

Lesson 11–3

Example 1 Find the Next Term

Multiple-Choice Test Item

Find the missing term in the geometric sequence: $-64, 16, -4, 1, \underline{\hspace{1cm}}$.

- A. -0.25 B. 0.25 C. 4 D. -4

Read the Test Item

Since $\frac{16}{-64} = -0.25$, $\frac{-4}{16} = -0.25$, and $\frac{1}{-4} = -0.25$, the sequence has a common ratio of -0.25 .

Solve the Test Item

To find the missing term, multiply the last given term by -0.25 .

$$1(-0.25) = -0.25$$

The answer is A.

Example 2 Find a Particular Term

Find the sixth term of a geometric sequence for which $a_1 = -10$ and $r = -1.5$.

$$\begin{aligned} a_n &= a_1 \cdot r^{n-1} && \text{Formula for } n\text{th term} \\ a_6 &= (-10) \cdot (-1.5)^{6-1} && n = 6, a_1 = -10, r = -1.5 \\ a_6 &= (-10) \cdot (-7.59375) && (-1.5)^5 = -7.59375 \\ a_6 &= 75.9375 \end{aligned}$$

The sixth term is 75.9375.

Example 3 Write an Equation for the n th Term

Write an equation for the n th term of the geometric sequence $-45, 15, -5, -\frac{5}{3}, \dots$.

In this sequence, $a_1 = -45$ and $r = \frac{15}{-45}$ or $-\frac{1}{3}$. Use the n th term formula to write an equation.

$$\begin{aligned} a_n &= a_1 \cdot r^{n-1} && \text{Formula for } n\text{th term} \\ a_n &= -45 \cdot \left(-\frac{1}{3}\right)^{n-1} && a_1 = -45, r = -\frac{1}{3} \end{aligned}$$

An equation is $a_n = -45 \cdot \left(-\frac{1}{3}\right)^{n-1}$.

Example 4 Find a Term Given the Fourth Term and the Ratio

Find the fifteenth term of a geometric sequence for which $a_4 = 25.5$ and $r = -2$.

First, find the value of a_1 .

$$\begin{aligned} a_n &= a_1 \cdot r^{n-1} && \text{Formula for } n\text{th term} \\ a_4 &= a_1 \cdot (-2)^{4-1} && n = 15, r = -2 \\ 25.5 &= -8a_1 && a_4 = 25.5 \\ -3.1875 &= a_1 && \text{Divide each side by } -8. \end{aligned}$$

Now find a_{15} .

$$\begin{aligned} a_n &= a_1 \cdot r^{n-1} && \text{Formula for } n\text{th term} \\ a_{15} &= -3.1875 \cdot (-2)^{15-1} && n = 15, a_1 = -3.1875, r = -2, \\ a_{15} &= -52,224 \end{aligned}$$

The fifteenth term is $-52,224$.

Example 5 Find Geometric Means

Find four geometric means between 15,625 and 5.

Use the n th term formula to find the value of r . In the sequence 15,625, $\underline{\quad ? \quad}$, $\underline{\quad ? \quad}$, $\underline{\quad ? \quad}$, $\underline{\quad ? \quad}$, 5, a_1 is 15,625 and a_6 is 5.

$$\begin{aligned} a_n &= a_1 \cdot r^{n-1} && \text{Formula for } n\text{th term} \\ a_6 &= 15,625 \cdot r^{6-1} && n = 6, a_1 = 15,625 \\ 5 &= 15,625r^5 && a_6 = 5 \\ 0.00032 &= r^5 && \text{Divide each side by } 15,625. \\ 0.2 &= r && \text{Take the fifth root of each side.} \end{aligned}$$

Since 5 is an odd root, there is only one solution to $0.00032 = r^5$, which is positive 0.2. Use $r = 0.2$ to find four geometric means.

$$\begin{aligned} r &= 0.2 \\ a_2 &= 15,625(0.2) \text{ or } 3125 \\ a_3 &= 3125(0.2) \text{ or } 625 \\ a_4 &= 625(0.2) \text{ or } 125 \\ a_5 &= 125(0.2) \text{ or } 25 \end{aligned}$$

The geometric means are 3125, 625, 125, and 25.