Lesson 10–3

Example 1 Use the Product Property

Use $\log_3 2 \approx 0.6310$ to approximate the value of $\log_3 54$.

$\log_3 54 = \log_3 (3^3 \cdot 2)$	Replace 54 with $27 \cdot 2$ or $3^3 \cdot 2$.
$= \log_3 3^3 + \log_3 2$	Product Property
$= 3 + \log_3 2$	Inverse Property of Exponents and Logarithms
$= 3 + 0.6310$ or 3.6310	Replace $\log_3 2$ with 0.6310.

Thus, $\log_3 54$ is approximately 3.6310.

Example 2 Use the Quotient Property

Use $\log_5 2 \approx 0.4307$ and $\log_5 7 \approx 1.2091$ to approximate $\log_5 3.5$.

$\log_5 3.5 = \log_5 \frac{7}{2}$	Replace 3.5 with the quotient $\frac{7}{2}$.
$= \log_5 7 - \log_5 2$	Quotient Property
$\approx 1.2091 - 0.4307$ or 0.7784	Replace $\log_5 7$ with 1.2091 and $\log_5 2$ with 0.4307.

Thus, $\log_5 3.5$ is approximately 0.7784.

CHECK Using the definition of logarithm and a calculator, $5^{0.7784} \approx 3.5$. ✓

Example 3 Use Properties of Logarithms

SOUND The loudness L of a sound in decibels is given by $L = 10 \log_{10} R$, where R is the sound's relative intensity. A rocket engine has a relative intensity of 10^{18} or 180 decibels. A jet taking off has a relative intensity of 10^{15} or 150 decibels. How does the intensity of 100 jets taking off at once compare to the intensity of the rocket engine?

Let L_1 be the loudness of one jet taking off. → $L_1 = 10 \log_{10} 10^{15}$

Let L_2 be the loudness of 100 jets taking off. → $L_1 = 10 \log_{10} (100 \cdot 10^{15})$

Then the increase in loudness is $L_2 - L_1$.

$L_2 - L_1 = 10 \log_{10} (100 \cdot 10^{15}) - 10 \log_{10} 10^{15}$	Substitute for L_1 and L_2.
$= 10 (\log_{10} 10^2 + \log_{10} 10^{15}) - 10 \log_{10} 10^{15}$	Product Property
$= 10 \log_{10} 10^2 + 10 \log_{10} 10^{15} - 10 \log_{10} 10^{15}$	Distributive Property
$= 10 \log_{10} 10^2$	Subtract.
$= 10 (2)$	Inverse Property of Exponents and Logarithms
$= 20$	Multiply

The sound of 100 jets taking off is perceived by the human ear to be 20 decibels louder than one jet taking off or 170 decibels. The rocket engine at 180 decibels is 10 decibels louder than the 100 jets taking off.

Example 4 Power Property of Logarithms

Given $\log_3 7 \approx 1.7712$, approximate the value of $\log_3 49$.

$\log_3 49 = \log_3 (7^2)$	Replace 49 with 7^2.
$= 2 \log_3 7$	Power Property
$\approx 2(1.7712)$ or 3.5424	Replace $\log_3 7$ with 1.7712.

Thus, $\log_3 49$ is approximately 3.5424.

Example 5 Solve Equations Using Properties of Logarithms

Solve each equation.

a. $2 \log_2 x + \log_2 3 = \log_2 27$

$2 \log_2 x + \log_2 3 = \log_2 27$	Original equation
$\log_2 x^2 + \log_2 3 = \log_2 27$	Power Property
$\log_2 (x^2 \cdot 3) = \log_2 27$	Product Property
$3x^2 = 27$	Property of Equality for Logarithmic Functions
$x^2 = 9$	Divide each side by 3.
$x = \pm 3$	Take the square root of each side.

CHECK Substitute each value into the original equation.

$2 \log_2 x + \log_2 3 = \log_2 27$	$2 \log_2 x + \log_2 3 = \log_2 27$
$2 \log_2 (3) + \log_2 3 \stackrel{?}{=} \log_2 27$	$2 \log_2 (-3) + \log_2 3 = \log_2 27$
$\log_2 (3^2) + \log_2 3 \stackrel{?}{=} \log_2 27$	Since $\log_2 (-3)$ is undefined, -3
$\log_2 (3^2 \cdot 3) \stackrel{?}{=} \log_2 27$	is an extraneous solution and must
$\log_2 27 = \log_2 27$	be eliminated.

The only solution is 3.

b. $\log_7 (2n + 3) - \log_7 (n - 1) = 1$

$\log_7 (2n + 3) - \log_7 (n - 1) = 1$	Original equation
$\log_7 \dfrac{2n+3}{n-1} = 1$	Quotient Property
$\dfrac{2n+3}{n-1} = 7^1$	Definition of logarithm
$(n-1)\left(\dfrac{2n+3}{n-1}\right) = (n-1)(7)$	Multiply each side by $n - 1$.
$2n + 3 = 7n - 7$	Simplify.
$3 = 5n - 7$	Subtract $2n$ from each side.
$10 = 5n$	Add 7 to each side.
$2 = n$	Divide each side by 5.

The solution is 2.

CHECK Substitute 2 into the original equation.

$\log_7 (2n + 3) - \log_7 (n - 1) = 1$	Original equation
$\log_7 [2(2) + 3] - \log_7 (2 - 1) \stackrel{?}{=} 1$	Substitute 2 for *n*.
$\log_7 (7) - \log_7 (1) \stackrel{?}{=} 1$	Simplify.
$1 - 0 \stackrel{?}{=} 1$	$\log_7 (7) = 1$, $\log_7 (1) = 0$
$1 = 1$ ✓	Simplify.