

Lesson 10-2

Example 1 Logarithmic to Exponential Form

Write each equation in exponential form.

a. $\log_{216} 6 = \frac{1}{3}$

$$\log_{216} 6 = \frac{1}{3} \quad \rightarrow \quad 6 = 216^{\frac{1}{3}}$$

b. $\log_5 \frac{1}{3125} = -5$

$$\log_5 \frac{1}{3125} = -5 \quad \rightarrow \quad \frac{1}{3125} = 5^{-5}$$

Example 2 Exponential to Logarithmic Form

Write each equation in logarithmic form.

a. $1331^{-\frac{1}{3}} = \frac{1}{11}$

$$1331^{-\frac{1}{3}} = \frac{1}{11} \quad \rightarrow \quad \log_{1331} \frac{1}{11} = -\frac{1}{3}$$

b. $8^4 = 4096$

$$8^4 = 4096 \quad \rightarrow \quad \log_8 4096 = 4$$

Example 3 Evaluate Logarithmic Expressions

Evaluate $\log_{\frac{1}{4}} \frac{1}{1024}$.

$$\log_{\frac{1}{4}} \frac{1}{1024} = y$$

Let the logarithm equal y .

$$\frac{1}{1024} = \left(\frac{1}{4}\right)^y$$

Definition of logarithm

$$4^{-5} = (4^{-1})^y$$

Rewrite each side with the same base.

$$4^{-5} = 4^{-y}$$

Power of a Power

$$-5 = -y$$

Property of Equality for Exponential Functions

$$5 = y$$

Divide each side by -1 .

$$\text{So, } \log_{\frac{1}{4}} \frac{1}{1024} = 5.$$

Example 4 Inverse Property of Exponents and Logarithms

Evaluate each expression.

a. $\log_3 3^{\frac{1}{2}}$

$$\log_3 3^{\frac{1}{2}} = \frac{1}{2}$$

$$\log_b b^x = x$$

b. $9^{\log_9(a+1)}$

$$9^{\log_9(a+1)} = a+1$$

$$b^{\log_b x} = x$$

Example 5 Solve a Logarithmic Equation

Solve $\log_{\frac{1}{8}} a = \frac{2}{3}$.

$$\log_{\frac{1}{8}} a = \frac{2}{3} \quad \text{Original equation}$$

$$a = \left(\frac{1}{8}\right)^{\frac{2}{3}} \quad \text{Definition of logarithm}$$

$$a = \left[\left(\frac{1}{2}\right)^3\right]^{\frac{2}{3}} \quad \frac{1}{8} = \left(\frac{1}{2}\right)^3$$

$$a = \left(\frac{1}{2}\right)^2 \quad \text{Power of a Power}$$

$$a = \frac{1}{4} \quad \text{Simplify.}$$

Example 6 Solve a Logarithmic InequalitySolve $\log_7 x > 3$. Check your solution.

$$\begin{array}{ll} \log_7 x > 3 & \text{Original inequality} \\ x > 7^3 & \text{Logarithmic to exponential inequality.} \\ x > 343 & \text{Simplify.} \end{array}$$

The solution set is $\{x \mid x > 343\}$.**CHECK** Try 2401 to see if it satisfies the inequality.

$$\begin{array}{ll} \log_7 x > 3 & \text{Original inequality.} \\ \log_7 (2401) > 3 & \text{Substitute 2401 for } x. \\ 4 > 3 \quad \checkmark & \log_7 2401 = 4 \text{ because } 7^4 = 2401. \end{array}$$

Example 7 Solve Equations with Logarithms on Each SideSolve $\log_6 (a^2 - 15) = \log_6 (2a)$. Check your solution.

$$\begin{array}{ll} \log_6 (a^2 - 15) = \log_6 (2a) & \text{Original equation} \\ a^2 - 15 = 2a & \text{Property of Equality for Logarithmic Functions} \\ a^2 - 2a - 15 = 0 & \text{Subtract } 2a \text{ from each side.} \\ (a - 5)(a + 3) = 0 & \text{Factor.} \\ a - 5 = 0 \quad \text{or} \quad a + 3 = 0 & \text{Zero Product Property} \\ a = 5 \quad \text{or} \quad a = -3 & \text{Solve each equation.} \end{array}$$

CHECK Substitute each value into the original equation.

$$\begin{array}{ll} \log_6 (5^2 - 15) = \log_6 [2(5)] & \text{Substitute 5 for } a. \\ \log_6 10 = \log_6 10 \quad \checkmark & \text{Simplify.} \end{array}$$

$$\log_6 [(-3)^2 - 15] = \log_6 [2(-3)] \quad \text{Substitute } -3 \text{ for } a.$$

Since $\log_6(-6)$ is undefined, -3 is an *extraneous* solution and must be eliminated. Thus, the solution is 5.

Example 8 Solve Inequalities with Logarithms on Each Side

Solve $\log_3(5x + 7) < \log_3(3x + 13)$. Check your solution.

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|------------------------------------|--|
| $\log_3(5x + 7) < \log_3(3x + 13)$ | Original inequality |
| $5x + 7 < 3x + 13$ | Property of Inequality for Logarithmic Functions |
| $2x + 7 < 13$ | Subtract $3x$ from each side. |
| $2x < 6$ | Subtract 7 from each side. |
| $x < 3$ | Divide each side by 2. |

We must exclude all values of x such that $5x + 7 \leq 0$ or $3x + 13 \leq 0$. Thus, the solution set is $x > -\frac{7}{5}$, $x > -\frac{13}{3}$, and $x < 3$. This simplifies to $-\frac{7}{5} < x < 3$.

CHECK Test $x = 2$ in the original inequality.

| | |
|--|--|
| $\log_3 [5(2) + 7] < \log_3 [3(2) + 13]$ | Substitute 2 for x . |
| $\log_3 17 < \log_3 19$ | Simplify. |
| $17 < 19 \checkmark$ | Property of Inequality for Logarithmic Functions |

Test other values to verify the reasonableness of your solution.