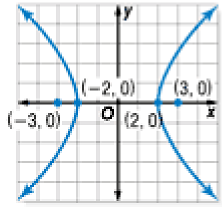
Lesson 8–5

Example 1 **Write an Equation for a Graph**

Write an equation for the hyperbola shown at the right.



The center is the midpoint of the segment connecting the vertices, or (0, 0).

The value of *a* is the distance from the center to a vertex, or 2 units. The value of *c* is the distance from the center to a focus, or 3 units.

$c^2 = a^2 + b^2$	Equation relating *a*, *b*, and *c* for a hyperbola
$3^2 = 2^2 + b^2$	$c = 3$, $a = 2$
$9 = 4 + b^2$	Evaluate squares.
$5 = b^2$	Solve for b^2.

Since the transverse axis is horizontal, the equation is of the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. Substitute the values for a^2 and b^2. An equation of the hyperbola is $\frac{x^2}{4} - \frac{y^2}{5} = 1$.

Example 2 **Write an Equation Given the Vertices and Conjugate Axis**

Write an equation for a hyperbola with vertices at (0, –8) and (0, 8) with conjugate axis of length 10 units.

The conjugate axis is a segment of length 2*b* units that is perpendicular to the transverse axis at the center. Since $2b = 10$, $b = 5$.

The transverse axis is a segment of length 2*a* whose endpoint are the vertices of the hyperbola. Since the distance between (0, –8) and (0, 8) is 16 units, $2a = 16$, and $a = 8$.

Since the vertices are (0, –8) and (0, 8), the center is (0, 0) and the axis is vertical. The equation of the hyperbola is of the form $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$. Substitute the values for a^2 and b^2. An equation of the hyperbola is $\frac{y^2}{64} - \frac{x^2}{25} = 1$.

Example 3 **Graph an Equation in Standard From**

Find the coordinates of the vertices and foci and the equations of the asymptotes for the hyperbola with equation $\frac{(y+4)^2}{9} - \frac{(x-1)^2}{36} = 1$. Then graph the hyperbola.

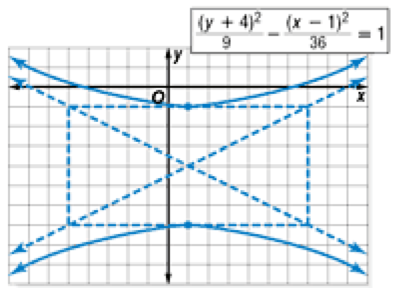
The center of this hyperbola is at (1, –4). According to the equation $a^2 = 9$ and $b^2 = 36$, so $a = 3$ and $b = 6$. The coordinates of the vertices are (1, –1) and (1, –7).

$c^2 = a^2 + b^2$	Equation relating *a*, *b*, and *c* for a hyperbola
$c^2 = 3^2 + 6^2$	$a = 3$, $b = 6$
$c^2 = 9 + 36$	Evaluate squares.
$c^2 = 45$	Solve for c^2.
$c = \sqrt{45}$ or $3\sqrt{5}$	Take the square root of each side.

The foci are at $(1, -4 + 3\sqrt{5})$ and $(1, -4 - 3\sqrt{5})$.

The equations of the asymptotes are $y - k = \pm\frac{a}{b}(x - h)$ or $y + 4 = \pm\frac{1}{2}(x - 1)$.

Draw a 12–unit by 6–unit rectangle. The asymptotes contain the diagonals of the rectangle. Graph the vertices. Use the asymptotes as a guide to draw the hyperbola that passes through the vertices. The graph does not intersect the asymptotes.



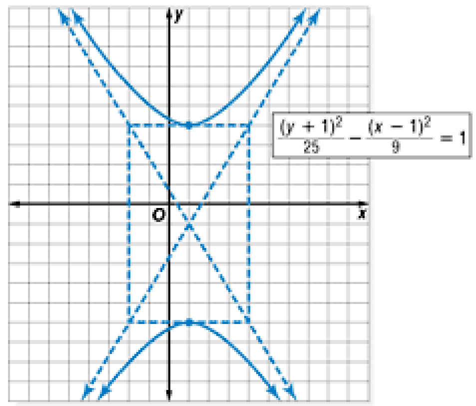
Example 4 Graph an Equation Not in Standard Form

Find the coordinates of the vertices and foci and the equations of the asymptotes for the hyperbola with equation $9y^2 - 25x^2 + 18y + 50x - 241 = 0$. Then graph the hyperbola.

Complete the square for each variable to write this equation in standard form.

$9y^2 - 25x^2 + 18y + 50x - 241 = 0$	Original equation
$9(y^2 + 2y + \quad) - 25(x^2 - 2x + \quad) = 241 + 9(\quad) - 25(\quad)$	Complete the squares.
$9(y^2 + 2y + 1) - 25(x^2 - 2x + 1) = 241 + 9(1) - 25(1)$	
$9(y + 1)^2 - 25(x - 1)^2 = 225$	Write the trinomials as perfect squares.
$\frac{(y+1)^2}{25} - \frac{(x-1)^2}{9} = 1$	Divide each side by 225.

The center of this hyperbola is at (1, –1). According to the equation, $a^2 = 25$ and $b^2 = 9$, so $a = 5$ and $b = 3$. Using the relationship between a, b, and c for a hyperbola, $c^2 = 25 + 9 = 34$, and $c = \sqrt{34}$. The coordinates of the vertices are (1, 4) and (1, –6) and the foci are at $(1, -1 - \sqrt{34})$ and $(1, -1 + \sqrt{34})$. The equations of the asymptotes are $y - k = \pm\frac{a}{b}(x - h)$ or $y + 1 = \pm\frac{5}{3}(x - 1)$.



Draw a 6–unit by 10–unit rectangle. The asymptotes contain the diagonals of the rectangle. Graph the vertices and use the asymptotes as a guide to draw the hyperbola that passes through the vertices. The graph does not intersect the asymptotes.