Lesson 1–6

Example 1 Solve an "and" Compound Inequality

Solve $-10 \le 4x - 2 \le 10$. Graph the solution set on a number line.

Method 1

Write the compound inequality using the word *and*. Then solve each inequality.

$$\begin{array}{lcl} -10 \le 4x - 2 & \text{and} & 4x - 2 \le 10 \\ -8 \le 4x & & 4x \le 12 \\ -2 \le x & & x \le 3 \end{array}$$

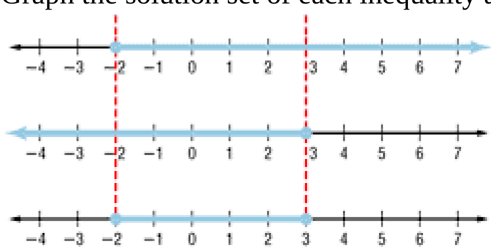
$$-2 \le x \le 3$$

Method 2

Solve both parts at the same time by adding 2 to each part of the inequality. Then divide each part by 4.

$$\begin{array}{rcccl} -10 & \le & 4x - 2 & \le & 10 \\ -10 + 2 & \le & 4x - 2 + 2 & \le & 10 + 2 \\ -8 & \le & 4x & \le & 12 \\ -\frac{8}{4} & \le & \frac{4x}{4} & \le & \frac{12}{4} \\ -2 & \le & x & \le & 3 \end{array}$$

Graph the solution set of each inequality and find their intersection.



$-2 \le x$

$x \le 3$

$-2 \le x \le 3$

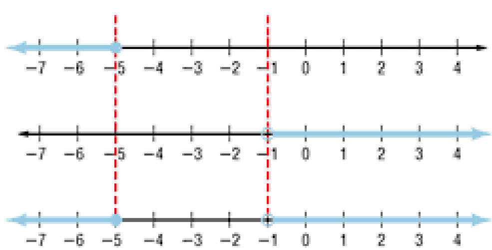
The solution set is $\{x \mid -2 \le x \le 3\}$.

Example 2 Solve an "or" Compound Inequality

Solve $2n + 6 \le -4$ or $5n - 1 > 4$. Graph the solution set on a number line.

Solve each pair separately.

$$\begin{array}{rcr} 2n + 6 \le -4 & \text{or} & 5n - 1 > 4 \\ 2n \le -10 & & 5n > 5 \\ n \le -5 & & n > 1 \end{array}$$



$n \le -5$

$n > 1$

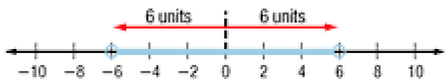
$n \le -5$ or $n > 1$

The solution set is $\{n \mid n \le -5 \text{ or } n > 1\}$.

Example 3 **Solve an Absolute Value Inequality (<)**

Solve $|c| < 6$. Graph the solution set on a number line.

$|c| < 6$ means that the distance between c and 0 on a number line is less than 6 units. To make $|c| < 6$ true, you must substitute numbers for c that are fewer than 6 units from 0.



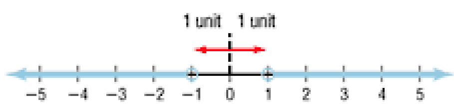
Notice that the graph of $|c| < 6$ is the same as the graph of $c > –6$ and $c < 6$.

All of the numbers between –6 and 6 are less than 6 units from 0. The solution set is $\{c \mid –6 < c < 6\}$.

Example 4 **Solve an Absolute Value Inequality (>)**

Solve $|t| > 1$. Graph the solution set on a number line.

$|t| > 1$ means that the distance between t and 0 is greater than 1 unit. To make $|t| > 1$ true, you must substitute values for t that are greater than 1 unit from 0.



Notice that the graph of $|t| > 1$ is the same as the graph of $t > 1$ or $t < –1$.

All of the numbers *not* between –1 and 1 are greater than 1 unit from 0. The solution set is $\{t \mid t > 1 \text{ or } t < –1\}$.

Example 5 **Solve a Multi–Step Absolute Value Inequality.**

Solve $|5s + 5| \leq 20$. Graph the solution set on a number line.

$|5s + 5| \leq 20$ is equivalent to $–20 \leq 5s + 5 \leq 20$. Solve each inequality or solve both parts of the inequality at the same time. The solution below shows solving both parts of the inequality at the same time.

$$\begin{aligned}
–20 &\leq 5s + 5 \leq 20 \\
–20 – 5 &\leq 5s + 5 – 5 \leq 20 – 5 \\
–25 &\leq 5s \leq 15 \\
\frac{–25}{5} &\leq \frac{5s}{5} \leq \frac{15}{5} \\
–5 &\leq s \leq 3
\end{aligned}$$

The solution set is $\{s \mid –5 \leq s \leq 3\}$.

Example 6 **Apply Absolute Value Inequalities**

ASTRONOMY The average distance of the planet Pluto from the Sun is 3647.5 million miles. However, since the orbit is not circular, the distance can differ by as much as 891.5 million miles.
Source: The World Almanac

a. Write an absolute value inequality to describe this situation.
Let d = the distance of Pluto from the Sun.

The distance can differ from the average	by as much as	891.5 miles.
$\lvert 3647.5 - d \rvert$	$\leq$	891.5

b. Solve the inequality to find the range of Pluto's distance from the Sun.
Rewrite the absolute value inequality as a compound inequality. Then solve for d.

$$-891.5 \leq 3647.5 - d \leq 891.5$$
$$-891.5 - 3647.5 \leq 3647.5 - d - 3647.5 \leq 891.5 - 3647.5$$
$$-4539 \leq -d \leq -2756$$
$$4539 \geq d \geq 2756$$

The solution set is $\{d \mid 2756 \leq d \leq 4539\}$. Thus, the distance of Pluto from the Sun will fall between 2756 million miles and 4539 million miles, inclusive.