $a = 5$ and $b = 2$ into the form $\frac{y^2}{a^2} + \frac{x^2}{b^2} = 1$. An equation of the ellipse is $\frac{y^2}{25} + \frac{x^2}{4} = 1$.

Example 3 Graph an Equation in Standard Form

Find the coordinates of the center and foci and the lengths of the major and minor axes of the ellipse with equation $\frac{(y-3)^2}{24} + \frac{(x+1)^2}{4} = 1$**. Then graph the ellipse.**

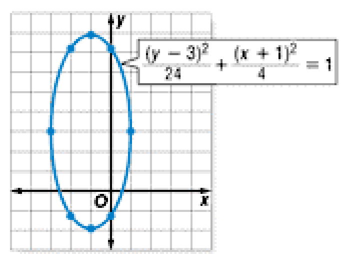
The center of this ellipse is at (–1, 3).
Since $a^2 = 24$, $a = 2\sqrt{6}$. Since $b^2 = 4$, $b = 2$.

The length of the major axis is $2(2\sqrt{6})$ or $4\sqrt{6}$ units, and the length of the minor axis is 2(2) or 4 units. Since the y^2 term has the greater denominator, the major axis is vertical.

$c^2 = a^2 - b^2$	Equation relating a, b, and c
$c^2 = (2\sqrt{6})^2 - 2^2$ or 20	$a = 2\sqrt{6}$, $b = 2$
$c = \sqrt{20}$ or $2\sqrt{5}$	Take the square root of each side.

The foci are at $(-1, 3 + 2\sqrt{5})$ and $(-1, 3 - 2\sqrt{5})$.

You can use a calculator to find some approximate values for x and y that satisfy the equation.



x	y
0	–1.2
0	7.2
–2	–1.2
–2	7.2

Graph the vertices, $(-1, 3 + 2\sqrt{6})$, $(-1, 3 - 2\sqrt{6})$, $(-1 - 2, 3)$, and $(-1 + 2, 3)$, and draw the ellipse that passes through them and the other points.

Example 4 Graph an Equation not in Standard Form

Find the coordinates of the center and foci and the lengths of the major and minor axes of the ellipse with equation $4y^2 + 5x^2 - 24y + 20x - 124 = 0$. Then graph the ellipse.

Complete the square for each variable to write this equation in standard form.

$4y^2 + 5x^2 - 24y + 20x - 124 = 0$	Original equation
$4y^2 - 24y + 5x^2 + 20x - 124 = 0$	Group *x*– and *y*–terms.
$4(y^2 - 6y + \square) + 5(x^2 + 4x + \square) = 124 + 4(\square) + 5(\square)$	Complete the squares.
$4(y^2 - 6y + 9) + 5(x^2 + 4x + 4) = 124 + 4(9) + 5(4)$	$\left(\frac{6}{2}\right)^2 = 9$, $\left(\frac{4}{2}\right)^2 = 4$
$4(y - 3)^2 + 5(x + 2)^2 = 180$	Write the trinomials as perfect squares.
$\frac{(y-3)^2}{45} + \frac{(x+2)^2}{36} = 1$	Divide each side by 180.

The center of this ellipse is at (–2, 3) and the foci are at (–2, 6) and (–2, 0). The length of the major axis is $6\sqrt{5}$ units, and the length of the minor axis is 12.