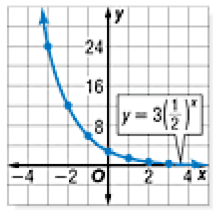
Lesson 10–1

Example 1 Graph an Exponential Function

Sketch the graph of $y = 3\left(\frac{1}{2}\right)^x$. Then state the function's domain and range.

Make a table of values. Connect the points to sketch a smooth curve.

x	$y = 3\left(\frac{1}{2}\right)^x$
–3	$3\left(\frac{1}{2}\right)^{-3} = 24$
–2	$3\left(\frac{1}{2}\right)^{-2} = 12$
–1	$3\left(\frac{1}{2}\right)^{-1} = 6$
0	$3\left(\frac{1}{2}\right)^{0} = 3$
1	$3\left(\frac{1}{2}\right)^{1} = 1.5$
2	$3\left(\frac{1}{2}\right)^{2} = 0.75$
3	$3\left(\frac{1}{2}\right)^{3} = 0.375$



The domain of the function is all real numbers, while the range is the set of all positive numbers.

Example 2 Identify Exponential Growth and Decay

Determine whether each function represents exponential *growth* or *decay*.

Function	Exponential Growth or Decay?
a. $y = 2\left(\frac{1}{6}\right)^x$	The function represents exponential decay, since the base, $\frac{1}{6}$, is between 0 and 1.
b. $y = 0.4(7)^x$	The function represents exponential growth, since the base, 7, is greater than 1.
c. $y = 3.4(0.1)^x$	The function represents exponential decay, since the base, 0.1, is between 0 and 1.

Example 3 Write an Exponential Function

Write an exponential function whose graph passes through the given points.

a. (0, –1) and (2, –9)

Since the graph of the function contains the point (0, –1), the *y*-intercept, and value of *a*, is –1.

Substitute the values for *x* and *y* from the point (2, –9) into an exponential function to find the value of *b*.

$y = ab^x$	Exponential function
$-9 = (-1)b^2$	Replace *x* with 2, *y* with –9, and *a* with –1.
$9 = b^2$	Divide each side by –1
$3 = b$	Take the square root of each side.

An exponential function whose graph passes through the points (0, –1) and (2, –9) is $y = (-1)(3)^x$.

b. (0, 6) and (2, 24)

Since the graph of the function contains the point (0, 6), the *y*-intercept, and value of *a*, is 6.

Substitute the values for *x* and *y* from the point (2, 24) into an exponential function to approximate the value of *b*.

$y = ab^x$	Exponential function
$24 = (6)b^2$	Replace *x* with 2, *y* with 24, and *a* with 6.
$4 = b^2$	Divide each side by 6
$2 = b$	Take the square root of each side.

An exponential function whose graph passes through the points (0, 6) and (2, 24) is $y = 6(2)^x$.

Example 4 Simplify Expressions with Irrational Exponents

Simplify each expression.

a. $3^{2\sqrt{5}} \cdot 3^{5\sqrt{5}}$

$3^{2\sqrt{5}} \cdot 3^{5\sqrt{5}} = 3^{2\sqrt{5} + 5\sqrt{5}}$	Product of a Power
$= 3^{7\sqrt{5}}$	Add like radicals.

b. $27^{\sqrt{2}} \div 3^{2\sqrt{2}}$

$27^{\sqrt{2}} \div 3^{2\sqrt{2}} = \left(3^3\right)^{\sqrt{2}} \div 3^{2\sqrt{2}}$	$3^3 = 27$
$= 3^{3\sqrt{2}} \div 3^{2\sqrt{2}}$	Power of a Power
$= 3^{3\sqrt{2} - 2\sqrt{2}}$	Quotient of a Power
$= 3^{\sqrt{2}}$	Subtract like radicals.

Example 5 Solve Exponential Equations

Solve each equation.

a. $\left(\frac{1}{2}\right)^{n-1} = 16$

$\left(\frac{1}{2}\right)^{n-1} = 16$	Original equation
$(2^{-1})^{n-1} = 2^4$	Rewrite $\frac{1}{2}$ as 2^{-1} and 16 as 2^4 so each side as the same base.
$2^{-n+1} = 2^4$	Power of a Power
$-n + 1 = 4$	Property of Equality for Exponential Functions
$-n = 3$	Subtract 1 from each side.
$n = -3$	Divide each side by –1.

The solution is –3.

CHECK

$\left(\frac{1}{2}\right)^{n-1} = 16$	Original equation
$\left(\frac{1}{2}\right)^{-3-1} \stackrel{?}{=} 16$	Substitute –3 for n.
$\left(\frac{1}{2}\right)^{-4} \stackrel{?}{=} 16$	Simplify.
$16 = 16$ ✓	Simplify.

b. $5^{5n+1} = 125^{n-2}$

$5^{5n+1} = 125^{n-2}$	Original equation
$5^{5n+1} = (5^3)^{n-2}$	Rewrite 125 as 5^3 so each side has the same base.
$5^{5n+1} = 5^{3n-6}$	Power of a Power
$5n + 1 = 3n - 6$	Property of Equality for Exponential Functions
$2n + 1 = -6$	Subtract $3n$ from each side.
$2n = -7$	Subtract 1 from each side.
$n = -3.5$	Divide each side by 2.

The solution is –3.5.

Example 6 Solve Exponential Inequalities

Solve $9^{a-5} < 729$.

$9^{a-5} < 729$	Original inequality
$(3^2)^{a-5} < 3^6$	Rewrite each side with a base of 3.
$3^{2a-10} < 3^6$	Power of a Power
$2a - 10 < 6$	Property of Inequality for Exponential Functions
$2a < 16$	Add 10 to each side.
$a < 8$	Divide each side by 2.

The solution set is $a < 8$.

CHECK Test a value of a less than 8; for example, $a = 5$.

$9^{a-5} < 729$	Original inequality
$9^{5-5} \overset{?}{<} 729$	Replace a with 5.
$9^0 \overset{?}{<} 729$	Simplify.
$1 < 729$ ✓	Simplify.